

PhD Macro II: What is a Macro Model?

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Spring 2022

Announcements

- ▶ Today: Introduce basic two-period consumption-savings model.
- ▶ Use to understand what we are doing with macro models.
- ▶ Key: macro models are
 - ▶ difference equations from a convex optimization problem
 - ▶ that are resolved by a specified equilibrium concept.
- ▶ i.e., we specify a what we think the world looks like.
- ▶ Then we show how people would figure out that world.
- ▶ Then we show how those decisions aggregate.

Basic two-period model

- ▶ A (very) basic consumption-savings model:

$$\max_{c_1, a_2, c_2} u(c_1) + \beta u(c_2) \quad (1)$$

$$\text{s.t. } c_1 + a_2 = (1 + r)a_1 + w_1 \quad (2)$$

$$c_2 = (1 + r)a_2 + w_2 \quad (3)$$

- ▶ What is this?:
 - ▶ concave return function (sum of concave functions is concave)
 - ▶ over convex set (budget constraint).
- ▶ Two (philosophical) ways to think about solving this problem:
 1. We are solving a decision problem of an agent, then aggregating to clear markets.
 2. We are deriving a set of difference (cont. time \implies differential) equations and finding an equilibrium.
- ▶ Keep **both** in mind (will return to this later).

Euler Equation

- ▶ We solve this and get an Euler Equation:

$$u'(c_1) = \beta(1 + r)u'(c_2) \quad (4)$$

- ▶ What does this say?
 1. Agents will *allocate* their budget between two periods according to this equation.
 2. This expression tells us the growth path of consumption, given c_0 .
- ▶ Euler equation: absolute, fundamental, key equation in every (dynamic) macro model.
- ▶ Note: Euler equation *need not* be over consumption.
- ▶ Budget constraint tells us path of assets/consumption for a given initial condition.

Euler Equation

- ▶ The Euler Equation tells us the evolution of consumption in an economy.
- ▶ That is, it determines the dynamics.
- ▶ The effect of taxes, the presence of frictions or wedges, adjustment costs, etc. can usually be distilled to the following:

$$u'(c_1) = (1 + \Delta)\beta(1 + r)u'(c_2) \quad (5)$$

- ▶ where Δ is a distortion in the economy, i.e. a friction that prevents the market from realizing the perfectly competitive equilibrium.
- ▶ These features change the marginal utility of consumption over time, and thus distort the path of consumption.

Key Insight II: Portfolio Allocation

- ▶ Let's return to the two-period model:

$$\max_{c_1, a_2, \ell, c_2} u(c_1, \ell) + \beta u(c_2, 1) \quad (6)$$

$$\text{s.t. } c_1 + a_2 = (1 + r)a_1 + w_1(1 - \ell) \quad (7)$$

$$c_2 = (1 + r)a_2 \quad (8)$$

- ▶ Now agents are optimizing over consumption **and** leisure.
- ▶ At first blush, this looks like it could become more difficult.

Portfolio Allocation

- ▶ When we solve this model, we get

$$u_1(c_1, \ell^*) = \beta(1+r)u_1(c_2, 0) \quad (9)$$

- ▶ But also

$$\frac{\partial V}{\partial \ell} = u_2(c_1^*, \ell) - w\lambda = 0 \quad (10)$$

$$u_2(c_1^*, \ell) = wu_1(c_1^*, \ell) \quad (11)$$

- ▶ and

$$c_1 + a_2 = (1+r)a_1 + w_1(1-\ell) \quad (12)$$

- ▶ Now we have an equation that determines dynamics (Euler Equation) & one that gives corresponding change in assets.
- ▶ **And a static** equation that determines the allocation of resources within a period (Portfolio Allocation).
- ▶ A lot of problems boil down to these two equations (possibly more with additional static choices).

Models as Dynamic Systems

- ▶ Two (philosophical) ways one might think about solving this problem:
 1. We are solving a decision problem of an agent, then aggregating to clear markets.
 2. We are deriving a difference equation and finding an equilibrium.
- ▶ Now, we'll briefly discuss the second interpretation.

Neoclassical Growth Model

- ▶ The baseline model for most of modern macro (value function representation):

$$V(k_t) = \max_{c_t} u(c_t) + \beta V(k_{t+1}) \quad (13)$$

$$\text{s.t. } c_t + k_{t+1} = k_t^\alpha + (1 - \delta)k_t \quad (14)$$

- ▶ We have a recursive formulation.
- ▶ What we will solve for:
 - ▶ Euler Equation;
 - ▶ Steady state capital and consumption.

Neoclassical Growth Model

- ▶ The baseline model for most of modern macro (value function representation):

$$V(k_t) = \max_{c_t} u(c_t) + \beta V(k_{t+1}) \quad (15)$$

$$\text{s.t. } c_t + k_{t+1} = k_t^\alpha + (1 - \delta)k_t \quad (16)$$

- ▶ We have a recursive formulation.
- ▶ What we will solve for:
 - ▶ Euler Equation;
 - ▶ Steady state capital and consumption.

Neoclassical Growth Model

$$V_t(k_t) = \max_{c_t} u(c_t) + \beta V_{t+1}(k_{t+1}) \quad (17)$$

$$\text{s.t. } c_t + k_{t+1} = k_t^\alpha + (1 - \delta)k_t \quad (18)$$

► Solving this:

$$\frac{\partial V_t}{\partial c_t} = -\lambda + u'(c_t) = 0 \quad (19)$$

$$\frac{\partial V_t}{\partial k_{t+1}} = -\lambda + \beta \frac{\partial V_{t+1}}{\partial k_{t+1}} = 0 \quad (20)$$

► Envelope condition:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial V_t}{\partial k_t} = \lambda(\alpha k_t^{\alpha-1} + (1 - \delta)) \quad (21)$$

Neoclassical Growth Model

- ▶ FOCs:

$$\frac{\partial V_t}{\partial c_t} = -\lambda + u'(c_t) = 0 \quad (22)$$

$$\frac{\partial V_t}{\partial k_{t+1}} = -\lambda + \beta \frac{\partial V_{t+1}}{\partial k_{t+1}} = 0 \quad (23)$$

- ▶ Envelope condition:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial V_t}{\partial k_t} = \lambda(\alpha k_t^{\alpha-1} + (1 - \delta)) \quad (24)$$

- ▶ Putting these together gives us the Euler Equation:

$$u'(c_t) = \beta(\alpha k_t^{\alpha-1} + (1 - \delta))u'(c_{t+1}) \quad (25)$$

- ▶ This & BC give us dynamics of neoclassical growth model.

Steady State

- ▶ What is a steady state and why do we care?
- ▶ It is challenging in general to characterize the solution to our model:
- ▶ Even if we specify a utility function, it will have no closed form solution unless $\delta = 1$.
- ▶ But we can characterize the solution in the steady-state, i.e., where variables are constant over time:
- ▶ $c_t = c_{t+1} = c^*$, $k_t = k_{t+1} = k^*$.

Steady State

- ▶ But we can characterize the solution in the steady-state, i.e., where variables are constant over time:
- ▶ $c_t = c_{t+1} = c^*$, $k_t = k_{t+1} = k^*$.
- ▶ pick $u(c) = \ln(c)$. then

$$\frac{1}{c_t} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta)) \frac{1}{c_{t+1}} \quad (26)$$

- ▶ In steady state:

$$\frac{1}{c^*} = \beta(\alpha k^{*\alpha-1} + (1 - \delta)) \frac{1}{c^*} \quad (27)$$

- ▶ Why would the Euler Equation in the steady-state only be a function of capital?

Steady State

- ▶ This leaves us with capital:

$$1 = \beta(\alpha k^{*\alpha-1} + (1 - \delta)) \quad (28)$$

$$k^* = \left(\frac{1}{\alpha\beta} - \frac{(1 - \delta)}{\alpha} \right)^{\frac{1}{\alpha-1}} \quad (29)$$

$$k^* = \left(\frac{\alpha\beta}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \quad (30)$$

- ▶ Now consumption from the budget constraint:

$$c^* + k^* = k^{*\alpha} + (1 - \delta)k^* \quad (31)$$

$$c^* = k^{*\alpha} - \delta k^* \quad (32)$$

$$c^* = \left(\frac{\alpha\beta}{1 - \beta(1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left(\frac{\alpha\beta}{1 - \beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} \quad (33)$$

- ▶ Why would consumption be determined by the budget constraint, not the Euler Equation?

Dynamics

- ▶ Outside of steady-state we need to think about dynamics, i.e., how model evolves or fluctuates (in presence of shocks).
- ▶ Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta))c_t \quad (34)$$

$$k_{t+1} = k_t^\alpha + (1 - \delta)k_t - c_t \quad (35)$$

- ▶ We have two dynamic variables: c and k .
- ▶ The behavior of this system will depend on their dynamics.

Dynamics

- ▶ Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta))c_t \quad (36)$$

$$k_{t+1} = k_t^\alpha + (1 - \delta)k_t - c_t \quad (37)$$

- ▶ The behavior of this system will depend on their dynamics.
- ▶ At steady-state:

$$1 = \frac{c_{t+1}}{c_t} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta)) \quad (38)$$

$$1 = \frac{k_{t+1}}{k_t} = k_t^{\alpha-1} + (1 - \delta) - \frac{c_t}{k_t} \quad (39)$$

- ▶ If both hold, we are in steady-state, if not, quantities can vary dynamically.

Dynamics

- ▶ Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta))c_t \quad (40)$$

$$k_{t+1} = k_t^\alpha + (1 - \delta)k_t - c_t \quad (41)$$

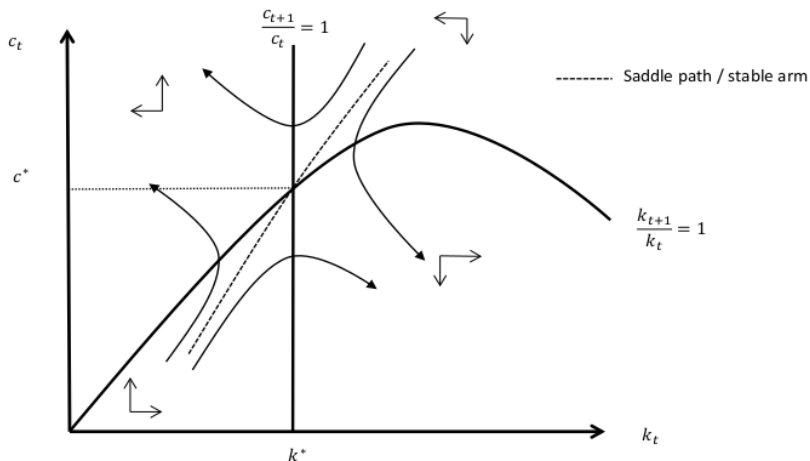
- ▶ Small value of c_t : second equation dictates that $k_t \uparrow$.
- ▶ Small value of k_t : first equation dictates that $c_t \uparrow$.
- ▶ Reverse is true.

Phase Diagram

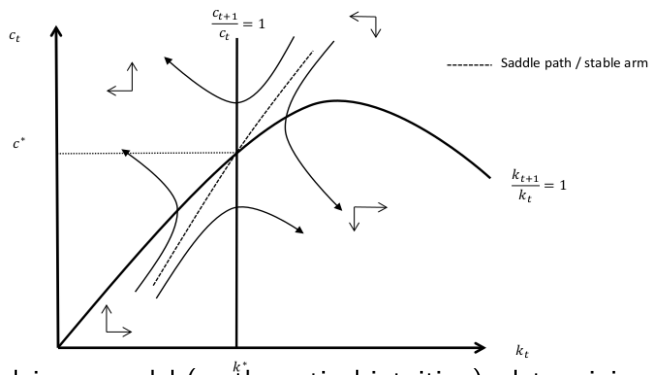
- Dynamics (figure from Eric Sim's notes):

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1 - \delta))c_t \quad (42)$$

$$k_{t+1} = k_t^\alpha + (1 - \delta)k_t - c_t \quad (43)$$



Phase Diagram



- ▶ Solving a model (mathematical intuition): determining rules that put us on the saddle path (dashed line).
- ▶ Same concept for a decentralized economy.
- ▶ Seeing these models as dynamic systems expands our toolbox for solving them.
- ▶ We will discuss this later.

Next Time

- ▶ Discuss important time series preliminaries.
- ▶ Be sure to start Matlab homework.
- ▶ See online for specific assignment.