Labor Market Policy in the Presence of a Participation Externality

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April 2021

Abstract

A participation externality emerges when vacancy creation depends on workforce composition. As marginal workers enter the labor market, they lower average quality which suppresses vacancy creation. They do not internalize this impact. This paper studies how this externality interacts with search externalities and the efficacy of policies at addressing it. These externalities interact because either party may retain an inefficient share of the surplus and workforce composition affects the expected surplus. We show that when chosen optimally, minimum wages and unemployment insurance partially address both externalities, but minimum wages primarily affect participation, while unemployment insurance primarily affects search externalities.

JEL Codes: E24, J64, E64
Keywords: Minimum wage; unemployment insurance; search and matching; labor market participation; optimal policy

1 Introduction

This paper investigates a participation externality that arises whenever firms create vacancies based on the composition of the workforce but new-entrant workers do not internalize the impact of their participation on that composition. We study this externality in a frictional environment in which

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workers differ in their ability levels. With this environment, we explore how this externality interacts with the previously understood search externalities and then assess the efficacy of labor market policies that could be used to address it.

We construct our model to mirror key features of the environment faced by low-wage workers. These workers differ by ability, while jobs are ex ante homogeneous, which reflects evidence that better workers have higher wages in similar jobs (Syverson (2011), Song et al. (2019)). As a result, workers will determine their participation based on their ability which may not meet the minimum threshold determined by extant labor market policies or other features of their environment.

As described in Pissarides (2000), in random search models there are two search (or congestion) externalities that reflect the extent to which the participants (here workers and firms) are under- or over-rewarded for their contribution to aggregate matching. Essentially, neither set of participants internalize the impact they have on their cohorts' search effectiveness by their own presence in the market. In an environment without a participation externality, Hosios (1990) shows that models are constrained efficient when the bargaining power of the firms is equal to the elasticity of the matching function with respect to vacancies. In our environment, this "Hosios Rule" causes the participation and search externalities to be orthogonal. In that case, as shown in Masters (2015), a participation threshold set to the level chosen by the social Planner will restore constrained efficiency. However, understanding the interaction between these externalities away from the Hosios Rule is an empirically important, but analytically challenging task. For this reason, we provide two versions of the model: a static model to provide insight and a dynamic model for quantitative analysis.

We use a static model to explore the relationship between the participation externality, the search externalities, and labor market policies. There, is it is straightforward to demonstrate the orthogonality of the participation and search externalities under the Hosios Rule. Away from the Hosios Rule, the externalities tend to counteract each other. If firm bargaining power is too low, the search externalities, amplify the impact of participation externality (and vice-versa). But, the participation externality itself gets weaker. It turns out that the sign of the impact of changes to the participation threshold on welfare depends only on the elasticity of matching function and not on bargaining power of the firm per se. This quasi-neutrality result emerges because lower firm bargaining power reduces the firms' ability to recoup their sunk cost of job posting but also raises their equilibrium matching probability.

We also consider potential policy interventions in the one-shot model. As supported by the analysis, the targeting principle points to an intervention that simply raises the participation threshold. However, it is typically assumed in such environments that the government cannot identify individual ability levels (or is precluded from discriminating on that basis). We look at two ubiquitous policies: unemployment insurance (UI) and the minimum wage. In this framework, both policies raise the participation threshold and can, therefore, be expected to address the participation externality. UI essentially pays people to stay out of the market while a binding minimum wage causes low-ability workers to drop out of the market. But in doing so, both create additional distortions. The minimum wage raises wages on the lowest ability groups that remain in the market which tends to suppress vacancy creation and therefore reduces the welfare gains accrued from excluding the lowest ability workers. Still, where it just binds, welfare is increasing in the minimum wage regardless of worker bargaining power. A UI scheme set to achieve the same participation threshold, raises wages by even more than does the minimum wage. As such it is better at addressing the excess vacancy creation associated with low worker bargaining power relative to the Hosios Rule. But, if it is the firms who have low bargaining power, the minimum wage will be doing the least additional harm as measured by welfare. Indeed, for low enough firm bargaining power, UI may not be welfare enhancing at any level.

After using the static model for insight, we introduce a dynamic version of the model. It endogenizes the workers' continuation values which increases the responsiveness of the match surplus to changes in bargaining power. This strengthens the search externalities in relation to the participation externality. Still, we show that the analytical predictions of the dynamic model are qualitatively similar to those of the static model. We use this model to provide quantitative analysis about the externalities, and labor market policies.

With the dynamic model, we answer 3 key questions: i) what is the constrained efficient allocation? ii) what are the optimal minimum wage and UI policies? iii) how do these policies affect the participation and search externalities, and how do the externalities interact?

We estimate our dynamic model on US data between 2012 and 2014, and provide targets to discipline parameters of the model, including bargaining power. We find that worker bargaining power is low relative to the Hosios Rule – the orthogonality result does not apply and, the participation and search externalities interact. As expected, the Planner's ability cut-off (at \$7.04) is higher than the cut-off in the laissez-faire economy (estimated at

$6.04) and achieves a welfare gain of 1.54pp. The minimum wage and UI both achieve higher levels of welfare than the laissez-faire economy, but do not achieve the same level as the Planner. Although the minimum wage can exclude low ability workers and raise wages for some workers who receive less than the "Hosios wage" it also overly inflates wages for many of the marginal entrants. In our quantitative analysis, the optimal minimum wage ($7.15) raises welfare by 0.32pp above the laissez-faire economy. By contrast, UI does not distort wages for marginal entrants, but raises wages for all workers. Our analysis indicates that the optimal UI (an additional $1.53 to a participation threshold of $7.57) raises welfare by 0.86pp.

These policies achieve these welfare gains by reducing both externalities. To understand the channels through which each policy operates, we restrict these policies so that they operate as participation thresholds, without affecting wages. Under this restriction, these policies reduce the participation externality, but do not affect the search externalities. Using this decomposition, we find that the minimum wage operates almost entirely by reducing the participation externality (0.31pp of the 0.32pp welfare gain), while UI is effective along both margins (0.25pp of the 0.86pp welfare gain is achieved by reducing the participation externality).

The remainder of the paper is laid out as follows. Section 2 places this paper within the recent literature on externalities and labor market policy in the Diamond-Mortensen-Pissarides (DMP) framework. Section 3 describes the static model. Section 4 develops the dynamic model. Section 5 discusses the data and the calibration of the dynamic model. The results and decomposition appear in Section 6. Section 7 concludes.

2 Literature

There are 4 other papers in which the participation externality studies here emerges: Gavrel (2011), Masters (2015), Julien and Mangin (2017a) and Mangin and Julien (2021). Both Julien and Mangin (2017a) and Mangin and Julien (2021) generalize the Hosios Rule for an environment with a participation decision and an opportunity cost of search. They show that constrained efficiency equates the bargaining power of the firm to the sum of the matching elasticity, a surplus elasticity and a participation elasticity. To focus on the participation externality, Masters (2015) eliminates the opportunity cost of search and shows that constrained efficiency can also be achieved through implementation of the standard Hosios Rule combined with a participation threshold that excludes the same low ability workers that the Social Planner would exclude. Gavrel (2011) provides a static model and assumes that the terms of trade are determined by a fixed division of output rather than the more usual Nash bargaining. Consequently, a transfer scheme, akin to unemployment insurance (UI) studied here, is able to raise the ability threshold for participation without affecting wages. It therefore, implements constrained efficiency under the Hosios Rule. Beyond this, none of these papers looks into how labor market policy performs in addressing the participation externality. And, none of them look into the interactions between the participation and search externalities.

Hungerbühler and Lehmann (2009) show that the minimum wage can emerge as part of an optimal tax and transfer scheme in a DMP environment when worker bargaining power is low. Their model has ex ante heterogeneous workers and a participation decision but abstracts from the participation externality explored here by allowing for complete market segmentation by ability level.

Several papers consider the minimum wage in a frictional model. Braun (2019) investigates the relationship between the minimum wage and people's propensity to commit property crime. She provides a model in which workers are ex ante heterogeneous in ability. There is no vacancy creation in her model and so there is no participation externality. Instead, an externality emerges from the wage setting protocol. Firms do not fully internalize their choice of wage offer on the worker's propensity to commit crime, which is different than our participation externality. Flinn (2006) considers an environment with vacancy creation, but no participation margin. Lavecchia (2020) and Lee and Saez (2012) both consider a participation decision in models where workers are heterogeneous with respect to skills. The former does not explore the effect of a participation externality, though one could arise in his environment. The latter considers a perfectly competitive environment, and hence will not feature the externalities in our model or their interactions.

The large literature on optimal UI is generally focused on the trade-off between providing consumption smoothing to risk averse workers and moral hazard emerging from the government's inability to observe job search effort. This trade-off is considered either in the partial equilibrium principle-agent framework (Shavell and Weiss (1979), Hopenhayn and Nicolini (1997)) or in a decentralized market context (Fredriksson and Holmlund (2001), Coles and Masters (2006), Acemoglu and Shimer (1999)). None of these look into the role of UI in addressing the participation externality considered in this paper.

3 Static Model

Throughout this section longer derivations have been moved to Appendix A.

3.1 Environment

The economy exists for one period. It is populated by a unit mass continuum of workers indexed by their ability level, $p \sim F(.)$ with continuous density, $f(.)$ on $[0, \bar{p}]$. There is also a large mass of firms that can each create one vacancy at a cost a. They determine the level of vacancy creation according to a free-entry condition. Workers and firms are both risk neutral. Workers receive value from leisure, z, whether they look for work or not. To ensure that gains from trade exist we assume that $\bar{p} > z + a$. A firm matched to a worker of ability p produces p units of output.

As firms are unable to observe a worker's ability until the moment that they meet, search is random. Workers are matched to a vacancy with probability $m(\theta)$ where, θ represents "labor market tightness" defined as the ratio of the mass of vacancies, v, to the mass of job-seeking workers, u. The matching function, $m(.)$, is strictly increasing and strictly concave with $m(0) = 0$, $m'(0) = 1$ and $\lim_{\theta\to\infty} m(\theta) = 1$. Then, a vacancy will meet a job-seeker with probability $m(\theta)/\theta$ which is assumed to be strictly decreasing in θ. This means that $\eta(\theta)$, the elasticity of $m(\theta)$, is less than one.

3.2 Efficiency

To make the Planner's problem comparable with that of the policy maker in the decentralized economy, the Planner will be constrained by both the matching frictions and the informational asymmetry.[1] The Planner will be able to control the mass of vacancies, v, and a threshold productivity level, $\hat{p}$, required for participation in the matching game. Thus, the mass of of participants, $u = 1 - F(\hat{p})$ and the market tightness, $\theta = v/[1 - F(\hat{p})]$. The trade-off the Planner faces is that raising $\hat{p}$ improves match quality but reduces their quantity.

The social welfare function is

$$W(\theta, \hat{p}) = zF(\hat{p}) + \int_{\hat{p}}^{\bar{p}} [m(\theta)p + (1 - m(\theta))z] \, dF(p) - [1 - F(\hat{p})]a\theta. \quad (3.1)$$

[1] In Appendix C we solve the model in which the Planner is subject to matching frictions alone. We also consider how that allocation might be supported in a decentralized economy.

The Planner's problem is to maximize W over θ and $\hat{p}$. The necessary conditions for an optimum imply

$$m'(\theta_p)\mathbb{E}_{[p \geq \hat{p}_p]}(p - z) = a \tag{3.2}$$

and

$$\frac{m(\theta_p)}{\theta_p}(\hat{p}_p - z) = a \tag{3.3}$$

where the subscript, p, refers to the Planner's solution. That such an interior solution exists follows from the assumption that $\bar{p} > z + a$. Equation 3.2 equates the marginal benefit from creating an additional vacancy to its marginal cost. Equation 3.3 equates the marginal cost in terms of output of raising $\hat{p}$ to the marginal benefit in terms of the saved vacancy costs.

3.3 Market Economy

In the market economy, wages are determined by generalized Nash bargaining in which the bargaining power of the worker is $\beta \in [0, 1]$. As is standard in the literature using the DMP framework, the threat point for each of the participants is assumed to be their outside option. For the workers that is z and for the firms it is 0. We assume that workers who are indifferent between market entry and sitting it out will choose the latter.

In the laissez-faire economy, firms will create vacancies up to the point where doing so allows them to break-even. The firm's vacancy creation cost is sunk but, if the meeting does not consummate to a match, the worker still receives z. Consequently, the surplus from a match with a type p worker is $p - z$. This implies that for workers with $p \leq z$ there will be no gains from trade with any employer and as a result will not participate in the labor market. For anyone else, since they get $z + \beta(p - z)$ if they match and z if they do not, participation is a strictly dominant strategy. Letting $(\theta^*, \hat{p}^*)$ represent the market economy equilibrium outcome, we have

$$\frac{m(\theta^*)}{\theta^*}(1 - \beta)\mathbb{E}_{[p \geq \hat{p}^*]}(p - z) = a \tag{3.4}$$

and

$$\hat{p}^* = z. \tag{3.5}$$

Comparing Equation 3.2 and Equation 3.4 for a given threshold of ability for participation, $\hat{p}$, the Hosios Rule (i.e. $\beta = 1 - \eta$) implies efficient vacancy creation. But, this does not resolve the participation externality. Comparing Equation 3.3 and Equation 3.5 clearly shows that the participation threshold in the laissez-faire economy is too low. As a result, even at the Hosios Rule, Equation 3.2 implies that the market tightness is also too low. In the market economy, firms cannot pre-commit to reject low ability workers with $p > z$. The prospect of hiring those workers lowers expected profits and reduces vacancy creation below the optimal level.

3.4 Participation externality

The inability of the Hosios Rule to bring about full constrained efficiency here is a consequence of a participation externality. In choosing whether or not to participate, a worker does not take into account the impact of that choice on the average quality of the unemployment pool. But, vacancy creation depends precisely on that average quality – the externality directly impacts firms and thereby indirectly impacts the other workers.

How this externality interacts with the search externalities, which have been more extensively studied (Pissarides, 2000), is of interest here. Search externalities arise on both sides of the market whenever an individual's private return from their search behavior does not accurately compensate them for their contribution to welfare. Absent a participation externality (e.g. when $F(.)$ is degenerate), the Hosios surplus sharing rule implies that social contributions and private returns on both sides of the market are aligned.[2] Alternatively, in a directed search equilibrium (e.g. Moen (1997)) participants take the terms of trade as given and the search externalities are made pecuniary.

We should point out that the focus on the participation externality has also guided our choice of model structure. In particular we have not incorporated any opportunity cost of search. A consequence is that our laissez-faire ability cut-off, $\hat{p}^*$, does not depend on the market tightness. By comparison, Julien and Mangin (2017b) introduces an explicit search cost which introduces an "output" externality that has its own interactions with the participation and search externalities. By assuming that unsuccessful job seekers and those that sit out of the market both get z, we are deliberately abstracting from the output externality. Doing so means that at the Hosios Rule the participation and search externalities are orthogonal.

To further reveal the role of the participation externality, we will consider how artificially raising the ability cut-off, $\hat{p}$, above z in the market equilibrium impacts welfare. The welfare measure is the same as in Equation 3.1

[2] This happens because firms create vacancies whenever the average expected return exceeds the cost, a. The Planner, meanwhile creates vacancies based on the marginal expected social return. The Hosios Rule equates those expected returns.

while for given $\hat{p}$ the equilibrium market tightness, θ^*, is given by

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) = a. \tag{3.6}$$

Then,

$$\frac{dW}{d\hat{p}} = \frac{\partial W}{\partial \hat{p}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\hat{p}}. \tag{3.7}$$

The first term, $\frac{\partial W}{\partial \hat{p}}$, captures the participation externality in isolation while the second term captures its interaction with the search externalities. From Equation 3.1,

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})\left[a\theta^* - m(\theta^*)(\hat{p}-z)\right]. \tag{3.8}$$

Using Equation 3.6 this can be written as

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - (\hat{p}-z)\right]. \tag{3.9}$$

It is clearly positive for all values of β when $\hat{p} = z$ but as $\hat{p}$ rises towards $\bar{p}$ it becomes negative. An alternative way to view the participation externality is that it emerges because of a hold-up problem. In the market economy, firms cannot pre-contract with their future employees so their incurred vacancy costs are sunk. The Planner, on the other hand, fully internalizes those costs. On that basis, raising $\hat{p}$ above z without affecting vacancy creation precludes firms from matching with those workers whose ability does not justify the vacancy creation cost. Of course, raising $\hat{p}$ too high can can begin to preclude them from matching with higher ability workers that the Planner would have the firms hire.

The term that captures the interaction of the participation and search externalities has two components. The first is the direct effect of changes in the market tightness on welfare,

$$\begin{aligned}\frac{\partial W}{\partial \theta} &= m'(\theta^*)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p) - \left[1-F(\hat{p})\right]a \\ &= \frac{m'(\theta^*)\left[1-F(\hat{p})\right]a\theta^*}{m(\theta^*)(1-\beta)} - \left[1-F(\hat{p})\right]a \\ &= \frac{\left[1-F(\hat{p})\right]a}{(1-\beta)}\left[\beta-(1-\eta(\theta^*))\right]. \end{aligned} \tag{3.10}$$

This term is clearly positive for values of β above the Hosios Rule and negative below it. This evidences the orthogonality of the participation and search externalities at the Hosios Rule. The second component in the interaction term is,

$$\frac{d\theta^*}{d\hat{p}} = \frac{(1-\beta)m(\theta^*)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{[1-F(\hat{p})]\,a(1-\eta(\theta^*))} > 0. \tag{3.11}$$

As the policy maker excludes more and more of the lower ability workers, the average ability of those that remain in the labor force increases. Free entry of vacancies means that the market tightness in the remaining market will also increase.

The upshot is that the search externalities exacerbate (resp. ameliorate) the participation externality if worker bargaining power, β, is too high (resp. low) relative to the Hosios Rule value. As stated above, that firms cannot pre-commit to reject low ability entrants means that market tightness is too low even at the Hosios Rule. Increasing β above the Hosios Rule value further reduces vacancy creation and puts more pressure on a government to somehow raise the participation threshold. However, as we see in Equation 3.9 increasing β reduces the direct value of raising $\hat{p}$.

Bringing all of the above together we obtain

$$\frac{dW}{d\hat{p}} = \left(\frac{\beta f(\hat{p})m(\theta^*)}{1-\eta(\theta^*)}\right)\left[\eta(\theta^*)\mathbb{E}_{[p\geq\hat{p}]}(p-z)-(\hat{p}-z)\right] \tag{3.12}$$

which obviously collapses to Equation 3.9 at the Hosios Rule. Equation 3.12 shows that changes in β do not directly affect the sign of the impact of $\hat{p}$ on welfare. The direct and indirect effects exactly cancel out. Raising β reduces the firms' ability to recoup their sunk cost, a, but also raises their equilibrium matching probability. Ultimately, what matters for the sign of the effect is the elasticity of the matching function. It is immediate from Equation 3.12 that when $\hat{p} = z$, welfare is increasing in $\hat{p}$ for all (interior) values of β – a direct intervention to raise the participation threshold for workers is, on aggregate, always beneficial. In Appendix A we show that an optimal value of $\hat{p}$ exists and solves

$$\eta(\theta^*)\mathbb{E}_{[p\geq\hat{p}]}(p-z) = (\hat{p}-z).$$

At the Hosios Rule this implements the Planner's solution.

3.5 Policy initiatives

3.5.1 Minimum Wages

An underlying assumption for the market economy is that the government cannot directly observe worker ability and, therefore, cannot directly manipulate $\hat{p}$ as analyzed above. As wages are observable to the government they can, however, be used as proxy for ability. That is, the policy maker could attempt to address the participation externality with a minimum wage. A further possible justification for a minimum wage follows from the search externalities. It is well known in the search and matching literature that if homogenous workers have low bargaining power (i.e. $\beta < 1-\eta$) a minimum wage set to the "Hosios wage", that would emerge under the Hosios Rule, will implement constrained efficiency.[3]

As a minimum wage set below z has no impact, we will only consider $\bar{w} \geq z$. There are no gains from trade between a worker with $p \leq \bar{w}$ and any firm – the worker leaves the workforce. When a minimum wage is below the worker's ability but higher than the bargained wage, $\beta p + (1-\beta)z$, pairwise Pareto optimality implies that the worker will be hired at the minimum wage. If the minimum wage lies below $\beta p + (1-\beta)z$, Nash's Independence of Irrelevant Alternatives axiom implies that the worker gets hired at the bargained wage as if there were no floor (Muthoo (1999)). So, contingent on getting hired, a type p worker gets paid

$$\max\left\{\bar{w}, \beta p + (1-\beta)z\right\}.$$

Let $\tilde{p}$ be the highest ability level that receives the minimum wage so that $\bar{w} = \beta\tilde{p} + (1-\beta)z$. Then,

$$\tilde{p} = \frac{\bar{w} - (1-\beta)z}{\beta}. \tag{3.13}$$

Whenever $\bar{w} > z$, the market equilibrium is characterized by

$$\hat{p}^* = \bar{w}$$

and

$$\frac{m(\theta^*)}{\theta^*}\left\{\frac{F(\tilde{p}) - F(\bar{w})}{1-F(\bar{w})}\mathbb{E}_{p\in[\bar{w},\tilde{p}]}(p-\bar{w}) + \frac{1-F(\tilde{p})}{1-F(\bar{w})}(1-\beta)\mathbb{E}_{[p\geq\tilde{p}]}(p-z)\right\} = a.$$

These imply that the participation threshold is effectively exogenous and θ^* is characterized by,

$$\frac{m(\theta^*)}{\theta^*\left(1-F(\bar{w})\right)}\left\{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right\} = a. \tag{3.14}$$

[3] Here too, it is straightforward to show that a minimum wage made contingent on worker ability, $\bar{w}(p)$, will implement constrained efficiency if $\beta < 1-\eta(\theta_p)$ and

$$\bar{w}(p) = \begin{cases} \hat{p}_p \text{ for } p \leq \hat{p}_p \\ b + (1-\eta(\theta_p))(p-b) \text{ for } p > \hat{p}_p \end{cases}$$

Meanwhile, the welfare measure is the same as in Equation 3.1 with $\hat{p}$ replaced by $\bar{w}$. We would like to see how welfare changes as the minimum wage binds and then increases further. We have

$$\frac{dW}{d\bar{w}} = \frac{\partial W}{\partial \bar{w}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\bar{w}} \tag{3.15}$$

where the two terms on the RHS capture the direct and indirect effects respectively. The goal is to assess the extent to which the minimum wage can address the participation externality as compared to direct manipulation of $\hat{p}$.

Mirroring Equation 3.8 we have,

$$\frac{\partial W}{\partial \bar{w}} = f(\bar{w})\left[a\theta^* - m(\theta^*)(\bar{w} - z)\right]. \tag{3.16}$$

The direct impact of the minimum wage captures the extent to which it effectively excludes lower ability workers and is identical to the direct impact of raising $\hat{p}$. As such, for values of $\bar{w}$ close enough to z this will be positive. As $\frac{m(\theta)}{\theta}$ cannot exceed 1, however, high values of $\bar{w}$ can reduce welfare by excluding too many workers.

From Equation 3.10 we have

$$\frac{\partial W}{\partial \theta} = m'(\theta^*)\int_{\bar{w}}^{\bar{p}}(p - z)dF(p) - \left[1 - F(\bar{w})\right]a. \tag{3.17}$$

And, from Equation 3.14,

$$\frac{d\theta^*}{d\bar{w}} = \frac{f(\bar{w})a\theta^* - m(\theta^*)\left[F(\tilde{p}) - F(\bar{w})\right]}{a\left[1 - \eta(\theta^*)\right]\left[1 - F(\bar{w})\right]}. \tag{3.18}$$

This cannot be signed in general. A binding minimum wage mimics $\hat{p}$ in that it increases the average quality of the labor force which puts upward pressure on θ^*. However, it also raises the wages of those with productivities between $\bar{w}$ and $\tilde{p}$ which tends to suppress vacancy creation. The negative term disappears when $\bar{w} = z$.

Combining all of the above by substitution into Equation 3.15 leads to,

$$\begin{aligned}\frac{dW}{d\bar{w}} = \frac{\beta f(\bar{w})m(\theta^*)}{1-\eta(\theta^*)}\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z)-(\bar{w}-z)\right] \\ +\frac{\beta f(\bar{w})m(\theta^*)\eta(\theta^*)}{[1-\eta(\theta^*)]\,[1-F(\bar{w})]}\int_{\bar{w}}^{\tilde{p}}(\tilde{p}-p)dF(p) \\ +\frac{f(\bar{w})m(\theta^*)(\bar{w}-z)}{1-\eta(\theta^*)}\left[\beta-(1-\eta(\theta^*))\right] \\ +\frac{m(\theta^*)\,[F(\tilde{p})-F(\bar{w})]\,[a-m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z)]}{a\,(1-\eta(\theta^*))}. \quad (3.19)\end{aligned}$$

The first term in Equation 3.19 is identical to Equation 3.12. The subsequent terms therefore capture the impact of changing the minimum wage vis-a-vis direct manipulation of $\hat{p}$. When the minimum wage just binds, (i.e. when $\bar{w} = z$) only the first term remains. This tells us that (locally) adjusting $\bar{w}$ and $\hat{p}$ only differ in their second order effects. In particular we see that a just-binding minimum wage is beneficial to welfare for all values of β.

When $\bar{w} > z$, the last three terms of Equation 3.19 matter. The second term is always positive and reflects the fact that workers with abilities in the range $[\bar{w}, \tilde{p}]$ have their wages raised to $\bar{w}$ – they get $\beta(\tilde{p}-p)$ more than when $\hat{p}$ is raised independently. The third term in Equation 3.19 pertains to the marginal worker whose ability is just at $\bar{w}$. As $\bar{w}$ increases that worker moves out of the labor force and loses the income $\bar{w} - z$. Of course, the marginal worker loses income when $\hat{p}$ is raised too but, because $\beta < 1$, not as much.[4] The extent to which this excess loss of income, $(1-\beta)(\bar{w}-z)$, is a good or bad thing for aggregate welfare depends, according to the Hosios Rule, on whether β exceeds $1-\eta(\theta^*)$ or not. The final term of Equation 3.19 comes from the indirect effect of $\bar{w}$ on welfare and is zero whenever $\theta^* = \theta_p$. Because $m(.)$ is strictly concave, this term is negative when $\theta^* < \theta_p$. Compared to simply raising $\hat{p}$, a minimum wage raises some wages and therefore suppresses vacancy creation. So, if market tightness is already too low, the welfare contribution from this term will be negative.

As welfare is increasing at $\bar{w} = z$ and converges to z for high values of $\bar{w}$, similar analysis to that for adjusting $\hat{p}$ implies that there will be an optimal minimum wage larger than z. Because of the wage distortions caused by the minimum wage, even at the Hosios Rule, the optimal minimum wage does

[4] With a binding minimum wage, the marginal worker earns $\bar{w} = \hat{p}$. If the government could directly set $\hat{p} > b$, the marginal worker would earn $w = \beta\hat{p} + (1-\beta)b$.

not implement the Planner's solution. In Appendix C we show that this is possible when combined with a vacancy subsidy.

3.5.2 Unemployment insurance (UI)

Rather than price them out of the market with a minimum wage, the government might attempt to pay low ability workers to sit out of the market (i.e. make a disability payment). As job search behavior is assumed to be unobservable to the government, however, even the high ability workers will sign up for such payments and forego them if they get a job. Such a policy effectively becomes UI in that everyone who fails to match will receive the payment.[5]

The scheme pays b at the end of the period to anyone who did not get a job. To avoid introducing a fiscal externality coming from taxation that is contingent on employment, UI payments will be paid for by a lump-sum tax, τ, on every worker. The payments will be received over and above the value of leisure, z, which does not need to be paid for.[6]

The equilibrium conditions Equation 3.4 and Equation 3.5 become

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p\geq\hat{p}^*]}(p-z-b) = a$$

and

$$\hat{p}^* = z + b. \tag{3.20}$$

Clearly, $\hat{p}^*$ moves one-to-one with b and becomes the de facto policy instrument, $\hat{p}$. Substituting b out of the equilibrium conditions yields the following characterization of equilibrium market tightness, θ^*:

$$m(\theta^*)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})\,dF(p) = a\theta^*(1-F(\hat{p})). \tag{3.21}$$

This differs from Equation 3.6 because, now as b (i.e. $\hat{p}$) increases, it raises wages by $(1-\beta)b$ at every worker ability level and reduces the expected

[5]Of course, unemployment insurance is something of a misnomer here. With risk-neutral workers there is no "insurance" value to the policy. The focus here is on how the payments interact with the participation externality. Introducing risk-aversion would only serve to obscure that interaction. We stick with the insurance moniker because it has historically been used in the context of similar policies.

[6]The value of z that emerges from the data may well contain some employment contingent payments. What matters here is the extent to which increasing those payments can address the participation externality.

return to vacancy creation. Direct manipulation of $\hat{p}$, by comparison, simply excludes low ability workers and does not raise wages.

As the UI payments are transfers and workers are risk-neutral, introducing UI does not change the welfare measure, Equation 3.1. So, to measure the impact on welfare of a change in b we need

$$\frac{dW}{db} = \frac{\partial W}{\partial \hat{p}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\hat{p}}. \tag{3.22}$$

Using Equation 3.21,

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (\hat{p}-z)\right]. \tag{3.23}$$

Comparing this to Equation 3.9 shows that when $b = 0$, the direct impact of raising it is identical to that of simply raising $\hat{p}$ without UI. But for higher values of b, $\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) < \mathbb{E}_{[p\geq\hat{p}]}(p-z)$. By raising wages, the positive impact of raising $\hat{p}$ through UI payments peters out more quickly than from raising it directly. UI, therefore, imperfectly targets the participation externality.

Now,

$$\begin{aligned}\frac{\partial W}{\partial \theta} &= m'(\theta)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p) - (1-F(\hat{p}))a \\ &= m'(\theta)\int_{\hat{p}^*}^{\bar{p}}(p-\hat{p})dF(p) - (1-F(\hat{p}))a + (1-F(\hat{p}))(\hat{p}-z) \\ &= \left(\frac{1-F(\hat{p})}{1-\beta}\right)\left\{\left[\eta-(1-\beta)\right]a + (1-\beta)m'(\theta)(\hat{p}-z)\right\} \end{aligned} \tag{3.24}$$

where the final expression uses Equation 3.21. Again the difference between the impact of raising UI payments and direct manipulation of the participation threshold stems from the way UI payments increase wages. Comparing Equation 3.10 and Equation 3.24, there is no distinction in that impact when $b = 0$. When UI payments are strictly positive, the additional positive term, $(1-\beta)m'(\theta)(\hat{p}-z)$, means that an increase in θ has a more positive (or less negative) impact on welfare than occurs when $\hat{p}$ is simply raised above z. This comes from workers being made better off both by the higher wages.

So, how does UI affect market tightness?

$$\begin{aligned}\frac{d\theta^*}{d\hat{p}} &= \frac{a\theta f(\hat{p}) - m(\theta)(1-\beta)(1-F(\hat{p}))}{m'(\theta)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p) - a(1-F(\hat{p}))} \\ &= \frac{(1-\beta)m(\theta^*)\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (1-F(\hat{p}))\right]}{(1-\eta(\theta^*))a(1-F(\hat{p}))}. \end{aligned} \tag{3.25}$$

This is the same as Equation 3.11 except for the negative $(1-F(\hat{p}))$ in the numerator. This negative pressure on vacancy creation from increases in b, again, stems from the increase in wages at every ability level which depresses vacancy creation. The net impact of an increase in UI payments on labor market tightness can be negative even when $b=0$.[7] The extent to which this is a good or bad thing will depend as usual on the workers' bargaining power.

Substituting the above analysis into Equation 3.22 yields

$$\begin{aligned}\frac{dW}{db} = &\left(\frac{\beta f(\hat{p})m(\theta)}{1-\eta(\theta)}\right)\left[\eta(\theta)\mathbb{E}_{[p\geq\hat{p}]}(p-z)-b\right] \\ &+\left(\frac{m(\theta)}{(1-\eta(\theta))a}\right)\left\{m'(\theta)b(1-\beta)\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-(1-F(\hat{p}))\right]\right. \\ &\qquad -f(\hat{p})(1-\beta)b(1-\eta(\theta))a \\ &\qquad \left. -(1-F(\hat{p})\left[\beta-(1-\eta)\right]a\right\}. \qquad (3.26)\end{aligned}$$

As $b=\hat{p}-z$ the first line is identical to Equation 3.12. The second term, therefore captures the impact of changing the UI payment vis-a-vis direct manipulation of $\hat{p}$. When $b=0$ the contents of the curly braces reduces to the third term only. This is zero at the Hosios Rule and negative (resp. positive) when β is high (resp. low). Increasing UI payments only has a second order effect on welfare only at the Hosios rule. More generally, an increase in UI payments increases worker continuation values, reduces the size of the surplus, raises wages and reduces ex post profits for all levels of ability. This puts downward pressure on vacancy creation. But, when bargaining power is low ($\beta<1-\eta$) laissez-faire vacancy creation is too high so increasing b from zero can be more beneficial for welfare than increasing $\hat{p}$ in isolation.

For strictly positive values of b, the second term in the curly braces in Equation 3.26 is strictly negative. The sign of the first term is generally ambiguous depending on the nature of the distribution of abilities (see Equation 3.5.2). The impact of further increases in UI payments on welfare relative to what increasing $\hat{p}$ does is therefore difficult to characterize in any general way. This will be explored more in the numerical analysis below.

[7] The term $f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-(1-F(\hat{p}))$ is negative if $\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p)$, sometimes called the "surplus function" is log-concave (see Bagnoli and Bergstrom (2005)). If the density of a distribution is log-concave so is the surplus function. Examples of such distributions include uniform and normal. In the calibration of the dynamic version of the model we use a log-normal distribution. The density of that distribution is not log-concave but its cdf is. Its surplus function is considered mixed in that parts can be log-concave and parts are log-convex.

The existence of an optimal value of b is shown in Appendix A. For high enough values of β, welfare may be decreasing in b at $b = 0$ and the optimal value is negative. Under that scenario wages are lower than under laissez-faire which counteracts the effect of the high β and increases vacancy creation. The cost of $b < 0$ is that it introduces some workers with ability below z into the market. The lower is $f(z)$ the lower that cost will be.

Because of the wage distortions caused by the UI payments, in isolation, they cannot implement the Planner's solution even at the Hosios Rule. In Appendix C we show that this is possible when UI is combined with a vacancy subsidy.

3.5.3 Comparing the efficacy of UI and the minimum wage

In the calibrated version of the model we quantify the extent to which each policy addresses the participation externality. Here we compare how they work. Both policies are readily implemented ways to raise the participation threshold, $\hat{p}$, which is too low in laissez-faire. Both, however, have side-effects. Under UI, a type p worker's wage is $\beta p + (1 - \beta)(z + b)$. Increases in b therefore increase wages at every ability level by $(1 - \beta)b$. Meanwhile with a minimum wage, a type p worker's wage is $\max \{\bar{w}, \beta p + (1 - \beta)z\}$.

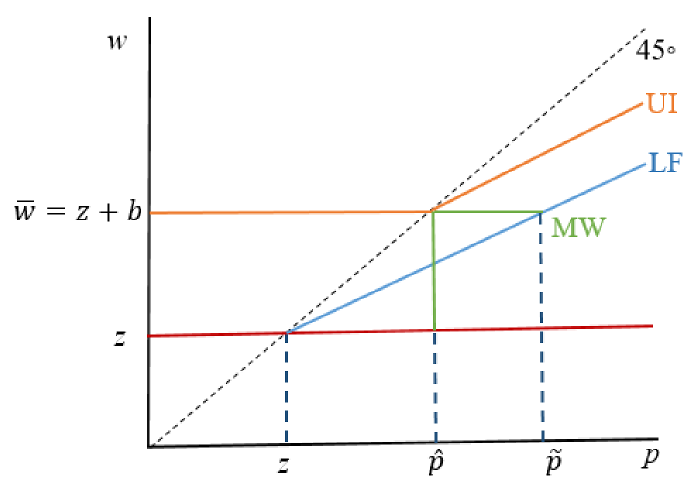

Figure 3.1: Comparing policy effectiveness.

The impact of both policies on worker (before tax) incomes by ability level is depicted in Figure 3.1. In the laissez-faire economy, the participation threshold is z, non-employed workers all receive z and the wages of the employed are represented by the light-blue line marked "LF". For comparability, both policies imply the same participation threshold, $\hat{p}$. With a minium wage, $\bar{w} = \hat{p}$, in place the non-employed receive z. The incomes of the employed with abilities in the range $[\hat{p}, \tilde{p}]$ are represented by the green line marked "MW". Employed workers with abilities above $\tilde{p}$ get the laissez-faire wage. With UI payment, $b = \hat{p} - z$, in place, the non-employed receive $z + b$. The wages of the employed are represented by the upward sloping portion of the orange line marked UI. The upshot from Figure 3.1 is that for the same participation threshold, workers wages are higher for every ability level with UI than with a minimum wage in place. This means that firms face lower expected profits and fewer vacancies will be created under the UI policy.

From this, the minimum wage comes out as the least disruptive way to raise $\hat{p}$ and, therefore, more directly able to target the participation externality. However, the impact on overall efficiency will depend on how each policy also interacts with the search externalities. By raising wages by more than the minimum wage at every ability level, UI will be better at addressing the excess vacancy creation associated with low worker bargaining power. But, if $\beta > 1-\eta$, the minimum wage will be doing the least additional harm as measured by welfare.

While both of these policies can generate efficiency gains over laissez-faire, they are not necessarily Pareto improving. Appendix C provides an analysis of who are likely winners and losers under these policies.

3.5.4 Further discussion

The emphasis on simplicity here has also ruled out a number of potential generalizations. Endogenous search intensity, for example, is very close in nature to what we study here. Because they have higher returns to employment, higher ability workers would search harder but would not internalize the impact of their intensity choice on the other market participants. At first blush, the potential for harm from low ability workers entering should be reduced because they would not search very hard. But, with a low matching rate, they would remain in the market for a long time and worsen the congestion felt by more able workers. A binding minimum wage would exclude the lowest ability workers but increase the search intensity of the marginal worker with p in $[\bar{w}, \tilde{p}]$. UI excludes the lowest ability workers and reduces the returns to search at every remaining ability level. How these changes affect welfare will depend on the functional form of the search cost function and the bargaining power of the workers vis-a-vis the Hosios Rule.

We have also abstracted from the possibility that workers are additionally heterogeneous in their values of non-market activities, z. This is a concern that is relevant to low-income markets and to policy makers. In that environment, the participation externality would be felt more diffusely as none of the low ability workers with high z would ever enter the market. The policies we consider would not change that. An earned-income tax credit (EITC) could induce some quite high ability workers with even higher z into the market but might also bring in more low ability workers. A minimum wage coupled with EITC could prevent the latter outcome.

4 Dynamic Model

A dynamic version of the model is required to ascertain the quantitative importance of the participation externality. The analytical results here largely mirror those of the static model so much of the work has been moved to Appendix B.

4.1 Environment

The dynamic model is cast in continuous time so all of the utility sources, p, z, a etc. become flows. The function $m(.)$ now represents a Poisson matching rate[8] and there is an exogenous job destruction rate, λ, as well as a common discount rate, r. Workers live for ever.

4.2 Efficiency

Risk-neutrality implies that welfare, W, amounts to discounted transitional benefits minus costs. Thus,

$$W = \int_0^{\infty} \left(z\left[F(\hat{p}_t) + u_t\right] + \left[1 - F(\hat{p}_t) - u_t\right] \mathbb{E}_{[p \geq \hat{p}_t]}\left(p\right) - \theta_t u_t a \right) e^{-rt} dt \quad (4.1)$$

where the subscript t, signifies time. The variable, $\hat{p}_t$, is the cut-off level of ability below which a worker is excluded from job matching.[9] The first term

[8]More specifically, $m(.)$ is strictly concave and strictly increasing on $\mathbb{R}_+$ with $\lim_{\theta \to \infty} m(\theta) = \infty$. And, $m(\theta)/\theta$ is strictly decreasing with $\lim_{\theta \to \infty} m(\theta)/\theta = 0$. To avoid a corner solution, $\lim_{\theta \to 0} m'(\theta) = \infty$.

[9]Technically, the definition of welfare here requires that the value of $\hat{p}_t$ increase over time so that the Planner can terminate matches as they become inviable. Because the Planner is subject to matching frictions, were $\hat{p}_t$ to decrease over time, the distribution of

in the parentheses sums the welfare obtained from non-market activities by excluded workers and job seekers. The second, sums output across matched workers. The final term is the cost of maintaining the vacancies.

The Social Planner chooses a path for the population share of job seekers, u_t, the labor market tightness, θ_t, and the ability exclusion threshold, $\hat{p}_t$, to maximize welfare. The Planner is constrained by the dynamics of unemployment,

$$\dot{u}_t = \lambda \left(1 - F(\hat{p}_t) - u_t\right) - m(\theta_t)u_t \tag{4.2}$$

where the dot over a variable indicates its rate of change with respect to time.

In Appendix B, from the first order conditions for θ and $\hat{p}$, we derive the following dynamic counterparts to Equation 3.2 and Equation 3.3,

$$\frac{m'(\theta_p)\mathbb{E}_{[p\geq\hat{p}_p]}(p-z)}{r+\lambda+m(\theta_p)-\theta_p m'(\theta_p)} = a \tag{4.3}$$

$$\frac{m'(\theta_p)\left[\left(m(\theta_p)+\lambda\right)\left(\hat{p}_p - z\right) + \lambda\mathbb{E}_{[p\geq\hat{p}_p]}(p-\hat{p}_p)\right]}{\lambda(m(\theta_p)+\lambda)} = a. \tag{4.4}$$

Notice that whenever $z > 0$ the Planner will at least exclude workers whose ability is less than z. They cause congestion to the other workers in the matching process and, if employed, will reduce total welfare. So, the constrained efficient value for $\hat{p}$ cannot be less than z.

In Appendix B we show that as long as $m(\theta)\eta(\theta) > r$ the Planner always chooses a value for $\hat{p}_p$ strictly larger than z. We call $m(\theta)\eta(\theta) > r$ the "Thick-Market" condition. Whenever $m(\theta)\eta(\theta) \leq r$, the Planner sets $\hat{p} = z$. This highlights a notable difference between the dynamic model and the static one. In the latter $\hat{p}_p$ is larger than z for any non-trivial equilibrium. This would also be true in the dynamic model in the limit as r approaches zero. In general, as long as r is small enough, the long-term benefit associated with improved selectivity of match formation outweighs the short-term loss of output associated with excluding some workers with $p > z$ from matching.

4.3 Laissez-faire market economy

In the dynamic market economy, wages are also determined by generalized Nash bargaining. Here, though, the worker's outside option is to continue to look for work. All dynamic market analysis is carried out for steady states.

current match productivities would temporarily differ from $F(.)$. However, the focus here is on steady states and as the economy converges on steady state the distinction between increasing and decreasing paths for $\hat{p}_t$ disappears.

Again, workers indifferernt between market entry and sitting it out choose the latter. The ability level below which individuals choose not to participate is $\hat{p}$. For any worker with $p \geq \hat{p}$ we have,

$$rV_u(p) = m(\theta)\left(V_e(p) - V_u(p)\right) + z \tag{4.5}$$

$$rV_e(p) = w(p) + \lambda\left(V_u(p) - V_e(p)\right) \tag{4.6}$$

where $V_u(p)$ is their value to unemployment and $V_e(p)$ is their value to employment. Then, given free entry of vacancies, the value of hiring a type p worker to the firm, $V_f(p)$, is given by

$$(r + \lambda)\, V_f(p) = p - w(p). \tag{4.7}$$

Eliminating the value functions reveals that the wage is a weighted average of worker ability and the effective flow income from non-employment:

$$w(p) = \frac{\beta\left(r + \lambda + m(\theta)\right)p + (1 - \beta)(r + \lambda)z}{r + \lambda + \beta m(\theta)}. \tag{4.8}$$

It should be clear that under laissez-faire, $\hat{p} = z$ – once a firm with a vacancy meets a worker the vacancy cost is sunk and ex post there are gains from trade whenever $p > z$. But, as long as the Thick-Market condition holds, the Planner would exclude some workers whose productivity is above z. Those workers failure to internalize the impact of their participation on vacancy creation leads to a participation externality.

Under free-entry, the vacancy creation condition is then:

$$\frac{m(\theta)}{\theta}\int_z^{\bar{p}} V_f(p)\frac{dF(p)}{1 - F(z)} = a. \tag{4.9}$$

Definition 1 *A free-entry market equilibrium is a list* $\{V_u(p), V_e(p), V_f(p), w(p), \theta^*, \hat{p}^*\}$ *such that:*

- $\hat{p}^* = z$
- *Given* θ^**, the value functions emerge from optimal search and matching*
- *For* $p \geq z$*,* $w(p)$ *is given by Equation 4.8.*
- θ^* *solves the free-entry condition, Equation 4.9.*

Straightforward algebra yields the following characterization of θ^*:

$$\frac{m(\theta^*)(1 - \beta)\mathbb{E}_{[p \geq z]}(p - z)}{\theta^*\left(r + \lambda + \beta m(\theta^*)\right)} = a.$$

It equates the the firms' marginal private value to marginal cost of vacancy creation. In Appendix B we show that the market equilibrium exists and is unique

4.4 Dynamic model policy analysis

Of interest here again is looking at the role of the participation externality, how it interacts with the search externalities and how either a minimum wage or UI can be used to address it. By its nature the dynamic model is more complex than the static one and results here are less amenable to succinct interpretation. Here we provide a synopsis of the results and remand the proofs to Appendix B.

4.4.1 Direct control of $\hat{p}$

In this scenario, the ability threshold for market entry, $\hat{p}$, is exogenous and the unique equilibrium market tightness, θ^*, is obtained from the free entry condition:

$$\frac{m(\theta^*)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z)}{\theta^*\left(r+\lambda+\beta m(\theta^*)\right)} = a. \tag{4.10}$$

The policy maker can adjust $\hat{p}$ to impact welfare, still obtained from Equation 4.1, subject to the law of motion for unemployment and the free entry condition, Equation 4.10.[10]

We find that welfare is increasing in $\hat{p}$ at $\hat{p} = z$ whenever

$$(1-\eta)\left[r+\lambda+m(\theta^*)\right]\left[\eta(\theta^*)m(\theta^*)-r\right] \\ + m(\theta^*)\left[\beta-(1-\eta(\theta^*))\right]\left(\left[r+\lambda+m(\theta^*)\right]\eta(\theta^*)-r\right) > 0$$

This is the General Thick-Market condition. It collapses to the basic Thick-Market condition at the Hosios Rule. If $\beta > 1-\eta(\theta^*)$ and $\eta(\theta^*)m(\theta^*) > r$ the second term is positive so this requirement is less stringent than under the basic condition. The opposite will be true if $\beta < 1-\eta(\theta^*)$.

Whenever the Generalized Thick-Market condition holds, we show that there will be an optimal value of $\hat{p} > z$. Recall that in the static model we were able to show that if η is invariant to θ, then optimal $\hat{p}$ is invariant to β. Here we are unable to confirm this result analytically but we do find this to be true in all of our numerical simulations. Comparison of Equation 4.3 and Equation 4.10 reveals that at the Hosios Rule, optimal $\hat{p}$ implements the Planner's solution.

[10] A possible cause for concern here is that condition Equation 4.10 was derived in steady-state. The only source of dynamics in the economy, however, comes from the fact that the measure of unemployment, u, is not a jump variable and u does not appear in Equation 4.10 – θ^* simply jumps to its new steady-state value whenever $\hat{p}$ (or any other policy variable) is changed.

4.4.2 The minimum wage

Again here, $\hat{p} = \bar{w}$. And again we let $\tilde{p}$, be the ability level above which workers are paid their bargained wage. For workers with $p > \tilde{p}$, the laissez-faire value functions, Equation 4.5, Equation 4.6 and Equation 4.7 still apply. For workers with $p \in [\bar{w}, \tilde{p}]$, they are Equation 4.5, Equation 4.6 and Equation 4.7 with $w(p)$ replaced by $\bar{w}$.

Definition 2 *A free-entry market equilibrium, with a binding minimum wage, is a list* $\{V_u(p), V_e(p), V_f(p), w(p), \theta^*, \tilde{p}\}$ *such that given* θ^* *and* $\tilde{p}$, *the value functions emerge from optimal search and matching and, for* $p \geq \tilde{p}$, $w(p)$ *is given by Equation 4.8. Then,* $\tilde{p}$ *solves* $w(\tilde{p}) = \bar{w}$ *and* θ^* *solves the free-entry condition, Equation 4.9.*

Straightforward algebra yields the following characterization of equilibrium in terms of θ^* and $\tilde{p}$:

$$a = \frac{m(\theta^*)}{\theta^* (1 - F(\bar{w}))} \left[\int_{\bar{w}}^{\tilde{p}} \frac{p - \bar{w}}{r + \lambda} dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} \frac{p - z}{r + \lambda + \beta m(\theta^*)} dF(p) \right] \tag{4.11}$$

$$\tilde{p} = \frac{[r + \lambda + \beta m(\theta^*)]\, \bar{w} - (1 - \beta)(r + \lambda) z}{\beta (r + \lambda + m(\theta^*))} \tag{4.12}$$

The uniqueness of $\tilde{p}$ follows because $w(p)$ is strictly increasing in p. In Appendix B we show that with $\bar{w} < \bar{p}$, equilibrium exists, it is unique and equilibrium values of θ^* and $\tilde{p}$ both decrease with worker bargaining power, β. We also show that whenever the Generalized Thick-Market condition holds, welfare is increasing in the minimum wage at $\bar{w} = z$ and an optimal strictly binding minimum wage exists.

4.4.3 Unemployment Insurance

We abstract from the complexities of real-world UI systems to focus on how an indefinite payment stream, b, interacts with the participation and search externalities. We assume that the payments are financed through a lump-sum tax, τ, on all workers.

Following the logic from the laissez-faire market economy above, for any $p \geq \hat{p}$ we now have,

$$\begin{aligned} rV_u(p) &= m(\theta)\,(V_e(p) - V_u(p)) + z + b - \tau \\ rV_e(p) &= w(p) - \tau + \lambda\,(V_u(p) - V_e(p)) \end{aligned}$$

and

$$(r+\lambda)\,V_f(p) = p - w(p).$$

Nash bargaining implies

$$w(p) = \frac{\beta\,(r+\lambda+m(\theta))\,p + (1-\beta)(r+\lambda)(z+b)}{r+\lambda+\beta m(\theta)}.$$

Here $\hat{p} = z + b$. The free-entry condition is identical to Equation 4.9 and the definition of equilibrium is identical to Definition 1. Equilibrium market tightness is now characterized by

$$a = \frac{m(\theta^*)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{\theta^*\,(r+\lambda+\beta m(\theta^*))}. \tag{4.13}$$

Existence and uniqueness of the equilibrium is demonstrated in Appendix B. We also show that at the Hosios Rule, welfare is increasing in b at $b = 0$ whenever the basic Thick-Market condition holds. As we found in the static model however, at $b = 0$,

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.13)} \neq \left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.10)}.$$

So, away from the Hosios Rule, the Generalized Thick-Market condition does not apply to UI payments.

5 Calibration

In this section, we discuss our calibration of our dynamic model, presented in Section 4. We first introduce our data and sample selection criteria. Then we discuss externally calibrated parameters. Last, we explain our identification strategy using key labor market moments and show the model fit.

5.1 Data

We use monthly Current Population Survey (CPS) data from January 2012 to December 2014[11] and restrict our sample to include only states where the minimum wage was \$7.25 in 2012, individuals who did not obtain a high school diploma or equivalent, and those who were between 25 and 54 years of age throughout the period under consideration. We drop anyone who reports ever being an unemployed new worker, unable to work, or retired.

[11] This is a window of time in which employment was reasonably stable but many individual states had not yet raised their minimum wages above the \$7.25 federal level.

5.2 Model Calibration

We follow a standard approach in the search and matching literature and preset a selection of parameters with estimated values from other papers with closely related models. We discuss these preset parameters as well as our functional form assumptions in Section 5.2.1. For parameters that are associated with key distinctions in our model, we estimate them by targeting the empirical specifications described in Section 5.2.2.

5.2.1 Externally Calibrated Parameters and Functional Forms

We start our calibration by making functional form assumptions that follow much of the related literature. We assume that the distribution of abilities, $F(.)$ is log-normal with parameters μ and σ. We also assume that the aggregate matching function is Cobb-Douglas with an elasticity of matching with respect to vacancies of η so that $m(\theta) = \bar{m}\theta^{\eta}$. Neither assumption departs from the bulk of previous work.

After making our functional form assumptions, we have 11 parameters to calibrate. We externally calibrate parameters with direct empirical interpretations $(\bar{w}, \lambda)$ and then we preset a selection of parameters to common values in the literature $(r, \eta, \bar{m})$. The remaining parameters $(\beta, \mu, \sigma, a, z)$ are calibrated using simulated method of moments in Section 5.2.2. Throughout our calibration, we assume that the period length is one month, though we focus on the steady state and thus do not discretize the model.

We set $\bar{w}$ to be the federal minimum from 2012 to 2014, \$7.25. In our empirical analysis, we restrict our sample to states in which the federal minimum wage is binding as discussed in Section 5.1. The separation rate, λ, directly translates into the flow rate from employment to unemployment. Thus we target this flow rate in our empirical sample, which turns out to be 0.0329. Following the real business cycle literature, we set the discount rate, r, to be consistent with 4% per year. We follow the results in Blanchard et al. (1990) and Petrongolo and Pissarides (2001) and set the elasticity of the matching function with respect to market tightness, η, to be 0.5. Because there is a one-to-one relationship between the advertising cost, a, and $\bar{m}$ in the current model, we choose to normalize $\bar{m}$ to unity. We present the values for our parameters in Table 5.2.

5.2.2 Calibrated Parameters

In this section, we discuss the moments that we use to estimate the remaining parameters of the model. We also describe which moments are most closely associated with each parameter, though they are jointly estimated.

We exploit the participation threshold to identify the parameters of the productivity process, μ and σ. In the absence of a non-binding minimum wage, non-participation is exclusively determined by the productivity distribution. What distinguishes μ from σ is that they have opposite effects on the measure of non-participants. An increase in σ further skews the distribution, leading to fewer participants, while an increase in μ shifts the distribution to the right, resulting in fewer non-participants. By contrast, increases in either μ or σ cause the average wage and the standard deviation of wages, moments we also target, to increase. Incorporating these three moments allow us to separately identify the productivity parameters. In the CPS, we find a non-participation rate of 20.6%, and a mean and standard deviation of the log-wage distribution of 2.45 and 0.32, respectively.

Our remaining challenge is to separately identify worker bargaining power, β, from non-labor market utility, z. From the wage equation (Equation 4.8), it is clear that an increase in β or z affects wages. However, an increase in β causes a proportional change across the wage distribution, while an increase in z causes an upward shift of all wages by the same amount. This difference allows us to identify each parameter. From Equation 4.12, an increase in either z and β cause a decrease in the measure of workers receiving the minimum wage. They differ in their effect on the dispersion of wages: an increase in β results in an increase in the standard deviation of wages, by causing wages to more closely reflect the skewness of the productivity distribution. By contrast, an increase in z leads to a uniform upward shift of the wage distribution, *reducing* the standard deviation of wages. Thus, in concert with the wage distribution described above, targeting the mass of workers at the minimum wage allows us to separately identify these parameters. In the data, the share of workers employed at the minimum wage is 3.32%, which we identify as workers who report earning $7.25 in the CPS.

For our last parameter to calibrate, the cost of vacancy creation, a, we follow a standard convention in the search literature. We match the unemployment rate, whose relation to a is clear from the free entry condition, Equation 4.9. While the national average unemployment rate for the whole workforce during this time period was 7.3%, we restrict our sample to strongly attached, but lower-educated workers. This results in an unemployment rate of 10.4%.

We present our calibration results in Table 5.1. The model nearly precisely matches all of the targeted moments.

We present the parameters that achieve this fit in Table 5.2. The first five rows are those that we estimate as described above, while the remaining are

Table 5.1: Model Fit.

Moment	Data	Model
Unemp. Rate	0.10	0.10
Non-Part. Rate	0.21	0.21
$E[ln(w)]$	2.45	2.45
$SD[ln(w)]$	0.32	0.34
$P(w = \bar{w})$	0.03	0.03

externally calibrated. In our model, average hourly productivity is $13.74, meaning that our estimate of leisure utility, $z = 6.04$ is 44% of average productivity. This is very close to Shimer (2005), but lower than Hagedorn and Manovskii (2008). Our estimated vacancy creation cost of $169.75 is high relative to previous estimates, but it is worth noting both that our sample yields a higher unemployment rate than the population average and that we normalize the matching scale parameter $\bar{m} = 1$, while many related papers separately estimate both and arrive at smaller values.

Table 5.2: Model parameters.

Parameter	Comment	Value
β	Barg. Power	0.30
μ	Mean of Prod. Dist	2.39
σ	Var. of Prod. Dist	0.50
$\bar{b}$	Non-Market Util.	6.04
a	Vacancy Cost	169.75
η	Matching Elast.	0.50
r	Discount Rate	0.0033
λ	Sep. Rate	0.03
$\bar{m}$	Matching Fun. Norm.	1.00
$\bar{w}$	Min. Wage	7.25

Notes: The first five parameters are calibrated using the moments described above. The bottom five are preset according to our description above.

5.2.3 Fit and non-targeted validation

It is no surprise that our model precisely matches our moments. Here, we show that the model is also capable of matching several non-targeted moments. In Table 5.3, we compare the modal wage in our model to the data, the minimum wage elasticity, and the job-finding rate. We come very close to matching both the modal wage ($10.34 in our model and $10.30 in the data) and the monthly job-finding rate (0.28 in our model and 0.34 in the data), but slightly overestimate the elasticity of employment with respect to the minimum wage (-0.76 in our model vs. -0.69 in the data), when we compare our results to the long-run estimates of Keil et al. (2001). We select this target because our model contains few of the short-run frictions that might limit a firm's employment response. This last target is contentious with estimates ranging from slightly positive to highly negative. We note that our sample is low-skilled workers that are more likely to be affected by changes in the minimum wage, and that this group tends to have the largest employment effects (Neumark and Wascher, 2006).

Table 5.3: Non-targeted moments.

Moment	Data	Model	Comment
Min. wage elasticity	-0.69	-0.76	From Keil et al. (2001)
Modal wage	10.30	10.34	CPS, 2012-2014
Monthly job-finding rate	0.34	0.28	From Shimer (2004)

6 Results

We now use our calibrated economy to consider the impact of minimum wage and unemployment policies on the participation and search externalities. We first compare welfare and labor market outcomes between a Laissez-faire benchmark economy and optimal policy economies. We also consider two additional benchmark economies: our economy as calibrated in Section 5 and an economy in which a Social Planner is able to directly control the participation decision of workers. Next, we use our results in Section 4 to quantify the gains by implementing an optimal minimum wage or optimal unemployment insurance. We then explore the channels through which each policy is able to simultaneously affect the participation and search externalities.

6.1 Optimal Policies in the Dynamic Model

We start by quantifying the gains from implementing an optimal minimum wage or unemployment insurance. In our experiments, a policy maker in the decentralized economy is able to directly control the minimum wage, $\bar{w}$, or the increase in unemployment utility from z to $z + b$. They are still bound, however, by the decentralized free entry condition, and can only indirectly change θ. We compare these economies to a laissez-faire economy, our calibrated baseline, and to a Social Planner's economy.

We focus on three measures to describe the impact of each policy. First, we assess the effect on the non-participation rate. We calculate this as the share of workers with lower ability than our ability cutoff in each economy, $F(\hat{p})$, where $\hat{p}$ is $\bar{w}$ for the calibrated and minimum wage economies, $z + b$ for the optimal UI economy, and $\hat{p}_p$ for the Planner's economy. In addition, we calculate the unemployment rate as the steady-state share of workers searching for a job under each policy. Finally, we quantify the gains achieved from switching policies by calculating welfare using Equation 4.1, where the participation threshold is set to $\hat{p}$ described above. We assume that each economy starts from the laissez-faire level of unemployment and calculate present discounted welfare along the transition to steady state. We compare the welfare gains due to each policy to the calibrated economy.

We present the results in Table 6.1. The "Calibrated economy" is the baseline model calibrated as described in Section 5. The "Planner's optimum" refers to an economy in which a Social Planner selects the optimal participation and labor market tightness according to Equation 4.3 and Equation 4.4. The "Optimal min. wage" economy refers to one in which the minimum wage is set according to Equation 4.11, leaving UI at its baseline value (i.e., $b = 0$). "Optimal UI" refers to an economy in which there is no minimum wage and additional UI is given to workers by solving the problem in Section 4.4.3. In each economy, the "Ability Threshold" column refers to the dollar value of $\hat{p}$. $P(w = \bar{w})$ denotes the mass of workers earning the minimum wage. We select the "laissez-faire" economy as a benchmark, because we implement our UI expansion without a minimum wage, making this the appropriate counterfactual.

Table 6.1: Outcome comparison with baseline parameters.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold (\$)	$P(w = \bar{w})$	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	0.000	100.000
Calibrated economy	0.104	0.206	0.079	7.250	0.033	100.314
Planners optimum	0.151	0.190	0.033	7.043	0.000	101.542
Optimal min. wage	0.105	0.198	0.078	7.149	0.030	100.317
Optimal UI	0.114	0.231	0.064	7.565	0.000	100.862

Our results indicate that both policies yield welfare improvements over their absence. Our optimal minimum wage is \$7.12, which indicates that the 2012 minimum wage of \$7.25 comes close to implementing the welfare maximizing minimum wage. Setting the minimum wage to \$7.12 yields a small welfare gain of 0.003pp, relative to the calibrated economy, but a moderate welfare gain of 0.317pp relative to the laissez-faire economy. The fifth row shows that if the policy maker instead funded additional UI payments of \$1.52 (\$7.56 - \$6.04), welfare would increase by 0.548pp relative to the calibrated economy, and yield an 0.862pp welfare gain over the laissez-faire benchmark. With both the participation threshold and vacancy creation under her control, the Planner is able to increase welfare by 1.228pp relative to our calibrated model and 1.542pp relative to the laissez-faire economy.

Notice that the ability thresholds for both policies are above the Planner's value. This is a consequence of low β relative to the Hosios value. As they both raise wages along with the participation threshold, these policies are able to simultaneously address the search and participation externalities. While the threshold is below the Planner's optimal level increasing it is beneficial on both fronts. Increasing the threshold beyond the Planner's level then sets up a trade off between these goals. As, for a given value of $\hat{p}$, UI raises wages more than does the minimum wage those forces balance out a higher value than for the minimum wage.

An important subtlety that we will explore in the next section is how each policy achieves these welfare gains. Compared with the laissez-faire economy, each alternative unsurprisingly reduces participation, varying from non-participation rate of 18.7% under the Planner to 23.1% under Optimal UI. The remaining effects on the labor market are surprisingly disperse across the different policies. The minimum wage policies under either the calibrated economy or our optimal minimum wage reduce the unemployment rate, while increasing θ. However, both the UI policy and the Planner's optimum result in increases in the unemployment rate and a reduction in θ, exactly the opposite of the minimum wage policies. This reflects an important difference in policies: while the minimum wage distorts vacancy creation, it does so by changing a subset of negotiated wages; by contrast, UI distorts wages across the wage distribution, leading to potentially more bite on vacancy creation.

6.2 Understanding the Externalities

In this section, we show how the search and participation externalities guide the choice of optimal minimum wage and unemployment insurance. With these policies in place there are three potential sources of inefficiency. First, the model contains the standard search externalities that are caused by inappropriate shares of the surplus being assigned to the firm and worker. Second, there is a participation externality, caused by the impact that average worker quality has on vacancy creation. And last, the policies distort wage formation which directly impacts vacancy creation. We first decompose the effect of the minimum wage among these channels and their interaction, before doing the same for UI. Throughout our decomposition, we will quantify the effect of each policy on the externalities by using the change in welfare as we use restrictions to isolate each externality.

6.2.1 The Participation and Search Externalities under the Minimum Wage

In our calibrated model, the minimum wage blunts the impact of both the participation and search externalities on welfare. It screens out low-productivity workers, reducing the participation externality, and increases the share of the surplus received by some workers, artificially helping to address the imabalance in the search externalities caused by $1-\beta<\eta$. This means that while the minimum wage may directly address the participation externality, the optimal minimum wage may not coincide with the optimal level of participation. This tension highlights both the challenge faced by a policy maker in the decentralized economy and the advantage offered by the minimum wage.

The challenge is that the policy maker is unable to directly control vacancy creation and participation. By contrast, the Social Planner is imbued with both these levers of policy. Unable to control vacancies, the decentralized policy maker who wishes to use the minimum wage is bound by the free entry condition:

$$a=\frac{m(\theta)}{\theta\left(1-F(\bar{w})\right)}\left[\int_{\bar{w}}^{\tilde{p}}\frac{p-\bar{w}}{r+\lambda}dF(p)+(1-\beta)\int_{\tilde{p}}^{\bar{p}}\frac{p-z}{r+\lambda+\beta m(\theta)}dF(p)\right]$$

which does not generically set θ to coincide with $\frac{\partial\mathcal{H}}{\partial\theta_t}=0$, the welfare maximizing value from the Planner's problem.[12] Further inspection of this expression yields two key insights: First, average worker quality increases as the minimum wage increases, reducing the participation externality. Second, even under the Hosios Condition, θ is not set optimally by the free

[12]Even if it were possible, the inability to control the participation margin would prevent the economy from generically achieving the first best.

entry condition. As a result, vacancy creation will be distorted if any wages are affected by the implementation of the policy, which we demonstrate in the following:

$$\int_{\bar{w}}^{\tilde{p}} \frac{p-\bar{w}}{r+\lambda} dF(p) < (1-\beta) \int_{\bar{w}}^{\tilde{p}} \frac{p-z}{r+\lambda+\beta m(\theta)} dF(p), \forall \tilde{p} > \bar{w}$$

The right hand side shows the surplus accrued by firms when matched with workers with productivity $p \in [\bar{w}, \tilde{p}]$ when all wages are Nash Bargained. The left hand side shows the surplus accrued by firms hiring workers on whom the minimum wage binds. Clearly, unless no wages are affected by the policy, i.e. $\bar{w} = \tilde{p}$, vacancy creation will be distorted and the minimum wage will affect the search externalities. Balancing this additional margin of impact along with the impact of excluding additional workers from the labor market determines the optimal minimum wage.

Assessing the resolution to this trade-off is integral to decomposing the channels by which the minimum wage operates on welfare. We do this by considering two counterfactual policy experiments that we compare to the outcome of our optimal minimum wage exercise (Table 6.1). For each policy, we consider an environment in which the policy maker is able to directly specify the participation threshold, $\hat{p}$, above which wages are subject to unconstrained Nash Bargaining. First, we allow the policy maker to optimize over the participation threshold, which she does by solving the problem described in Section 4.4.1. Next, we set the participation threshold equal to the optimal minimum wage, $\hat{p} = \bar{w}^*$. We then use both the laissez-faire and the optimal minimum wage economies as benchmarks to understand how the minimum wage affects each externality.

Comparing these economies allow us to determine the extent to which the minimum wage contributes to welfare by reducing each of the participation and search externalities. First, the change in welfare garnered by moving from the laissez-faire economy to an optimal minimum wage economy yields the cumulative effect of the minimum wage. Second, comparing the laissez-faire economy to the economy in which $\hat{p} = \bar{w}^*$ yields the effect that the optimal minimum wage has on the participation externality. The logic behind this conclusion is that we are removing the impact that the minimum wage has on vacancy creation, outside of the impact on participation. The effect of the minimum wage on the search externalities introduces an additional complication. We can calculate the *net* effect that the minimum wage has on welfare from helping balance out the search externalities by comparing the change in welfare moving from the $\hat{p} = \bar{w}^*$ participation economy to our optimal minimum wage economy. However, this understates the impact of the minimum wage on the search externalities.

While quantifying the net effect of the minimum wage on welfare relative to the participation policy is a straightforward calculation, changing the minimum wage affects the participation externality in the process. This interaction is critical to understanding the impact of the minimum wage in our environment. First, moving from an optimal participation policy to a minimum wage dampens the reduction in the participation externality. We can quantify the amount by comparing the $\hat{p}^*$ economy to the $\hat{p} = \bar{w}^*$ economy. The difference in welfare is the reduction in the participation externality forgone by the policy maker to further reduce the net negative effect of the search externalities. And as a result, this interaction should be included to the effect of the minimum wage on the search externalities. We report this decomposition in Table 6.2.

Table 6.2: Decomposition of the effect of the minimum wage.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold ($)	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	100.000
Optimal Participation	0.105	0.196	0.078	7.125	100.310
Min. Wage Participation ($\hat{p} = \bar{w}^*$)	0.104	0.198	0.078	7.149	100.310
Optimal Min. Wage	0.105	0.198	0.078	7.149	100.317

This table shows the value of the minimum wage, both in improving the average quality of worker in the market and in distorting the free entry condition so that θ approaches its optimal level. Comparing the third and first rows shows that by increasing the participation threshold, the minimum wage leads to a 0.31pp increase in welfare. The difference between the fourth and third rows shows that in addition to reducing the participation externality, the minimum wage also reduces the cost of the imbalance between the search externalities by 0.007pp. This reduction comes at very little expense from the participation externality: a policy that sets $\hat{p}$ optimally increases welfare less than 0.0005pp more than setting the participation threshold to $\bar{w}$, revealed by the difference between the third and second rows.

This table also highlights the key tension faced by the policy maker: above the optimal level of participation, raising the minimum wage trades-off exacerbating the participation externality with balancing out the search externalities. While this interaction is relatively inconsequential in this experiment, our optimal UI experiment will show that this interaction is an important consideration.

6.2.2 The Participation and Search Externalities under Unemployment Insurance

We now turn our attention to unemployment insurance. We first outline the key differences between the minimum wage and UI. Then we consider a similar set of counterfactuals to the ones in the previous section. We decompose the change in welfare due to the imposition of UI described in Section 4.4.3 into the share caused by a reduction in the participation externality and the share caused by reducing the net cost of search externalities.

Unemployment insurance differs from the minimum wage by affecting the flow utility of unemployment for workers of all productivity levels. This directly changes the participation decision of workers, but also affects vacancy creation:

$$a = \frac{m(\theta)}{\theta\,(1 - F(\hat{p}))}(1-\beta)\int_{\hat{p}}^{\bar{p}} \frac{p - (z+b)}{r + \lambda + \beta m(\theta)} dF(p)$$

where b denotes the amount of UI and $\hat{p} = z+b$ is the participation threshold. This expression encodes information about how UI affects the externalities. First, in our baseline model used to find optimal UI, $\hat{p} = z + b$. This means that increasing UI directly improves the pool of participating workers. Second, it decreases the surplus of every match, effectively transferring a larger share of welfare to workers. This means that UI is able to address the imablance in the search externalities by reducing vacancy creation when the Hosios Condition is not satisfied and $1 - \beta > \eta$.

We use these insights to decompose the effect of UI. As with the minimum wage, we consider two counterfactual economies in which the policy maker can directly target the participation threshold. We first restrict the participation threshold to equal that emerging under optimal UI. That is, we allow $\hat{p} = z + b$, but assume that this has no bearing on worker leisure utility. As a result, the participation threshold changes, but wages above the threshold are unaffected. Next, we first allow the policy maker to optimize over the participation threshold. We compare these economies against the laissez-faire and our optimal UI economies.

Like our minimum wage decomposition, this allows us to tease out the impact of UI on the participation and search externalities. As before, moving from the laissez-faire economy to the optimal UI economies tallies the cumulative effect of both policies. Taking the difference between the laissez-faire economy and the $\hat{p} = z + b$ shows the share of this effect that is caused by a reduction in the participation externality. Then as before, the difference between the $\hat{p} = z + b$ economy and the optimal participation economy is the reduction in the participation externality forgone to reduce the net cost of search externalities. Finally, this difference added to the difference between the $\hat{p} = z + b$ and the optimal UI economies is the change in welfare accrued from reducing the net cost of search externalities. We show this decomposition in Table 6.3.

Table 6.3: Decomposition of the effect of unemployment insurance.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold ($)	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	100.000
Optimal Participation	0.105	0.196	0.078	7.125	100.310
UI Participation ($\hat{p} = z + b^*$)	0.102	0.231	0.082	7.565	100.252
Optimal UI	0.114	0.231	0.064	7.565	100.862

What this table shows is that unlike the minimum wage, UI operates primarily by reducing the net cost of search externalities. Of the cumulative 0.862pp increase in welfare, relative to the laissez-faire economy, only about a third (0.252pp) is caused by reducing the participation externality (difference between ”laissez-faire” and ”UI Participation”). By contrast, UI reduces the net cost of the search externalities enough to cause a 0.61pp increase in welfare (difference between ”Optimal UI” and ”UI Participation”). This change was muted by the interaction between the search and participation externalities introduced by the change in UI. Increasing UI above the optimal participation threshold aggravated the participation externality and led to a 0.058pp decline in welfare, a sizable impact relative to the minimum wage policy. In the absence of this interaction, the overall effect of UI on the net cost of search externalities would be 0.668pp.

7 Conclusion

In this paper, we explore the participation externality and the role of policy in ameliorating it. In random search models with homogeneous workers, the only source of inefficiency is that which arises from the search externalities. In these environments, imposing the Hosios Rule implements constrained efficiency. In our model, heterogeneity in worker ability leads to a participation externality, in which market entry by low-ability workers depresses vacancy creation and leads a decentralized economy to diverge from the optimum even under the Hosios Rule.

We show that the participation externality has a quantitatively important effect and interacts with other sources of inefficiency. When either UI expansions or the minimum wage are implemented optimally, they can partially address the participation externality and do so in concert with addressing the search externalities. UI operates primarily by reducing the net cost of search externalities, while the minimum wage more directly targets the participation externality. However, both do so inefficiently and do not achieve the same welfare gains as a Social Planner could have done. Yet, they still achieve sizable gains over the laissez-faire economy.

Our findings show that the participation externality is a worthy consideration for a policymaker. While our findings do not support a sizable increase in the minimum wage or the generosity of UI, they do indicate that the presence is welfare improving. A richer environment with additional sources of heterogeneity may find even more support for these policies. At the end of the day, however, it is up to policy makers to assess whether the efficiency gains from the imposition of an appropriate policy are worth the social costs associated with employment loss for the lowest ability participants.

8 Appendix A: Static model proofs and derivations

8.1 Equation 3.11

From Equation 3.6 we have

$$\begin{aligned}
\frac{d\theta^*}{d\hat{p}} &= \frac{f(\hat{p})\left[\frac{m(\theta^*)}{\theta^*}(1-\beta)(\hat{p}-z)-a\right]}{\left(\frac{m'(\theta^*)\theta^*-m(\theta^*)}{\theta^{*2}}\right)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p)} \\
&= \frac{\left[\frac{m(\theta^*)}{\theta^*}(1-\beta)(\hat{p}-z)-a\right]}{\left(\frac{m'(\theta^*)\theta^*-m(\theta^*)}{\theta^{*2}}\right)\left(\frac{(1-F(\hat{p}))a\theta^*}{m(\theta^*)}\right)} \\
&= \left(\frac{f(\hat{p})}{1-F(\hat{p})}\right)\left(\frac{a\theta^*-(1-\beta)m(\theta^*)(\hat{p}-z)}{a(1-\eta(\theta^*)}\right)
\end{aligned}$$

Using Equation 3.6 in the numerator this becomes

$$\begin{aligned}\frac{d\theta^*}{d\hat{p}} &= \left(\frac{f(\hat{p})(1-\beta)m(\theta^*)}{1-F(\hat{p})}\right)\left(\frac{\mathbb{E}_{[p\geq\hat{p}]}(p-z)-(\hat{p}-z)}{a(1-\eta(\theta^*)}\right) \\ &= \frac{(1-\beta)m(\theta^*)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{[1-F(\hat{p})]\,a(1-\eta(\theta^*))} > 0\end{aligned}$$

8.2 Equation 3.12

Substituting from Equation 3.9, Equation 3.10 and Equation 3.11 into Equation 3.7 yields

$$\begin{aligned}\frac{dW}{d\hat{p}} &= \left(\frac{f(\hat{p})}{(1-\beta)(1-\eta(\theta^*))}\right)\left\{\begin{array}{c}[a\theta^* - m(\theta^*)(\hat{p}-z)]\,(1-\beta)(1-\eta(\theta^*))+ \\ [\beta-(1-\eta(\theta^*))]\,[a\theta^*-(1-\beta)m(\theta^*)(\hat{p}-z)]\end{array}\right\} \\ &= \left(\frac{\beta f(\hat{p})}{(1-\beta)(1-\eta(\theta^*))}\right)[a\theta^*\eta(\theta^*)-(1-\beta)m(\theta^*)(\hat{p}-z)] \\ &= \left(\frac{\beta f(\hat{p})m(\theta^*)}{1-\eta(\theta^*)}\right)\left[\eta(\theta^*)\mathbb{E}_{[p\geq\hat{p}]}(p-z)-(\hat{p}-z)\right].\end{aligned}$$

8.3 Existence of optimal $\hat{p}$

From Equation 3.1 as $\hat{p}$ approaches $\bar{p}$ welfare converges to z. Using Equation 3.6 when $\hat{p}=z$ we have

$$W(\theta^*, z) = z + \beta m(\theta^*)\int_z^{\bar{p}}[p-z]\,dF(p) > z.$$

Let $\theta^*(\hat{p})$ solve Equation 3.6 for given $\hat{p}$. The implicit function theorem tells us that this will be continuous in $\hat{p}$. As welfare is increasing in $\hat{p}$ at z, continuity of $W(\theta^*(\hat{p}),\hat{p})$ in $\hat{p}$ implies that there is some maximal value $\hat{p}_M$ such that for any $\hat{p} > \hat{p}_M$, $W(\theta^*(\hat{p}),\hat{p}) < W(\theta^*(z), z)$. The extreme value theorem then implies that there exists an optimal value of $\hat{p}$ in $[z, \hat{p}_M]$.

Unfortunately, when $F(.)$ has infinite support, $\hat{p}_M$ can be infinite and so then we can only really say the optimal $\hat{p}$ exists on the positive *extended* real line. For any practical purpose, however, as long as we truncate the distribution at any finite $\bar{p}$, $\hat{p}_M$ will be finite.

8.4 Equation 3.18

Let

$$\Gamma(\bar{w}) \equiv \frac{1}{(1-F(\bar{w}))}\left\{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right\}$$

Then

$$\frac{d\theta^*}{d\bar{w}} = \frac{-\frac{m(\theta)}{\theta}\frac{d\Gamma(\bar{w})}{d\bar{w}}}{\frac{m'(\theta)\theta - m}{\theta^2}\Gamma(\bar{w})} = \frac{\theta\frac{d\Gamma(\bar{w})}{d\bar{w}}}{[1-\eta(\theta)]\,\Gamma(\bar{w})}. \tag{8.1}$$

And,

$$\frac{d\Gamma(\bar{w})}{d\bar{w}} = \frac{1}{(1-F(\bar{w}))^2}\left[\begin{array}{c}\left\{\frac{1}{\beta}(\tilde{p}-w)f(\tilde{p}) - (F(\tilde{p})-F(\bar{w})) - \frac{1-\beta}{\beta}(\tilde{p}-z)f(\tilde{p})\right\}(1-F(\bar{w})) \\ +f(\bar{w})\left\{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right\}\end{array}\right]$$

Then using Equation 3.13 and substituting back into Equation 8.1 we then obtain

$$\begin{aligned}\frac{d\theta^*}{d\bar{w}} = \frac{\theta^*}{1-\eta(\theta^*)}&\left[\frac{f(\bar{w})}{1-F(\bar{w})} - \frac{F(\tilde{p})-F(\bar{w})}{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)}\right] \\ &= \frac{f(\bar{w})a\theta^* - m(\theta^*)\left[F(\tilde{p})-F(\bar{w})\right]}{a\left[1-\eta(\theta^*)\right]\left[1-F(\bar{w})\right]}.\end{aligned}$$

where the final line uses Equation 3.14

8.5 Equation 3.19

Substituting from Equation 3.16, Equation 3.17 and Equation 3.18 into equation Equation 3.15 and simplifying yields

$$\begin{aligned}\frac{dW}{d\bar{w}} &= f(\bar{w})\left[a\theta - m(\theta)(\bar{w}-z)\right] \\ &\quad + \frac{\left[m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - a\right]\left[f(\bar{w})a\theta - m(\theta)(F(\tilde{p})-F(\bar{w}))\right]}{a\left(1-\eta(\theta^*)\right)}\end{aligned}$$

so,

$$\begin{aligned}\frac{dW}{d\bar{w}} = \frac{f(\bar{w})}{1-\eta(\theta^*)}&\left\{m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] - \eta(\theta^*)\left[a\theta^* - m(\theta^*)(\bar{w}-z)\right]\right\} \\ &+ \frac{m(\theta^*)\left[F(\tilde{p})-F(\bar{w})\right]\left[a - m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z)\right]}{a\left(1-\eta(\theta^*)\right)}.\end{aligned}$$

Now let

$$\Psi \equiv m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] - \eta(\theta^*)\left[a\theta^* - m(\theta^*)(\bar{w}-z)\right]$$

which are the contents of the curly braces. This can be rewritten as

$$\Psi = \beta m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] + \eta(\theta^*)\left[(1-\beta)m(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - a\theta^*\right] + m(\theta^*)(\bar{w}-z)\left[\beta - (1-\eta(\theta^*))\right].$$

Using Equation 3.14 the second term of this expression becomes

$$\begin{aligned} & \frac{\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\left[(1-\beta)\int_{\bar{w}}^{\bar{p}}(p-z)dF(p) - \int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) - (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right] \\ = & \frac{\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\left[(1-\beta)\int_{\bar{w}}^{\tilde{p}}(p-z)dF(p) - \int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p)\right] \\ = & \frac{\beta\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\int_{\bar{w}}^{\tilde{p}}(\tilde{p}-p)dF(p) \end{aligned}$$

where the final equality uses the definition of $\tilde{p}$, Equation 3.13. Substituting this back into Ψ and then substituting Ψ back into $\frac{dW}{d\bar{w}}$ yields the desired representation, Equation 3.19.

8.6 Equation 3.26

Substituting from Equation 3.23, Equation 3.24 and Equation 3.23 into Equation 3.22 yields

$$\begin{aligned} \frac{dW}{db} = & f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (\hat{p}-z)\right] \\ & + \frac{m(\theta^*)\left[(\eta-(1-\beta))\,a + (1-\beta)m'(\theta)(\hat{p}-z)\right]\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (1-F(\hat{p}))\right]}{(1-\eta(\theta^*))a} \end{aligned}$$

Bringing this over the common denominator of $(1-\eta(\theta^*))a$, dropping arguments, setting $(\hat{p}-z) = b$ and separating out the terms yields a numerator of

$$fm(1-\beta)\mathbf{E}_p(1-\eta)a - fmb(1-\eta)a + m\left[\eta-(1-\beta)\right]af\mathbf{E}_p - m\left[\eta-(1-\beta)\right]a(1-F) + mm'b(1-\beta)m'b\left[f\mathbf{E}_p - (1-F)\right].$$

Here $\mathbf{E}_p \equiv \mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})$. The first 3 terms can be written as

$$fma\left[\eta\beta\mathbf{E}_p - b(1-\eta)\right]$$

Let $E_z \equiv \mathbb{E}_{[p\geq\hat{p}]}(p-z)$ then $E_z = E_p - b$ and

$$\eta\beta E_p - b(1-\eta) = \beta\left[\eta E_z - b\right] - (1-\beta)(1-\eta)b$$

Substitution back into the original expression yields the desired representation.

8.7 Existence of optimal b.

At some point b can get so large that financing it costs more than the total output of the economy. This occurs when

$$b\left(F(z+b)+[1-m(\theta)]\,[1-F(z+b)]\right) \\ > zF(z+b)+\int_{z+b}^{\bar{p}}[m(\theta)p+(1-m(\theta))z]\,dF(p),$$

which puts an upper bound b_M on b. As equilibrium welfare is continuous in b, when it is also increasing at $b=0$, the theorem of the maximum implies that an optimal value of b exists between 0 and b_M.

9 Appendix B: Dynamic model proofs and derivations

9.1 Derivation of Equation 4.3 and Equation 4.4

The implied Hamiltonian is,

$$\mathcal{H}=[1-F(\hat{p}_t)-u_t]\,\mathbb{E}_{[p\geq\hat{p}_t]}(p)+[F(\hat{p}_t)+u_t]\,z-\theta_t u_t a \\ +\mu_t\left[\lambda\left(1-F(\hat{p}_t)-u_t\right)-m(\theta_t)u_t\right] \quad (9.1)$$

where μ is a co-state variable. The necessary conditions for an optimum are

$$\frac{\partial\mathcal{H}}{\partial\theta_t}=0,\quad \frac{\partial\mathcal{H}}{\partial u_t}=r\mu_t-\dot{\mu}_t,\quad \frac{\partial\mathcal{H}}{\partial\hat{p}_t}=0,\quad \frac{\partial\mathcal{H}}{\partial\mu_t}=\dot{u}_t.$$

In a steady state, $\dot{\mu}_t=\dot{u}_t=0$, $\theta_t=\theta$, $\hat{p}_t=\hat{p}$, $u_t=u$ for all t. And, after some simplification, the necessary conditions respectively become,

$$\begin{aligned} a+\mu m'(\theta) &= 0 \\ \mathbb{E}_{[p\geq\hat{p}]}(p-z)+\theta a+\mu\left(r+\lambda+m(\theta)\right) &= 0 \\ \frac{(1-F(\hat{p})-u)}{1-F(\hat{p})}\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-\mathbb{E}_{[p\geq\hat{p}]}(p-z)-\mu\lambda &= 0 \\ \left(\lambda+m(\theta)\right)u-\lambda\left(1-F(\hat{p})\right) &= 0. \end{aligned}$$

Equation 4.3 and Equation 4.4 follow after eliminating u and μ.

9.2 Thick-Market condition

From the derivation of Equation 4.4, welfare is increasing in $\hat{p}$ if and only if LHS of Equation 4.4 is negative. Fixing $\hat{p} = z$ in Equation 4.3 yields the efficient value of θ under that restriction. Then, using Equation 4.3 to substitute out a in Equation 4.4, LHS of Equation 4.4 becomes

$$\frac{(\mathbb{E}[p|p \geq z] - z)\,\lambda m'(\theta)(r - \theta m'(\theta))}{(m(\theta) + \lambda)\,(r + \lambda + m(\theta) - \theta m'(\theta))}.$$

The result follows because $m'(\theta)\theta = m(\theta)\eta(\theta)$.

9.3 Existence and uniqueness of equilibrium

We start with the binding minimum wage economy as it is the most complex.

9.3.1 Equilibrium with binding minimum wage

Define the RHS of Equation 4.11 as Ψ. We need to look at what happens to Ψ as θ approaches both 0 and ∞. Then we will consider what happens between the two extremes.

As $\theta \to 0$: $\lim_{\theta\to 0} \tilde{p}(\theta, \bar{w}) = \frac{\bar{w}-(1-\beta)z}{\beta}$ which is finite so under the assumptions made on the matching function $\lim_{\theta\to 0} \Psi(\theta, \bar{w}) = \infty$.

As $\theta \to \infty$: $\lim_{\theta\to\infty} \tilde{p}(\theta, \bar{w}) = \bar{w}$ so $\lim_{\theta\to\infty} \Psi(\theta, \bar{w}) = 0$.

As $a > 0$, the previous results, along with the fact that $\Psi(., \bar{w})$ is continuous, imply existence of equilibrium. Uniqueness will follow from the monotonicity of Ψ with respect to θ. Now,

$$\frac{d\Psi}{d\theta} = \frac{\partial\Psi}{\partial\theta} + \frac{\partial\Psi}{\partial\tilde{p}}\frac{\partial\tilde{p}}{\partial\theta}.$$

But, given $\bar{w}$, $\tilde{p}(\theta, \bar{w})$ is the bilaterally efficient value of productivity above which firms negotiate wages with workers rather than pay the minimum wage. So, from the envelope theorem, $\frac{\partial\Psi}{\partial\tilde{p}} = 0$. Then,

$$\frac{\partial\Psi}{\partial\theta} = -\frac{a(1-\eta(\theta))}{\theta} - \frac{\beta(1-\beta)m'(\theta)m(\theta)}{\theta\,[1-F(\bar{w})]\,[r+\lambda+\beta m(\theta)]^2}\int_{\tilde{p}}^{\bar{p}} [p - z]\,dF(p) < 0. \tag{9.2}$$

9.3.2 Laissez-faire equilibrium

This is a special case of the binding minimum wage equilibrium in which $\hat{p} = z$.

9.3.3 Equilibrium with UI payments

This is a special case of the binding minimum wage equilibrium in which $\bar{w}$ is zero and z is replaced by $z + b$.

9.3.4 Equilibrium with direct control of $\hat{p}$

This is a special case of the binding minimum wage equilibrium in which $\tilde{p} = \bar{w} = \hat{p}$

9.4 General Thick-Market condition

The Hamiltonian associated with choosing $\hat{p}$ remains Equation 9.1. But θ^* is now obtained from Equation 4.10. The steady state necessary conditions for an optimal participation threshold of ability are

$$\frac{\partial \mathcal{H}}{\partial u} = r\mu, \quad \frac{d\mathcal{H}}{d\hat{p}} \equiv \frac{\partial \mathcal{H}}{\partial \hat{p}} + \frac{\partial \mathcal{H}}{\partial \theta}\frac{d\theta^*}{d\hat{p}} = 0, \quad \frac{\partial \mathcal{H}}{\partial \mu} = 0.$$

The LHS of second condition represents how welfare depends on $\hat{p}$. The first term is the direct effect and the second is the indirect (or general equilibrium) effect. We are interested in evaluating $\left.\frac{d\mathcal{H}}{d\hat{p}}\right|_{\hat{p}=z}$. First, notice that the first and third conditions hold for all $\hat{p}$ and θ. So, from the derivation of Equation 4.3 and Equation 4.4 we obtain

$$u = \frac{(1 - F(z))\lambda}{(\lambda + m(\theta^*))} \quad \text{and} \quad \mu = -\frac{\mathbb{E}_{[p \geq z]}(p - z) + a\theta^*}{(r + \lambda + m(\theta^*))}.$$

Now,

$$\left.\frac{\partial \mathcal{H}}{\partial \hat{p}}\right|_{\hat{p}=z} = \frac{-f(z)}{1 - F(z)} \left\{\mathbb{E}_{[p \geq z]}[p - z]u + (1 - F(z))\,\mu\lambda\right\}.$$

Substituting for u and μ yields,

$$\left.\frac{\partial \mathcal{H}}{\partial \hat{p}}\right|_{\hat{p}=z} = \frac{f(z)\lambda\left[a\theta^*(\lambda + m(\theta^*)) - r\mathbb{E}_{[p \geq z]}(p - z)\right]}{(\lambda + m(\theta^*))(r + \lambda + m(\theta^*))}.$$

Then, using Equation 4.10 we obtain

$$\left.\frac{\partial \mathcal{H}}{\partial \hat{p}}\right|_{\hat{p}=z} = \frac{f(z)\lambda\mathbb{E}_{[p \geq z]}(p - z)\left[m(\theta^*)(1 - \beta) - r\right]}{(\lambda + m(\theta^*))(r + \lambda + \beta m(\theta^*))}.$$

Next,

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z} = -u(a+\mu m(\theta^*)).$$

Substituting for u and μ, and using Equation 4.10 we obtain

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z} = \frac{a(1-F(z))\lambda\left[\beta-(1-\eta)\right]}{(\lambda+m(\theta^*))(1-\beta)}.$$

To obtain $\frac{d\theta^*}{d\hat{p}}$, let

$$\Gamma(\theta,\hat{p}) \equiv m(\theta)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - a\theta\left(r+\lambda+\beta m(\theta)\right)$$

so that from the equilibrium condition, Equation 4.10, $\Gamma(\theta^*,\hat{p})=0$. Then

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.10)} = \frac{-\frac{\partial\Gamma}{\partial\hat{p}}}{\frac{\partial\Gamma}{\partial\theta}}.$$

Now

$$\frac{\partial\Gamma}{\partial\hat{p}} = \frac{m(\theta^*)(1-\beta)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{1-F(\hat{p})}$$

and

$$\frac{\partial\Gamma}{\partial\theta} = m'(\theta^*)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - a\left(r+\lambda+\beta\left[m(\theta^*)+\theta^*m'(\theta^*)\right]\right).$$

Using Equation 4.10, we obtain

$$\begin{aligned}\frac{\partial\Gamma}{\partial\theta} &= \frac{m'(\theta^*)a\theta^*\left(r+\lambda+\beta m(\theta^*)\right)}{m(\theta^*)} - a\left(r+\lambda+\beta\left[m(\theta^*)+\theta^*m'(\theta^*)\right]\right)\\ &= -a\left[(1-\eta(\theta^*))(r+\lambda)+\beta m(\theta^*)\right].\end{aligned}$$

So,

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{\hat{p}=z} = \frac{m(\theta^*)(1-\beta)f(z)\mathbb{E}_{[p\geq z]}(p-z)}{a(1-F(z))\left[(1-\eta(\theta^*))\,(r+\lambda)+\beta m(\theta^*)\right]}. \tag{9.3a}$$

Substituting all of these back into $\frac{d\mathcal{H}}{d\hat{p}}$ yields

$$\begin{aligned}\left.\frac{d\mathcal{H}}{d\hat{p}}\right|_{\hat{p}=z} &= \frac{f(z)\lambda\mathbb{E}_{[p\geq z]}(p-z)}{\lambda+m(\theta^*)}\times\\ &\quad\left[\frac{m(\theta^*)(1-\beta)-r}{r+\lambda+\beta m(\theta^*)} + \frac{m(\theta^*)\left[\beta-(1-\eta)\right]}{(1-\eta(\theta^*))\,(r+\lambda)+\beta m(\theta^*)}\right]\end{aligned}$$

After bringing the contents of the square brackets over a positive common denominator, the numerator becomes

$$\begin{aligned}&(1-\eta)\left[r+\lambda+m(\theta^*)\right]\left[\eta(\theta^*)m(\theta^*)-r\right]\\ &\qquad + m(\theta^*)\left[\beta-(1-\eta(\theta^*))\right]\left(\left[r+\lambda+m(\theta^*)\right]\eta(\theta^*)-r\right).\end{aligned}$$

9.5 Dependence of $\tilde{p}$ and θ^* on β

First, let

$$X \equiv \int_{\bar{w}}^{\tilde{p}} (p-w)dF(p) \quad \text{and} \quad Y \equiv \int_{\tilde{p}}^{\bar{p}} (p-z)dF(p).$$

Then, we can rewrite Equation 4.11 and Equation 4.12 as

$$\begin{aligned} \mathbb{\Gamma} &\equiv m(\theta^*)\left[\frac{X}{r+\lambda} + (1-\beta)\frac{Y}{r+\lambda+\beta m(\theta^*)}\right] - a\theta^*[1-F(\bar{w})] = 0 \\ \Phi &\equiv \beta\left(r+\lambda+m(\theta^*)\right)\tilde{p} - \left(r+\lambda+\beta m(\theta^*)\right)\bar{w} + (1-\beta)(r+\lambda)z = 0. \end{aligned}$$

Taking the total derivative we have

$$\begin{pmatrix} \frac{\partial \mathbb{\Gamma}}{\partial \theta^*} & \frac{\partial \mathbb{\Gamma}}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{pmatrix} \begin{pmatrix} d\theta^* \\ d\tilde{p} \end{pmatrix} = -d\beta \begin{pmatrix} \frac{\partial \mathbb{\Gamma}}{\partial \beta} \\ \frac{\partial \Phi}{\partial \beta} \end{pmatrix}.$$

After dropping the argument in m we obtain

$$\begin{aligned} \frac{\partial \mathbb{\Gamma}}{\partial \theta^*} &= m'\left[\frac{X}{r+\lambda} + (1-\beta)\frac{Y}{r+\lambda+\beta m}\right] - \frac{\beta(1-\beta)m'mY}{(r+\lambda+\beta m)^2} - a[1-F(\bar{w})] \\ &= \left(\frac{\theta^* m' - m}{\theta^*}\right)\left[\frac{X}{r+\lambda} + (1-\beta)\frac{Y}{r+\lambda+\beta m}\right] - \frac{\beta(1-\beta)m'mY}{(r+\lambda+\beta m)^2} \\ &= \left(\frac{\theta^* m' - m}{\theta^*}\right)\left(\frac{X}{r+\lambda}\right) + \frac{(1-\beta)Y\left[\left(\theta^* m' - m\right)(r+\lambda) - \beta m^2\right]}{\theta^*\left(r+\lambda+\beta m\right)^2} < 0 \end{aligned}$$

where the second line uses Equation 4.11. Next,

$$\frac{\partial \mathbb{\Gamma}}{\partial \tilde{p}} = \frac{\left(\tilde{p}-\bar{w}\right)f(\tilde{p})}{r+\lambda} - \frac{(1-\beta)(\tilde{p}-z)f(\tilde{p})}{r+\lambda+\beta m}$$

which is 0 from Equation 4.12.

$$\frac{\partial \Phi}{\partial \theta^*} = \beta(\tilde{p}-\bar{w})m'(\theta^*)$$

and

$$\frac{\partial \Phi}{\partial \tilde{p}} = \beta(r+\lambda+\beta m(\theta^*)).$$

These tell us that the determinant of the Jacobean above is negative.

$$\frac{\partial \Gamma}{\partial \beta} = -\frac{mY}{r+\lambda+\beta m} - \frac{(1-\beta)m^2 Y}{(r+\lambda+\beta m)^2}$$
$$= -\frac{mY(r+\lambda+m)}{(r+\lambda+\beta m)^2} < 0$$

and

$$\frac{\partial \Phi}{\partial \beta} = (r+\lambda+m)\tilde{p} - m\bar{w} - (r+\lambda)z = \frac{(r+\lambda)(\bar{w}-z)}{\beta} > 0.$$

Cramer's rule tells us that

$$\frac{d\theta^*}{d\beta} = \frac{-\begin{vmatrix} \frac{\partial \Gamma}{\partial \beta} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \beta} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}} = \frac{-\begin{vmatrix} - & 0 \\ + & + \end{vmatrix}}{\begin{vmatrix} - & 0 \\ + & + \end{vmatrix}} < 0.$$

And,

$$\frac{d\tilde{p}}{d\beta} = \frac{-\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \beta} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \beta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}}.$$

We know that the denominator is negative and the numerator boils down to

$$\frac{(1-\beta)Y(r+\lambda)(\bar{w}-z)(m-\theta^* m')}{\beta\theta^*(r+\lambda+\beta m)} > 0.$$

9.6 Existence of optimal steady state $\hat{p}$

From Equation 4.1 with $\hat{p}_t = \hat{p}$ for all t, as $\hat{p}$ approaches $\bar{p}$, steady state welfare will converge to z. Meanwhile, welfare evaluated at $\hat{p} = z$,

$$W_z = z\left[F(z) + u_z\right] + \left[1 - F(z) - u_z\right]\mathbb{E}_{[p \geq z]}\left[p\right] - a\theta u_z$$

where u_z is the steady state measure of unemployed workers:

$$u_z = \frac{\lambda(1-F(z))}{\lambda + m(\theta)}.$$

From Equation 4.11,

$$a\theta = \frac{m(\theta)(1-\beta)\mathbb{E}_{[p\geq z]}[p-z]}{(r+\lambda+\beta m(\theta))}.$$

These imply

$$W_z = z + \frac{m(\theta)\left[r+\beta\left(\lambda+m(\theta)\right)\right][1-F(z)]\mathbb{E}_{[p\geq z]}[p-z]}{(r+\lambda+\beta m(\theta))\left(\lambda+m(\theta)\right)} > z.$$

Continuity of W in $\hat{p}$ means that there must be a $\hat{p}_m$ such that for all $\hat{p} > \hat{p}_m$, welfare is below W_z. As welfare is increasing in $\hat{p}$ at $\hat{p} = z$, the extreme value theorem then tells us that there is an optimal value of $\hat{p}$ in $(z, \hat{p}_m)$.

9.7 Welfare analysis for minimum wage

9.7.1 Existence of optimal $\bar{w}$

Follows from existence of optimal $\hat{p}$. Simply replace $\hat{p}$ with $\bar{w}$.

9.7.2 Derivation of equations used to obtain optimal $\bar{w}$.

The relevant Hamiltonian for the optimization problem is identical to that for the Planner (Equation 9.1) with $\hat{p}$ replaced by $\bar{w}$. The necessary conditions for an optimum are then Equation 4.11 along with

$$\frac{\partial\mathcal{H}}{\partial u_t} = r\mu_t - \dot{\mu}_t, \quad \frac{\partial\mathcal{H}}{\partial\bar{w}_t} + \frac{\partial\mathcal{H}}{\partial\theta_t}\left.\frac{d\theta_t}{d\bar{w}_t}\right|_{(4.11)} = 0, \quad \frac{\partial\mathcal{H}}{\partial\mu_t} = \dot{u}_t.$$

From these after imposing steady state we obtain

$$\mathbb{E}_{[p\geq\bar{w}]}\left[p\right] + \theta^* a - z + \mu\left(r+\lambda+m(\theta^*)\right) = 0 \tag{9.4}$$

$$\frac{-f(\bar{w})}{1-F(\bar{w})}\left\{\left[1-F(\bar{w})-u\right]\bar{w} + \mathbb{E}_{[p\geq\bar{w}]}\left[p\right]u - \left(1-F(\bar{w})\right)\left(z-\mu\lambda\right)\right\} - u\left(a+\mu m'(\theta^*)\right)\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.11)} = 0 \tag{9.5}$$

$$\left(\lambda+m(\theta^*)\right)u - \lambda\left(1-F(\bar{w})\right) = 0 \tag{9.6}$$

where μ is the co-state variable on Equation 4.2. To obtain $\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.11)}$ define the RHS of Equation 4.11) as $\Psi(\theta^*, \bar{w})$ then,

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.11)} = \frac{-\left[\frac{\partial\Psi}{\partial\bar{w}} + \frac{\partial\Psi}{\partial\tilde{p}}\frac{\partial\tilde{p}}{\partial\bar{w}}\right]}{\frac{\partial\Psi}{\partial\theta^*} + \frac{\partial\Psi}{\partial\tilde{p}}\frac{\partial\tilde{p}}{\partial\theta^*}} = -\frac{\frac{\partial\Psi}{\partial\bar{w}}}{\frac{\partial\Psi}{\partial\theta^*}} \tag{9.7}$$

where the final equality follows from the envelope theorem because $\tilde{p}$ is the efficient ability level above which wages are negotiated (i.e. $\frac{\partial \Psi}{\partial \tilde{p}} = 0$).

Now, let $\bar{f} \equiv f(\bar{w})$, $\bar{F} \equiv F(\bar{w})$, $\tilde{F} \equiv F(\tilde{p})$, $\bar{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \bar{w}]}[p]$ and $\tilde{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \tilde{p}]}[p]$. Then, after dropping arguments in the matching function and its derivative, Equation 9.4 implies

$$\mu = -\frac{\bar{\mathbf{E}} - z + \theta^* a}{r + \lambda + m}.$$

And, Equation 9.6 implies

$$u = \frac{(1 - \bar{F})\lambda}{m + \lambda}.$$

Substituting for u and μ into Equation 9.5 yields,

$$\bar{f}\left\{(r + \lambda + m)m\bar{w} + r\lambda\bar{\mathbf{E}} - (\lambda + m)\left[\lambda\theta^* a + (r + m)z\right]\right\} + (1 - \bar{F})\lambda\left\{\left[\bar{\mathbf{E}} - z\right]m' - a(r + \lambda + m - \theta^* m')\right\}\frac{\frac{\partial \Psi}{\partial \bar{w}}}{\frac{\partial \Psi}{\partial \theta^*}} = 0 \quad (9.8)$$

From Equation 4.11,

$$\frac{\partial \Psi}{\partial \bar{w}} = \frac{(r + \lambda)\bar{f}\theta^* a - m(\tilde{F} - \bar{F})}{(r + \lambda)\theta^*(1 - \bar{F})}$$

and $\frac{\partial \Psi}{\partial \theta^*}$ is obtained from Equation 9.2 above.

Then

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.11)} = \frac{(r + \lambda)\bar{f}\theta^* a - m(\tilde{F} - \bar{F})}{(r + \lambda)\theta^*(1 - \bar{F})\left\{\frac{a(1-\eta(\theta^*))}{\theta^*} + \frac{\beta(1-\beta)m'(\theta)m(\theta)}{\theta[1-F(\bar{w})][r+\lambda+\beta m(\theta)]^2}\int_{\tilde{p}}^{\bar{p}}[p - z]\,dF(p)\right\}} \quad (9.9)$$

To obtain the optimal minimum wage we numerically substitute this into Equation 9.8.

9.7.3 General Thick-Market condition applies to minimum wage

As the Hamiltonian for the welfare effects of the minimum wage is identical to that of the Planner (and therefore that for direct $\hat{p}$ adjustments), we know that

$$\left.\frac{\partial \mathcal{H}}{\partial \bar{w}}\right|_{\bar{w}=z} = \left.\frac{\partial \mathcal{H}}{\partial \hat{p}}\right|_{\hat{p}=z}$$

and that

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\bar{w}=z} = \left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z}.$$

Using Equation 4.11 when $\bar{w} = z$ from Equation 9.9 we obtain

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.11)} = \frac{m(\theta^*)(1-\beta)f(z)\mathbb{E}_{[p\geq z]}(p-z)}{a\,[1-F(z)]\,[(1-\eta(\theta^*))(r+\lambda)+\beta m(\theta^*)]}. \tag{9.10}$$

Now from Equation 9.3a and Equation 9.10 it is also clear that,

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{\substack{Equation\ 4.11)\\ \bar{w}=z}} = \left.\frac{d\theta^*}{d\hat{p}}\right|_{\substack{Equation\ 4.10)\\ \hat{p}=z}}.$$

9.8 Derivation of equations used to obtain optimal b.

Because b is a transfer the relevant Hamiltonian for the optimization problem is identical to that for the Planner (Equation 9.1) with $\hat{p}$ replaced by $z + b$. The necessary conditions for an optimum are then equilibrium condition, Equation 4.11, along with

$$\frac{\partial \mathcal{H}}{\partial u_t} = r\mu_t - \dot{\mu}_t, \quad \frac{\partial \mathcal{H}}{\partial \mu_t} = \dot{u}_t \quad \frac{\partial \mathcal{H}}{\partial b_t} + \frac{\partial \mathcal{H}}{\partial \theta_t}\left.\frac{d\theta_t}{db_t}\right|_{(4.13)} = 0.$$

From the first and second optimality conditions, after imposing steady state, we obtain

$$\mu = -\frac{\mathbb{E}_{[p\geq\hat{p}]}[p-z]+\theta a}{r+\lambda+m(\theta)}$$

and

$$u = \frac{(1-F(\hat{p}))\,\lambda}{\lambda+m(\theta)}.$$

Now we construct the third optimality condition element by element. First,

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial b} &= f(\hat{p})\left\{\frac{[1-F(\hat{p})-u]\,\mathbb{E}_{[p\geq\hat{p}]}\,(p-\hat{p})}{1-F(\hat{p})} - \mathbb{E}_{[p\geq\hat{p}]}[p-z] - \mu\lambda\right\}\\
\frac{\partial \mathcal{H}}{\partial \theta} &= -u(a+\mu m'(\theta)).
\end{aligned}$$

Now, substitute for μ and u and recognize that

$$\mathbb{E}_{[p\geq\hat{p}]}[p-z] - \mathbb{E}_{[p\geq\hat{p}]}\,(p-\hat{p}) = \hat{p}-z.$$

Then, using Equation 4.13 and letting $\hat{f} \equiv f(\hat{p})$, $\hat{F} \equiv F(\hat{p})$, $m \equiv m(\theta)$, and $\hat{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \hat{p}]}(p - \hat{p})$ we obtain

$$\frac{\partial \mathcal{H}}{\partial b} = \frac{\hat{f}\left[m(1-\beta) - r\right]\lambda \hat{\mathbf{E}}}{(\lambda + m)(r + \lambda + \beta m)} - \frac{(r+m)(\hat{p} - z)}{r + \lambda + m}$$

and

$$\frac{\partial \mathcal{H}}{\partial \theta} = \frac{(1-\hat{F})\lambda m \left\{\left[\beta - (1-\eta)\right](r+\lambda+m)\hat{\mathbf{E}} + \eta(r+\lambda+\beta m)(\hat{p} - z)\right\}}{\theta^*(\lambda+m)(r+\lambda+\beta m)(r+\lambda+m)}$$

After rewriting Equation 4.13 as

$$m(1-\beta)\hat{\mathbf{E}} - a\theta^*(r+\lambda+\beta m) = 0 \tag{9.11}$$

we have,

$$\left.\frac{d\theta^*}{db}\right|_{(4.13)} = \frac{-\frac{\partial \text{LHS}(9.11)}{\partial b}}{\frac{\partial \text{LHS}(9.11)}{\partial \theta^*}}$$

where

$$\frac{\partial \text{LHS}(9.11)}{\partial b} = \frac{m(1-\beta)\left[\hat{f}\hat{\mathbf{E}} - (1-\hat{F})\right]}{1-\hat{F}}$$

and

$$\begin{aligned}\frac{\partial \text{LHS}(9.11)}{\partial \theta^*} &= m'(1-\beta)\hat{\mathbf{E}} - a\left(r+\lambda+\beta m+\beta\theta^* m'\right)\\ &= -\left[(1-\eta)(r+\lambda)+\beta m\right]a.\end{aligned}$$

So,

$$\left.\frac{d\theta^*}{db}\right|_{(4.13)} = \frac{m(1-\beta)\left[\hat{f}\hat{\mathbf{E}} - (1-\hat{F})\right]}{a\left(1-\hat{F}\right)\left[(1-\eta)(r+\lambda)+\beta m\right]}. \tag{9.12}$$

To obtain the optimal value of b, we numerically substitute these into the formula

$$\frac{\partial \mathcal{H}}{\partial b} + \frac{\partial \mathcal{H}}{\partial \theta}\left.\frac{d\theta^*}{db}\right|_{(4.13)} = 0$$

and solve for b. Existence of optimal b follows from the logic of that section in the static model.

10 Appendix C: Additional Material

10.1 Static model: Planner assignment with type knowledge

Here the Planner can send workers to matching locations based on their type. The Planner can also control the number of vacancies created at those locations. As workers with $p \leq z$ contribute at least as much to aggregate welfare by not working, the Planner will not create vacancies at locations at which $p < z$. Moreover, as vacancy creation is costly, the match output has to be expected to cover that cost too. Notice that,

$$\lim_{\theta \to 0} \frac{m(\theta)}{\theta} = m'(0) = 1.$$

This means that for the marginal worker where the firm has to have a guaranteed hire this can happen only when $\theta = 0$. So the actual threshold ability above which the Planner will create vacancies is $a + z$. The ratio of vacancies to workers in each active location, $\theta(p) = v(p)/f(p)$. The Planner effectively chooses $\theta(p)$ to maximize aggregate welfare:

$$W = F(z)z + \int_{z+a}^{\bar{p}} \{m(\theta(p))p + [1 - m(\theta(p))]\, z - a\theta(p)\}\, dF(p)$$

This can be solved at each p to obtain $m'(\theta_p(p))(p - z) = a$ for all p in $(a + z, \bar{p}]$.

This allocation can be decentralized in directed search equilibrium as follows. Firms commit vacancies to a market which is indexed by worker ability, p, the wage, w, and market tightness, θ. A worker of type p then solves

$$\max_{\theta, w} m(\theta)w + (1 - m(\theta))z$$

$$\text{s.t.:} \qquad \frac{m(\theta)}{\theta}(p - w) - a = 0$$

The Lagrangian is

$$\mathcal{L} = m(\theta)w + (1 - m(\theta))z + \mu\,[m(\theta)(p - w) - a\theta]$$

and the first order conditions are

$$m'(\theta)\,(w - z) + \mu\left[m'(\theta)(p - w) - a\right]$$

$$m(\theta) - \mu m(\theta) = 0$$

from which we obtain

$$m'(\theta^*(p))(p - z) = a$$

To avoid negative expected profits firms will not create vacancies in markets with $p \leq z + a$.

It is important to notice the level of commitment required to support this allocation. Firms not only commit to a wage, they commit to hiring only the appropriate worker type for the market they are operating in. Otherwise, it would be in any worker's interest to enter the market in which their ability equals the wage being paid in that market. Ex post the firm would still hire the worker. That is, this market allocation mechanism would not be incentive compatible.

10.2 Static model: Winners and losers

Another way to compare these policies is in terms of who are the winners and who are the losers vis-a-vis laissez-faire. The minimum wage is quite straightforward on this. People with $p \in [0, z]$ are unaffected. Those with $p \in [z, \hat{p}]$ are clearly made worse off. Looking only at Figure 3.1 it looks like those with $p \in [\hat{p}, \tilde{p}]$ see an increase in income but of course those workers' propensity to get a job could be impacted by the change in θ^*. As it happens though, whenever a binding minium wage increases welfare, those workers must be better off – someone has to be and they are the first beneficiaries. Workers with $p > \tilde{p}$ will be better off if θ^* increases with $\bar{w}$. From Equation 3.18 it is clear that this happens at least for values of $\bar{w}$ close to z.

Unfortunately, Figure 3.1 is somewhat deceptive when it comes to UI on this matter as it does not reflect the tax, τ, levied on workers. Balanced budget requires that

$$\tau = [1 - m(\theta_b^*)(1 - F(\hat{p}^*))]\, b$$

where θ_b^* is equilibrium market tightness with UI payments, b, in place. As $\tau < b$, people with $p \in [0, z]$ are better off by $m(\theta_b^*)(1 - F(\hat{p}^*))b$. For people with $p \in (z, \hat{p}]$ their expected income changes by

$$m(\theta_b^*)(1 - F(\hat{p}^*))b - m(\theta_0^*)\beta(p - z)$$

where θ_0^* is the equilibrium market tightness under laissez-faire. This obviously depends on how θ^* changes with b but, for low enough values of p in this range, it will be positive. The net gain to these folks decreases with p but whether it actually goes negative for $p = \hat{p}^*$ will depend on the actual parametric arrangement. For people with $p \in (\hat{p}, \bar{p}]$ their expected income changes by

$$[m(\theta_b^*) - m(\theta_0^*)]\,\beta(p-z) + b\,[1 - F(\hat{p}^*) - \beta]\,.$$

From Equation 3.25 we know that θ^* can decline with b even at $b = 0$ so this effect cannot be signed in general.

From Figure 3.1 we do know that market tightness will be higher with a minimum wage than with UI that implements the same value of $\hat{p}$. Ultimately, exactly who wins and looses by these policies, though, depends on the particular parametric arrangement. What does emerge from the preceding discussion is that the minimum wage benefits higher productivity workers more than it does the low productivity workers. Even when the minimum wage increases welfare, it is never Pareto improving. Meanwhile UI necessarily benefits the low ability workers and could benefit all workers at modest enough payouts.

10.3 Static model: Implementation of constrained efficiency with vacancy subsidies

The question here is whether or not by introducing a vacancy subsidy, s, (also paid for by lump-sum taxation on all workers) combined with either UI or a minimum wage can implement constrained efficiency.

With UI the equilibrium conditions become

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p \geq \hat{p}^*]}(p - z - b) = a - s$$

and

$$\hat{p}^* = z + b.$$

Eliminating b means the policy maker's problem becomes

$$\max_{\hat{p},s,\theta}\left\{zF(\hat{p}) + \int_{\hat{p}}^{\bar{p}}[m(\theta)p + (1-m(\theta))z]\,dF(p) - [1-F(\hat{p})]a\theta\right\}$$
$$\text{s.t.}\quad m(\theta)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p) - [1-F(\hat{p})]\theta(a-s) = 0.$$

It is immediate from this that s can be chosen so that the constraint does not bind and the problem reduces to that of the Social Planner.

Similarly with the minimum wage, the policy maker's problem is now

$$\max_{\bar{w},s,\theta}\left\{zF(\hat{p})+\int_{\bar{w}}^{\bar{p}}[m(\theta)p+(1-m(\theta))z]\,dF(p)-[1-F(\bar{w})]a\theta\right\}$$

$$\text{s.t.}\quad m(\theta)\left\{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p)+(1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right\}-[1-F(\hat{p})]\theta(a-s)=0.$$

Again, with a free choice of s, the constraint will not bind.

With either set of policy instruments, the government can implement constrained efficiency. The only caveat here is that the government has to raise taxes to pay for s (and b in the case of UI). If $z<\tau$ workers who do not participate in the market or who do not get a job could end up with negative utility. Were we to require that $\tau\leq z$, full constrained efficiency may not be achievable.

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