Worker Selectivity and Fiscal Externalities from Unemployment Insurance*

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Abstract

By making workers more selective, unemployment insurance (UI) increases re-employment wages and thereby generates a positive fiscal externality. We provide a sufficient-statistics formula for evaluating the size of this fiscal externality and argue theoretically that it is likely to be small. In a standard sequential search model, the effect of UI on wages is proportional to its effect on the job-finding hazard; the slope of the relationship between these elasticities depends on a small number of estimable statistics, key among them observed worker flows. Plausible calibrations of the model imply that the magnitude of the wage elasticity is small relative to the job-finding elasticity. Although ignoring the wage effect of UI would over-estimate its fiscal cost and under-estimate its welfare benefit, the model predicts the magnitude of this bias to be small.

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1 Introduction

Unemployment insurance (UI) has the potential to increase re-employment wages by making workers more selective about which jobs they accept. This insight, which follows directly from standard labor market search theory, has profound policy implications. The standard analysis of optimal UI focuses on the tradeoff between its consumption smoothing benefits and its search disincentive costs. However, if UI increases wages and is itself financed with proportional taxes, this effect on wages amounts to a positive fiscal externality, which alters the optimal provision of UI. This paper seeks to evaluate the magnitude of this effect.

The natural approach to quantifying the wage effect of UI is, of course, to estimate it directly in the data. In this paper, we propose an alternative, complementary approach, which suggests a *model-based* formula for the size of the wage elasticity and does not require estimating it directly. We believe such a model-based approach is useful and complements the existing empirical literature. While there is robust evidence regarding the effects of UI on re-employment rates, the literature estimating the wage effects of UI, while having made enormous progress, is less abundant and, in our reading, has reached less of a consensus: see e.g. Card *et al.* (2007), Lalive (2007), Van Ours and Vodopivec (2008), Schmieder *et al.* (2016), Nekoei and Weber (2017), Jäger *et al.* (2020), and Griffy (2021). Closely related is the fact that measuring the fiscal externality linked to this wage effect ideally requires knowing the *long-term* effects of UI on wages, which are even more difficult to obtain. In short, there are reasons to think that in many data settings, the effect of UI on the job-finding hazard is easier to estimate than its effect on subsequent wages, in particular its long-term effect.

Our approach instead provides a way to infer the effect of UI on wages theoretically. We show that the same search model that implies the existence of the wage effect also turns out to place very particular restrictions on its magnitude, given the value of other observables. We use a classic framework: workers sample wage offers sequentially, deciding to accept or reject, and can also search on the job while employed. The central insight is that, in such a framework, workers face a tradeoff between the rate at which they expect to find a job and the wage that they expect to receive. Unemployment insurance alters this tradeoff by making workers more selective about the wages they accept, thereby raising the average accepted wage while lowering the job-finding probability. This immediately implies that, to the extent that an increase in UI raises re-employment wages, it must also lower the job-finding probability. We confirm that intuition by deriving a simple sufficient statistics formula that analytically links the effect of UI on wages (henceforth the *wage elasticity*) to its effect on the job-finding probability (*job-finding elasticity*). In other words, our formula gives the *slope* of the wage/job-finding tradeoff faced by workers.

We show analytically that two channels determine the slope of this tradeoff. First, it depends on how dispersed wage offers are. If wages are very dispersed, a small increase in the worker's reservation wage can generate a large increase in the average accepted wage without a large accompanying drop in the job-finding rate. The opposite occurs when wages are very concentrated. Our formula shows, in fact, that the wage elasticity is proportional to the job-finding elasticity; the factor of proportionality depends on a particular measure of wage dispersion - the mean-min ratio (i.e. the ratio of the mean wage to the reservation wage). This is very convenient because, as shown by Hornstein *et al.* (2011), this particular measure of wage dispersion itself admits a simple sufficient statistics formula, which depends on the replacement rate of unemployment insurance and easily estimable worker flows. In fact, Hornstein *et al.* (2011) show that this characterization puts an upper bound on the model-implied mean-min ratio. In our setting this implies, all else equal, an upper bound on the wage elasticity for a given job-finding elasticity. Intuitively, workers find jobs rather quickly in the data. Disciplined by this statistic, the model implies that workers do not have much to be selective about. Hornstein *et al.* (2011) use this "unpleasant search arithmetic" to argue that frictional wage dispersion cannot be very large. We take this logic a step further by exploring its implications for the effects of unemployment insurance - a link that is, to our knowledge, novel in the literature.

Second, the slope of the wage/job-finding tradeoff depends on the efficacy of on-the-job search. Unemployment insurance raises initial re-employment wages through increased worker selectivity, but its impact on the average wage is muted by the speed with which workers progress up the job ladder after their initial job placement. If on-the-job search is fast relative to separations from unemployment - as is the case in the data - the model predicts that initial wages have a rather small effect on steady-state wages. This second channel (on-the-job search) is of crucial importance because of how it interacts with the first (wage dispersion). Our formula implies that all else equal, the wage elasticity is larger (i) the larger is wage dispersion, and (ii) the smaller is the efficacy of on-the-job search. As was shown by Hornstein *et al.* (2011), on-the-job search goes a long way in helping the model generate larger wage dispersion. Our result implies that this *does not* translate into a larger wage elasticity, precisely because faster on-the-job search also generates the second, offsetting effect. On-the-job search allows for higher wage dispersion, hence a higher effect of UI on accepted wages, but it also mutes the long-term effect of higher initial wages. On net, therefore, our formula implies that the wage elasticity cannot be too large, regardless of whether on-the-job search is fast or slow.

We next apply our result to quantify the fiscal cost and therefore the welfare gain from unemployment insurance, providing an extension of the standard Baily-Chetty (Baily (1978), Chetty (2006)) formula. Assuming that UI is financed by a proportional tax, the welfare gain from UI now depends on both the job-finding elasticity and the wage elasticity, but our result implies that the latter is proportional to the former. In our formula, the wage effect of UI thus shows up simply as a wedge on the job-finding elasticity; this wedge depends on a number of estimable statistics but does not require estimating the wage elasticity directly.

Finally, we numerically assess the importance of this wedge. In doing so, we seek to answer the following question. Suppose that a researcher computed the fiscal cost of UI and its resulting welfare benefit by applying the Baily-Chetty formula, but mistakenly assumed that UI had no effect on wages. By how much would they overstate the fiscal cost of UI and understate its welfare benefit? Our results imply that the magnitude of this bias, for either the fiscal cost or the welfare benefit, would not be very large. Ignoring the wage effect of UI would overestimate the fiscal cost of increasing UI by 3-6%, and underestimate the welfare gain from increasing UI by 1-7%. This is because, for a plausible range of parameter values, we find that the wage elasticity is about 1/10 of the job-finding elasticity. Moreover, our numerical results imply that worker ability to search on the job only dampens the magnitude of the wage effect, by making initial job placement less consequential for average wages.

This paper proceeds as follows. In section 2, we lay out the basic model environment. Section 3 contains our main result. Section 4 draws its implications for welfare gains from UI, and section 5 describes our parameter calibration and numerical results. Section 6 concludes and discusses the implications of our results in context of the existing literature.

2 Environment and preliminaries

In this section we lay out the model environment, which largely follows the conventional sequential search model.[1] Time is continuous, and the time horizon is infinite. There is a continuum of workers, each of whom evaluates consumption streams according to

$$\mathbb{E}\int_0^\infty e^{-rt} v\left(c\left(t\right)\right) dt \tag{1}$$

where $r > 0$ is the discount rate, and the flow utility of consumption v satisfies $v' > 0$, $v'' \leq 0$. When unemployed, the worker receives wage offers at Poisson rate λ_u, which are drawn from a cumulative distribution F with density f. When employed, the worker receives wage offers at Poisson rate $\lambda_e < \lambda_u$, which are also drawn from F. An employed worker becomes unemployed at Poisson rate δ. Workers do not save or borrow. An unemployed worker receives government-provided unemployment benefits b. Employed workers are taxed at a proportional rate τ, so that a worker employed at wage w receives consumption $(1-\tau)\,w$.

Let U be the value of an unemployed worker, and let $W(w)$ be the value of a worker employed at wage w. These values are given, respectively, by the Bellman equations

$$rU = v\left(b\right) + \lambda_u \int_0^\infty \max\left\{W\left(w\right) - U, 0\right\} dF\left(w\right) \tag{2}$$

and

$$\left(r+\delta\right) W\left(w\right) = v\left(\left(1-\tau\right) w\right) + \delta U + \lambda_e \int_0^\infty \max\left\{W\left(w'\right) - W\left(w\right), 0\right\} dF\left(w'\right) \tag{3}$$

It is standard to show that $W(w)$ is increasing in w, and therefore an employed worker switches jobs whenever $w' > w$, and the unemployed worker's job acceptance decision rule

[1] All the derivations are included in the Appendix to make the analysis self-contained, but these derivations are standard in the literature.

is characterized by a reservation wage, denoted by w_R. This reservation wage is the solution to $W(w_R) = U$. We show in Appendix A.1 that this reservation wage satisfies

$$v((1-\tau)w_R) = v(b) + (\lambda_u - \lambda_e)\int_{w_R}^{\infty} \frac{(1-\tau)v'((1-\tau)x)(1-F(x))}{r+\delta+\lambda_e(1-F(x))}dx \tag{4}$$

Given w_R, we can proceed to define several key equilibrium objects. First, the job-finding rate, denoted by h_u, is equal to

$$h_u = \lambda_u(1-F(w_R)) \tag{5}$$

and the steady-state unemployment rate is then given by

$$u = \frac{\delta}{\delta+\lambda_u(1-F(w_R))} \tag{6}$$

Next, define G to be the steady-state cumulative distribution of wages among employed workers. We show in Appendix A.2 that G is given by

$$G(w) = \frac{\delta}{\delta+\lambda_e(1-F(w))} \cdot \frac{F(w)-F(w_R)}{1-F(w_R)} \tag{7}$$

The average steady-state wage across employed workers, denoted $\overline{w}$, is then given by

$$\overline{w} = \int_{w_R}^{\infty} w dG(w) \tag{8}$$

3 The wage/job-finding tradeoff: main result

Our main result concerns the comparative statics of $\overline{w}$ and h_u with respect to b. Define the elasticities

$$\epsilon_{h,b} \equiv \frac{\partial \ln h_u}{\partial \ln b}; \quad \epsilon_{w,b} \equiv \frac{\partial \ln \overline{w}}{\partial \ln b}$$

Differentiation of (5) and (8) then gives

Proposition 1 *The elasticities of the job-finding rate and average wage with respect to b satisfy*

$$\epsilon_{w,b} = -\frac{1}{1+\kappa_e}\left(\frac{\mu-1}{\mu}\right)\epsilon_{h,b}, \tag{9}$$

where $\mu = \overline{w}/w_R$ *and* $\kappa_e = \lambda_e(1-F(w_R))/\delta$.

Proof. See Appendix A.3. ■

Interpretation of the formula (9). While simple, the formula in (9) is rich in economic intuition. To start with, it shows that $\epsilon_{w,b}$ is proportional to $\epsilon_{h,b}$. This indicates that the worker faces a wage/job-finding tradeoff: to the extent that an increase in b raises the accepted wage, it must also lower the job-finding probability. In turn, the ratio between the wage elasticity $\epsilon_{w,b}$ and the job-finding elasticity $\epsilon_{h,b}$ is shown to depend on two key statistics. First, it depends on how dispersed wages are, as captured by the mean-min ratio μ. All else equal, if wages are very concentrated, a given increase in b would lead to a lower job-finding rate without much of an increase in the average accepted wage; the opposite is true if wages are very dispersed. Second, it depends on the efficacy of on-the-job search relative to the job separation rate, as captured by the quantity κ_e. After an unemployed worker finds a job, they climb the job ladder via on-the-job search, a process interrupted by job separations. If κ_e is large, upward job switches are frequent relative to separations back into unemployment; in this case, the average steady-state wage is not very sensitive to the initially accepted re-employment wage, and hence not very sensitive to unemployment insurance.

Discussion of assumptions. It is instructive to note that the formula (9) relies on a rather minimal set of assumptions. In particular, it uses the fact that workers follow a reservation-wage rule, but not the fact that the reservation wage satisfies (4) (which we do, however, use below to characterize μ). In essence, equation (9) follows from the mathematical link between the objects $Prob\,(w \geq w_R)$ and $\mathbb{E}\,(w|w \geq w_R)$, which implies that the comparative statics of these two objects are also linked.

The crucial assumption is that unemployment benefits affect both $\overline{w}$ and h_u only through the reservation wage. Let us consider the most prominent violations of this assumption in the literature. First and foremost, the job-finding probability could depend on search effort, which responds to unemployment benefits. This is by far the most common way of modeling the distortionary effects of UI in the literature. In our framework, accounting for such an effect would amount to having λ_u endogenous and dependent on effort; in this case an increase in b would reduce λ_u in addition to increasing w_R, thus generating an additional reduction in h_u unaccompanied by an increase in $\overline{w}$. It is then clear that the "=" in equation (9) would need to be replaced by "$\leq$," as the job-finding elasticity is now larger (in absolute value) for the same wage elasticity. Second, the distribution of wages F faced by the worker may also not be invariant to b. In fact, as forcefully argued by Schmieder *et al.* (2016) and Nekoei and Weber (2017), human capital depreciation over the course of the unemployment spell plays an important role in rationalizing small wage elasticities observed in some studies. If human capital depreciates with unemployment duration, then higher UI, by raising unemployment duration, can worsen the distribution of future wage offers. If this is the case, the wage elasticity would once again be smaller than implied by (9). We conclude that the most prominent relaxations of our assumptions would imply that (9) likely provides an *upper bound* on the wage elasticity, given the job-finding elasticity.

Characterizing the mean-min ratio. To make further progress, we derive an expression for μ following the procedure in Hornstein *et al.* (2011). That paper shows that, in a sequential search setting, the mean-min ratio of wages is tightly linked in equilibrium to several estimable statistics, notably the replacement rate of non-market activity and the magnitudes of worker flows.[2] Hornstein *et al.* (2011) derive their formula for μ for a model with risk-neutral workers, as well as for a model with risk-averse workers and no on-the-job search. Fortunately, their technique extends in a straightforward way to the environment with both risk aversion and on-the-job search, which yields the following result.

Lemma 1 *Assume that utility takes the CRRA form,*

$$v(x) = \frac{x^{1-\gamma} - 1}{1-\gamma}, \quad \gamma \geq 0 \tag{10}$$

If $r \approx 0$, *then the mean-min ratio* $\mu = \overline{w}/w_R$ *satisfies*

$$\mu \approx \left[\frac{1 + \frac{\kappa_u - \kappa_e}{1+\kappa_e}}{\rho^{1-\gamma} + \frac{\kappa_u - \kappa_e}{1+\kappa_e} \left(1 + \frac{1}{2}\gamma(\gamma-1)\xi^2\right)} \right]^{\frac{1}{1-\gamma}}, \tag{11}$$

where $\rho = b/((1-\tau)\overline{w})$, $\kappa_u = \lambda_u (1 - F(w_R))/\delta$, $\kappa_e = \lambda_e (1 - F(w_R))/\delta$, *and*

$$\xi = \frac{\sqrt{var(w)}}{\overline{w}}$$

is the coefficient of variation of wages.

Proof. See Appendix A.4. ■

As in Hornstein *et al.* (2011), the key lesson from the expression in (11) is that observed worker flows put a lot of discipline on the model-implied mean-min ratio. If workers find jobs relatively quickly (as would be represented by a high κ_u), this indicates that they do not have much to be selective about, which implies that wages are not very dispersed. In turn, if this is the case, our formula (9) would imply that making workers slightly more selective would not raise their wages much, or in other words, a given decrease in h_u would be associated with only a modest increase in $\overline{w}$.

On-the-job search, as captured by κ_e, works in the opposite direction: if workers can continue to search on the job even while employed, they will be willing to accept jobs quickly even if there are much better jobs available. As a result, a higher κ_e can reconcile a high κ_u with a high μ. Importantly, however, while fast on-the-job search, all else equal, implies a higher mean-min ratio, it does not imply a higher $\epsilon_{w,b}$. This is precisely because, as shown in (9), a higher κ_e also has an independent, offsetting effect on the wage elasticity. It allows the model to be consistent with wider wage dispersion, but it also mutes the importance

[2] An alternative is to estimate the mean-min ratio directly from the data, though this requires taking a stand on what component of observed wage dispersion is frictional; see e.g. Hornstein *et al.* (2007). We instead take advantage of the analytical characterization of the mean-min ratio afforded by the model itself.

of initially accepted wages for average steady-state wages. So, while wage dispersion is of crucial importance for the magnitude of the wage elasticity, a high value of the former is not sufficient for a high value of the latter.

4 Fiscal cost and welfare gain from UI

In this section, we apply the formula derived above to quantify the fiscal externality from an increase in unemployment insurance and, consequently, its effect on welfare. As is common in the literature, consider a worker who starts out unemployed. A benevolent government is maximizing the discounted expected utility U of the worker by choosing τ and b, subject to the worker's optimal behavior, captured by (4), and subject to the budget constraint. The budget constraint states that the present discounted value of taxes collected from the worker must equal in expectation to the present discounted value of unemployment benefits paid to the worker. When $r \approx 0$, this problem is equivalent to maximizing the average steady-state flow utility,

$$uv(b) + (1-u)\int_0^\infty v((1-\tau)w)dG(w) \tag{12}$$

subject to the steady-state budget constraint

$$(1-u)\,\tau\overline{w} \approx ub \tag{13}$$

and subject to (4), (7), and (8). We show this equivalence formally in Appendix A.5. The approximation holds because the present discounted utility of an individual worker is approximately equal to steady-state average flow utility; similarly, the value of wages the worker expects to receive over the lifetime is approximately the same as the average wage in the cross-section in steady state.[3] From the expression (6) for the steady-state unemployment rate, (13) can further be rewritten as

$$h_u\tau\overline{w} \approx \delta b \tag{14}$$

We are interested in calculating $\frac{dU}{db}$, the welfare gain from an increase in b, taking into account that τ is a function of b through (14) and w_R is a function of b and τ through (4).[4] In Appendix A.6, we show that the welfare gain per unemployed worker satisfies

$$\frac{1}{u}\frac{dU}{db} \approx v'(b) - \epsilon_{\tau,b}\int_{w_R}^\infty \left(\frac{w}{\overline{w}}\right) v'\left((1-\tau)\,w\right)dG\left(w\right) \tag{15}$$

[3] In particular, this means that focusing on a worker who is initially unemployed is without loss of generality.

[4] The sufficient statistics formula that is derived here is, as usual, a local result; therefore, we are making local statements about the effects of a UI increase rather than statements about the globally optimal UI level.

where the elasticity $\epsilon_{\tau,b} = \frac{b}{\tau}\frac{d\tau}{db}$ is obtained by treating τ as a function of b in (14). This formula, similar to the prior literature, shows that the welfare gain from increasing UI equals to its consumption benefit minus its average consumption cost to the employed due to increased taxes. To get a welfare metric in consumption terms that is comparable across calibrations, we normalize by the average marginal utility of the employed, defining the normalized welfare gain as

$$\mathcal{W}_b = \frac{v'(b) - \epsilon_{\tau,b} \int_{w_R}^{\infty} \left(\frac{w}{\bar{w}}\right) v'((1-\tau) w)\, dG(w)}{\int_{w_R}^{\infty} v'((1-\tau) w)\, dG(w)} \tag{16}$$

In order to compute $\mathcal{W}_b$, we must first compute $\epsilon_{\tau,b}$, the fiscal cost of an increase in UI. We note from (14) that

$$\epsilon_{\tau,b} = 1 - \epsilon_{h,b} - \epsilon_{w,b} \tag{17}$$

Rather than separately estimate both $\epsilon_{h,b}$ and $\epsilon_{w,b}$ directly, we can now take advantage of the fact that $\epsilon_{h,b}$ and $\epsilon_{w,b}$ are linked analytically by (9). It follows that we can rewrite (17) as

$$\epsilon_{\tau,b} = 1 - (1 - \Phi)\, \epsilon_{h,b}, \tag{18}$$

where $\Phi = \frac{1}{1+\kappa_e}\left(\frac{\mu - 1}{\mu}\right)$ from Proposition 1, and μ is furthermore described by the characterization in Lemma 1. The key components necessary for computing the wedge Φ are the replacement rate of unemployment benefits and measures of worker flows, as well as the coefficient of variation of wages, all of which can be calibrated from available data. With regard to elasticities, only $\epsilon_{h,b}$ needs to be estimated.

5 Numerical analysis

We now proceed to implement the formulas in (18) and (16) numerically in order to evaluate the two key objects of interest: the marginal fiscal cost of UI, measured by $\epsilon_{\tau,b}$, and the marginal welfare gain from UI, measured by $\mathcal{W}_b$. We then use our numerical results to conduct the following counterfactual thought experiment. Suppose that a researcher computed the marginal fiscal cost and marginal welfare gain from UI by directly using equation (17) for $\epsilon_{\tau,b}$, but mistakenly assumed that $\epsilon_{w,b} = 0$. Such a calculation would overstate the marginal fiscal cost of UI and understate its marginal welfare benefit. How large would the magnitude of this bias be?

We assume, as above, that the utility function v is of the CRRA form (10), with risk-aversion parameter γ. A Taylor expansion procedure standard in the literature[5] gives

$$\mathcal{W}_b \approx 1 + \gamma (1 - \rho) - \epsilon_{\tau,b} \tag{19}$$

[5]See e.g. Baily (1978), Gruber (1997), Chetty (2006, 2009). Appendix A.6 contains the derivation of the approximate expression (19).

where $\rho = b/((1-\tau)\overline{w})$ is the replacement rate of UI with respect to the average after-tax wage. As explained above, $\epsilon_{\tau,b}$ can be computed according to the formula (18). We then compute, for the same parameter values, the "mis-specified" welfare gain

$$\widetilde{\mathcal{W}}_b \approx 1 + \gamma(1-\rho) - \widetilde{\epsilon}_{\tau,b} \tag{20}$$

where $\widetilde{\epsilon}_{\tau,b}$ is the fiscal externality a researcher would compute if ignoring the effect of UI on wages, i.e. if they used the same $\epsilon_{h,b}$ but setting $\epsilon_{w,b} = 0$:

$$\widetilde{\epsilon}_{\tau,b} = 1 - \epsilon_{h,b} \tag{21}$$

We then measure the importance of the wage effect by inspecting the magnitudes of $\epsilon_{\tau,b}/\widetilde{\epsilon}_{\tau,b}$ for the fiscal cost, and $\mathcal{W}_b/\widetilde{\mathcal{W}}_b$ for the welfare gain.

5.1 Calibration

Our baseline choices of parameter values come from observed worker flows and the existing empirical literature. Using data on monthly job-finding rates, job separation rates, and job-to-job transitions, we estimate $\kappa_u = 14.3$ and $\kappa_e = 2.3$ as defined in Lemma 1. Details of the calibration of these two parameters are provided in Appendix B. The coefficient of variation of wages, ξ, is set to 0.5, consistent with the upper bound of the range of estimates in Hornstein *et al.* (2007). For the baseline value of the replacement rate, we adopt the commonly used value of $\rho = 0.4$.[6] We set the elasticity of unemployment duration with respect to UI to the standard estimate from Chetty (2008): a 10% increase in unemployment benefits is associated with a 5% decrease in the job-finding hazard, so that $\epsilon_{h,b} = -0.5$. Our baseline value for the risk aversion parameter is $\gamma = 2$. Nonetheless, since there is not a consensus on the appropriate value of risk aversion, we conduct sensitivity analysis for a wide range of values for γ. Similarly, we conduct robustness checks with respect to the other model parameters; when varying each parameter, the others are kept at their baseline values.

5.2 Numerical results

For our baseline parameter calibration described above, we obtain a wedge of $\Phi \equiv -\epsilon_{w,b}/\epsilon_{h,b} = 0.1$, implying a wage elasticity equal to $\epsilon_{w,b} = 0.05$. This then implies a fiscal elasticity of $\epsilon_{\tau,b} = 1.44$ and a welfare gain of $\mathcal{W}_b = 0.75$. The mis-specified model ignoring the wage effect of UI, as described above, would result in $\tilde{\epsilon}_{\tau,b} = 1.5$ and a corresponding welfare gain of $\widetilde{\mathcal{W}}_b = 0.7$. In other words, a 1% increase in b raises the average steady-state wage by approximately 0.05%, or about 1/10 the amount by which it lowers the job-finding proba-

[6] Setting $\rho = 0.4$ amounts to assuming, as we did in this paper, that the only source of consumption during unemployment is unemployment insurance. We note that higher values of ρ would only imply lower values of μ and therefore lower values of the wage elasticity, all else equal. See the discussion of figures 1c and 1d below.

bility. A researcher mistakenly assuming a zero wage effect of UI would overestimate the marginal fiscal cost of UI by about 3.5% and underestimate its marginal welfare benefit by about 6.8%.

Figure 1 illustrates the model-implied effects of UI for alternative model parameterizations. The top-left graph, figure 1a, displays the model-implied elasticity of taxes with respect to b under various values of risk aversion, keeping all other parameters fixed. The curve labeled "True" displays the tax elasticity $\epsilon_{\tau,b}$ given by (18). The curve labeled "Misspecified" displays the tax elasticity $\tilde{\epsilon}_{\tau,b}$ given by (21), i.e. the tax elasticity implied by the model if ignoring the effect of UI on wages. The wedge resulting from the wage effect is small, even for very large values of risk aversion; a researcher ignoring the effect of UI on wages would overestimate its marginal fiscal cost by at most 6%. The size of the wedge is even smaller for more conservative values of risk aversion. Figure 1b displays the implications for welfare. The curves labeled "True" and "Misspecified" display the values of $\mathcal{W}_b$ (given by (19)) and $\widetilde{\mathcal{W}}_b$ (given by (20)), respectively. The bias resulting from neglecting the wage effect of UI is likewise small in this case. Figures 1c and 1d illustrate that this result is also robust to the chosen value of the replacement rate ρ.[7]

Of particular interest are Figures 1e and 1f, which show how the model-implied effect of UI depends on the relative efficacy of on-the-job search, κ_e. They illustrate, in particular, that the wage effect of UI and the welfare gain from UI are largest when this efficacy of on-the-job search is low. Intuitively, when κ_e is small, a better initial re-employment wage is more consequential since it persists for a longer period of time. Note that this occurs despite the fact that a higher κ_e would make the model consistent with higher wage dispersion, which, as explained above, would (all else equal) amplify the effect of UI on wages. This numerical result indicates that the former effect outweighs the latter, showcasing a tension between the model's ability to generate large wage dispersion and its ability to generate a large wage elasticity.

6 Discussion

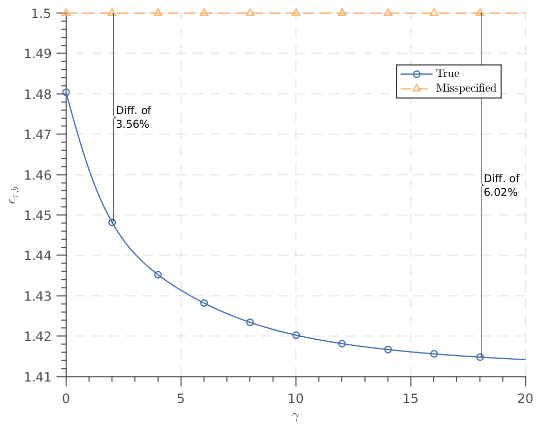
The potential of UI to improve re-employment job quality has long been recognized in principle, and is an important part of the discussion of its optimal design. A growing literature (Card *et al.* (2007), Lalive (2007), Van Ours and Vodopivec (2008), Schmieder *et al.* (2016), Nekoei and Weber (2017), Jäger *et al.* (2020), Griffy (2021)) has addressed the wage effects of UI empirically. This paper complements this literature by more deeply examining this effect theoretically. Our focus is on the relationship between two elasticities: the wage elasticity and the job-finding elasticity. The main message is that the standard search framework puts sharp restrictions on what *combinations* of the two elasticities are consistent with the

[7]Note that the wedge is lower for higher values of ρ. Our chosen value of $\rho = 0.4$ is likely to be a conservative (i.e. lower-bound) value for unemployment consumption, both in light of the business cycle literature (Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Hornstein *et al.* (2011)) and because it abstracts from sources of unemployment consumption other than UI (see e.g. Aguiar and Hurst (2005)).

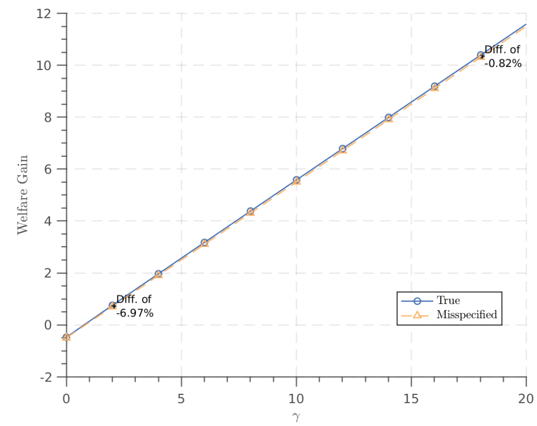
theory. Among other things, our result implies that - from a theoretical point of view - one should not be surprised to find small wage elasticities in the data, even in the absence of duration-dependence effects on human capital highlighted by Schmieder *et al.* (2016) and Nekoei and Weber (2017). Importantly, however, the search framework does not rule out a large wage elasticity per se; instead, it puts bounds on the wage elasticity for any given job-finding elasticity. This is important, since the *relative* magnitude of the two elasticities is important for optimal UI.

Our result also draws a clear connection between the fiscal externality from UI and the "unpleasant search arithmetic" of Hornstein *et al.* (2011), a connection that, to our knowledge, is new to the literature. If wages are not very dispersed, then workers do not have much to be selective about; as a consequence, little can be gained by making workers more selective. Because the standard search framework places bounds on model-implied wage dispersion, it also places bounds on the positive fiscal externality from UI. Finally, our formula draws attention to the central role of on-the-job search in driving the effects of UI on average wages. A high rate of on-the-job search relative to separations into unemployment mutes the importance of initial wages for steady-state wages, because workers manage to "escape" bad initial job placement quickly. This underscores the importance on accounting for job-to-job transitions in studying optimal UI.

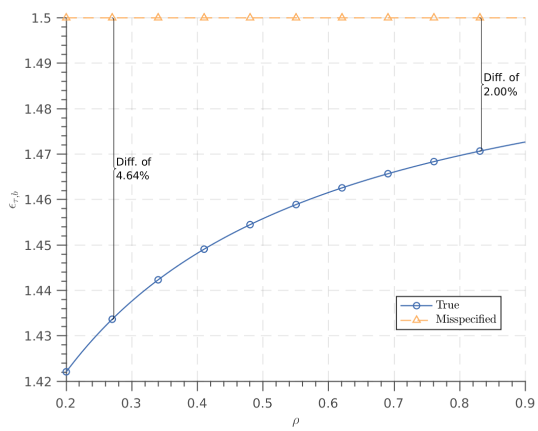
All of these results beg the question of how the model would need to be modified in order to generate larger wage elasticities. First, we note that - as discussed earlier - the most obvious modifications of the standard model would in fact imply that our formula provides an *upper bound* on the wage elasticity for a given job-finding elasticity. For example, if UI also reduces search intensity, this reduces the job-finding rate without an accompanying increase in wages. In addition, if, as suggested by Schmieder *et al.* (2016) and Nekoei and Weber (2017), human capital also depreciates over the unemployment spell, the longer unemployment duration generates a negative effect on wages that would offset the positive match quality effect of higher selectivity. In both cases, the model-implied wage elasticity, for a given job-finding elasticity, would be smaller than implied by our formula. Generating a higher wage elasticity would require UI to increase wages without an accompanying decrease in the job-finding rate. This may be the case, e.g. if wages are bargained, so that UI has a direct effect on wages through the worker's outside option; we should note, however, that the recent findings of Jäger *et al.* (2020) call this channel into question. Finally, an intriguing question is how alternative models of job mobility would affect the conclusions here. As we have argued, the very classic search framework implies that on-the-job search dampens the long-run effects of initial job placement. Our findings suggest that alternative models of how workers climb the job ladder would have quite profound implications for optimal UI.



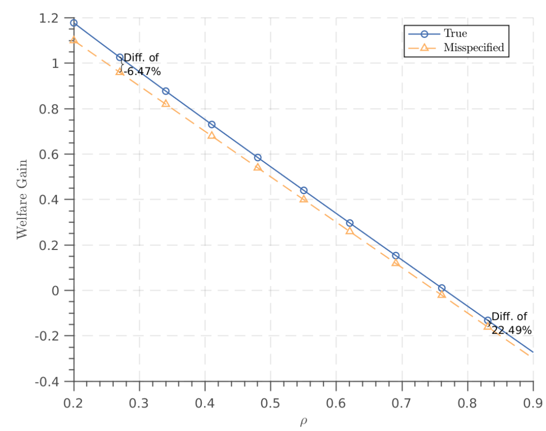
(a) Fiscal cost of UI, different values of γ.



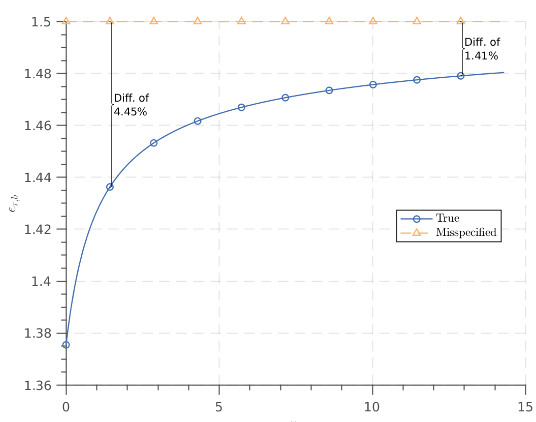
(b) Welfare gains from UI, different values of γ.



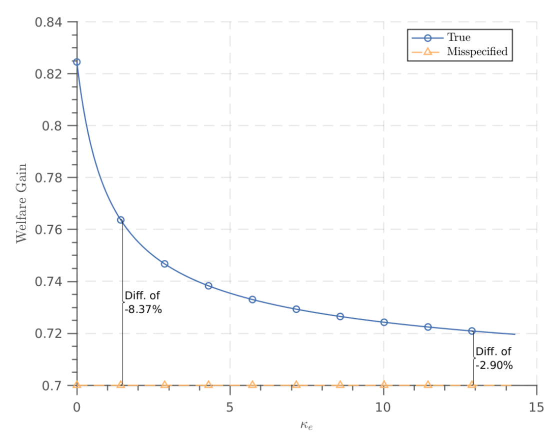
(c) Fiscal cost of UI, different values of ρ.



(d) Welfare gains from UI, different values of ρ.



(e) Fiscal cost of UI, different values of κ_e.



(f) Welfare gains from UI, different values of κ_e.

Figure 1: The effects of accounting for worker selectivity under various parameter values.

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A Proofs and derivations

A.1 The reservation wage equation

Derivation of (4). Since $W(w)$ is strictly increasing in w, we can rewrite (2) and (3) as

$$rU = v(b) + \lambda_u \int_{w_R}^{\infty} W(w) - U dF(w) \tag{22}$$

and

$$(r+\delta) W(w) = v((1-\tau) w) + \delta U + \lambda_e \int_{w}^{\infty} W(w') - W(w) dF(w') \tag{23}$$

Differentiating (23) with respect to w and rearranging, we obtain

$$W'(w) = \frac{(1-\tau) v'((1-\tau) w)}{r+\delta+\lambda_e (1-F(w))} \tag{24}$$

and therefore

$$W(w') - W(w) = \int_{w}^{w'} \frac{(1-\tau) v'((1-\tau) x)}{r+\delta+\lambda_e (1-F(x))} dx \tag{25}$$

It then follows that

$$\begin{aligned} \int_{w_R}^{\infty} W(w') - W(w_R) dF(w') &= \int_{w_R}^{\infty} \int_{w_R}^{w'} \frac{(1-\tau) v'((1-\tau) x)}{r+\delta+\lambda_e (1-F(x))} dx dF(w') \\ &= \int_{w_R}^{\infty} \int_{x}^{\infty} \frac{(1-\tau) v'((1-\tau) x)}{r+\delta+\lambda_e (1-F(x))} dF(w') dx \\ &= \int_{w_R}^{\infty} \frac{(1-\tau) v'((1-\tau) x)(1-F(x))}{r+\delta+\lambda_e (1-F(x))} dx \end{aligned} \tag{26}$$

Using $U = W(w_R)$ and using (26) in (22), we obtain

$$rW(w_R) = v(b) + \lambda_u \int_{w_R}^{\infty} \frac{(1-\tau) v'((1-\tau) x)(1-F(x))}{r+\delta+\lambda_e (1-F(x))} dx \tag{27}$$

Similarly, using (26) in (23) evaluated at $w = w_R$, we obtain

$$rW(w_R) = v((1-\tau) w_R) + \lambda_e \int_{w_R}^{\infty} \frac{(1-\tau) v'((1-\tau) x)(1-F(x))}{r+\delta+\lambda_e (1-F(x))} dx \tag{28}$$

Combining (27) with (28) gives (4). ■

A.2 Steady-state wage distribution

Derivation of (7). In steady state, inflows into employment at wages less than or equal to w equal outflows. Inflows are equal to

$$\lambda_u \left(F\left(w\right) - F\left(w_R\right)\right) u.$$

Outflows, into both unemployment and higher-wage jobs, are equal to

$$\left(\delta + \lambda_e \left(1 - F\left(w\right)\right)\right) G\left(w\right)\left(1 - u\right).$$

Setting the two expressions equal to each other, and substituting for u from (6), gives (7). ■

A.3 Wage/job-finding tradeoff

Proof of Proposition 1. Differentiating (5) with respect to b gives

$$\epsilon_{h,b} = \frac{d \ln h_u}{d \ln w_R} \frac{\partial \ln w_R}{\partial \ln b} = -\frac{w_R f\left(w_R\right)}{1 - F\left(w_R\right)} \frac{\partial \ln w_R}{\partial \ln b} \tag{29}$$

Next, we note that

$$\overline{w} = w_R + \int_{w_R}^{\infty} \left(w - w_R\right) dG\left(w\right) \tag{30}$$

$$= w_R + \int_{w_R}^{\infty} \left(1 - G\left(w\right)\right) dw \tag{31}$$

$$= w_R + \int_{w_R}^{\infty} \frac{\left[\delta + \lambda_e \left(1 - F\left(w_R\right)\right)\right]\left(1 - F\left(w\right)\right)}{\left[\delta + \lambda_e \left(1 - F\left(w\right)\right)\right]\left(1 - F\left(w_R\right)\right)} dw \tag{32}$$

where the second line is obtained from integrating by parts and the third line is obtained by substituting for G from (7). Differentiating (32) with respect to w_R gives us

$$\frac{d\overline{w}}{dw_R} = \frac{f\left(w_R\right)}{1 - F\left(w_R\right)} \int_{w_R}^{\infty} \frac{1}{1 - F\left(w_R\right)} \cdot \frac{\delta\left(1 - F\left(w\right)\right)}{\delta + \lambda_e \left(1 - F\left(w\right)\right)} dw \tag{33}$$

$$= \frac{f\left(w_R\right)}{1 - F\left(w_R\right)} \left(\frac{\delta}{\delta + \lambda_e \left(1 - F\left(w_R\right)\right)}\right)\left(\overline{w} - w_R\right), \tag{34}$$

where the last line follows by substituting again from (32). Now, multiplying both sides by $\frac{w_R}{\overline{w}} \frac{\partial \ln w_R}{\partial \ln b}$ and using (29) gives (9). ■

A.4 Mean-min ratio of wages

Proof of Lemma 1. First, we observe that

$$\int_{w_R}^{\infty} \frac{(1-\tau) v'((1-\tau) x)(1-F(x))}{r+\delta+\lambda_e (1-F(x))} dx \approx \int_{w_R}^{\infty} \frac{(1-\tau) v'((1-\tau) x)(1-F(x))}{\delta+\lambda_e (1-F(x))} dx \tag{35}$$

$$= \frac{1-F(w_R)}{\delta+\lambda_e (1-F(w_R))} \int_{w_R}^{\infty} (1-G(x))(1-\tau) v'((1-\tau) x)\, dx \tag{36}$$

$$= \frac{1-F(w_R)}{\delta+\lambda_e (1-F(w_R))} \int_{w_R}^{\infty} [v((1-\tau) w) - v((1-\tau) w_R)]\, dG(x), \tag{37}$$

where the first line follows from the approximation $r \approx 0$, the second line substitutes G from (7), and the third line uses integration by parts. We can then re-write the reservation wage equation (4) as

$$v((1-\tau) w_R) \approx v(b) + \frac{\zeta_u - \zeta_e}{1+\zeta_e} [\mathbb{E}\{v((1-\tau) w)\} - v((1-\tau) w_R)] \tag{38}$$

where the expectation is taken over the distribution G. Next, we use a second-order Taylor approximation of $v(z)$ around $v(\overline{z})$,

$$v(z) \approx v(\overline{z}) + v'(\overline{z})(z-\overline{z}) + \frac{1}{2} v''(\overline{z})(z-\overline{z})^2 \tag{39}$$

Setting $z = (1-\tau) w$ and $\overline{z} = (1-\tau)\overline{w}$, taking expectations of (39) gives us

$$\mathbb{E}\{v((1-\tau) w)\} \approx v((1-\tau)\overline{w}) + \frac{1}{2}(1-\tau)^2 v''((1-\tau)\overline{w})\, var(w) \tag{40}$$

Substituting this expression for $\mathbb{E}\{v((1-\tau) w)\}$ into (38), assuming CRRA utility (10), and denoting $b = \rho(1-\tau)\overline{w}$ gives (11). ■

A.5 The government objective and budget constraint

This section formally confirms the approximate equivalence of the government's problem to maximizing (12) subject to (13). First, when $r \approx 0$, we have

$$\begin{aligned} rU &\approx v(b) + \lambda_u \int_{w_R}^{\infty} \frac{(1-\tau)v'((1-\tau)x)(1-F(x))}{\delta+\lambda_e(1-F(x))} dx \\ &= v(b) + \frac{\lambda_u(1-F(w_R))}{\delta+\lambda_u(1-F(w_R))} \left[(\lambda_u - \lambda_e) + \frac{\delta+\lambda_e(1-F(w_R))}{1-F(w_R))}\right] \int_{w_R}^{\infty} \frac{(1-\tau)v'((1-\tau)x)(1-F(x))}{\delta+\lambda_e(1-F(x))} dx \\ &= uv(b) + (1-u)\left[v(w_R) + \int_{w_R}^{\infty} (1-\tau)v'((1-\tau)x)(1-G(x))dx\right] \\ &= uv(b) + (1-u)\int_{w_R}^{\infty} v((1-\tau)w)dG(w), \end{aligned} \tag{41}$$

showing that the government objective is approximately equivalent to (12). Next, consider the budget constraint. Let Ω_u be the present discounted revenue to the government from an unemployed worker, and let Ω_e be the present discounted revenue from a worker employed at wage w, both for a given τ and b, and taking into account that the worker responds optimally to this τ and b via (4). These values then satisfy

$$r\Omega_u = -b + \lambda_u \int_{w_R}^{\infty} \left(\Omega_e\left(w\right) - \Omega_u\right) dF\left(w\right) \tag{42}$$

and

$$r\Omega_e\left(w\right) = \tau w + \delta\left(\Omega_u - \Omega_e\left(w\right)\right) + \lambda_e \int_{w}^{\infty} \left(\Omega_e\left(w'\right) - \Omega_e\left(w\right)\right) dF\left(w'\right) \tag{43}$$

Differentiation of (43) gives

$$\Omega_e'\left(w\right) = \frac{\tau}{r + \delta + \lambda_e\left(1 - F\left(w\right)\right)} \tag{44}$$

and therefore

$$\begin{aligned}
\left(r+\delta\right)\Omega_e\left(w\right) &= \tau w + \delta\Omega_u + \lambda_e \int_{w}^{\infty}\int_{w}^{w'} \frac{\tau}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dx dF\left(w'\right) && (45)\\
&= \tau w + \delta\Omega_u + \lambda_e \int_{w}^{\infty}\int_{x}^{\infty} \frac{\tau}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dF\left(w'\right) dx && (46)\\
&= \tau w + \delta\Omega_u + \lambda_e \int_{w}^{\infty} \frac{\tau\left(1-F\left(x\right)\right)}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dx && (47)
\end{aligned}$$

Budget balance requires $\Omega_u = 0$. Setting this in (42) and (47), and substituting (47) into (42), we obtain

$$b = \frac{\lambda_u \tau}{r+\delta} \int_{w_R}^{\infty} \left[w + \lambda_e \int_{w}^{\infty} \frac{\left(1-F\left(x\right)\right)}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dx \right] dF\left(w\right) \tag{48}$$

We will now show that (48) approaches (14) when $r \to 0$. Define

$$J\left(w\right) = w + \lambda_e \int_{w}^{\infty} \frac{\left(1-F\left(x\right)\right)}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dx \tag{49}$$

It transpires, from differentiating J, that

$$J\left(w\right) = J\left(w_R\right) + \int_{w_R}^{w} \frac{\delta}{r+\delta+\lambda_e\left(1-F\left(x\right)\right)} dx \tag{50}$$

and therefore (48) can be written as

$$
\begin{aligned}
b =& \frac{\lambda_u \tau}{r+\delta} \int_{w_R}^{\infty} \left[J(w_R) + \int_{w_R}^{w} \frac{\delta}{r+\delta+\lambda_e(1-F(x))} dx \right] dF(w) && (51)\\
=& \frac{\lambda_u (1-F(w_R)) \tau}{r+\delta} \left[w_R + \lambda_e \int_{w_R}^{\infty} \frac{(1-F(x))}{r+\delta+\lambda_e(1-F(x))} dx \right] && (52)\\
&+ \frac{\lambda_u \tau}{r+\delta} \int_{w_R}^{\infty} \int_{w_R}^{w} \frac{\delta}{r+\delta+\lambda_e(1-F(x))} dx dF(w) && (53)\\
=& \frac{\lambda_u (1-F(w_R)) \tau}{r+\delta} \left[w_R + \lambda_e \int_{w_R}^{\infty} \frac{(1-F(x))}{r+\delta+\lambda_e(1-F(x))} dx \right] && (54)\\
&+ \frac{\lambda_u \tau}{r+\delta} \int_{w_R}^{\infty} \frac{\delta(1-F(x))}{r+\delta+\lambda_e(1-F(x))} dx && (55)\\
=& \frac{\lambda_u (1-F(w_R)) \tau}{r+\delta} \left[w_R + \frac{\lambda_e(1-F(w_R))+\delta}{1-F(w_R)} \int_{w_R}^{\infty} \frac{(1-F(x))}{r+\delta+\lambda_e(1-F(x))} dx \right] && (56)
\end{aligned}
$$

When $r \approx 0$, this becomes

$$
\begin{aligned}
b \approx& \frac{\lambda_u (1-F(w_R)) \tau}{\delta} \left[w_R + \frac{\lambda_e(1-F(w_R))+\delta}{1-F(w_R)} \int_{w_R}^{\infty} \frac{(1-F(x))}{\delta+\lambda_e(1-F(x))} dx \right] && (57)\\
=& \frac{\lambda_u (1-F(w_R)) \tau}{\delta} \left[w_R + \int_{w_R}^{\infty} (1-G(x)) dx \right] && (58)\\
=& \frac{h_u \tau}{\delta} \overline{w} && (59)
\end{aligned}
$$

from (5), (7), and (31).

A.6 Welfare gains from unemployment insurance

From (27), the value of an unemployed worker U satisfies

$$
rU = v(b) + \lambda_u \int_{w_R}^{\infty} (1-\tau) v'((1-\tau)x) A(x) dx \tag{60}
$$

where, for convenience, we defined the function $A(x) = \frac{1-F(x)}{r+\delta+\lambda_e(1-F(x))}$. Totally differentiating with respect to b gives

$$
\begin{aligned}
\frac{dU}{db} =& v'(b) - \lambda_u (1-\tau) v'((1-\tau) w_R) A(w_R) \left[\frac{\partial w_R}{\partial b} + \frac{\partial w_R}{\partial \tau} \frac{d\tau}{db} \right] \\
&- \lambda_u \frac{d\tau}{db} \int_{w_R}^{\infty} [(1-\tau) x v''((1-\tau)x) + v'((1-\tau)x)] A(x) dx
\end{aligned} \tag{61}
$$

Next, we derive expressions for $\frac{\partial w_R}{\partial b}$ and $\frac{\partial w_R}{\partial \tau}$, which come from differentiating (4) with respect to b and τ, respectively. This gives

$$\frac{\partial w_R}{\partial b} = \frac{v'(b)}{(1-\tau)\, u'((1-\tau)\, w_R)\,[1+(\lambda_u-\lambda_e)\, A(w_R)]} \tag{62}$$

and

$$\frac{\partial w_R}{\partial \tau} = \frac{w_R v'((1-\tau)\, w_R) - (\lambda_u-\lambda_e)\int_{w_R}^{\infty}[(1-\tau)\, x v''((1-\tau)\, x) + v'((1-\tau)\, x)]\, A(x)\, dx}{(1-\tau)\, v'((1-\tau)\, w_R)\,[1+(\lambda_u-\lambda_e)\, A(w_R)]} \tag{63}$$

Substituting (62) and (63) into (61) and simplifying gives

$$\begin{aligned}\frac{dU}{db} =& \frac{r+\delta}{r+\delta+\lambda_u(1-F(w_R))} v'(b) - \frac{\lambda_u(1-F(w_R))}{r+\delta+\lambda_u(1-F(w_R))}\frac{d\tau}{db} w_R v'((1-\tau)\, w_R) \\ & - \lambda_u \frac{d\tau}{db}\frac{r+\delta+\lambda_e(1-F(w_R))}{r+\delta+\lambda_u(1-F(w_R))}\int_{w_R}^{\infty}\big[(1-\tau)\, x v''((1-\tau)\, x) + v'((1-\tau)\, x)\big]\, A(x)\, dx\end{aligned} \tag{64}$$

Next, we use integration by parts, together with $r \approx 0$, to get

$$\begin{aligned}&\int_{w_R}^{\infty}\big[(1-\tau)\, x v''((1-\tau)\, x) + v'((1-\tau)\, x)\big]\, A(x)\, dx \\ &\qquad \approx \frac{1-F(w_R)}{\delta+\lambda_u(1-F(w_R))}\left[-w_R v'((1-\tau)\, w_R) + \int_{w_R}^{\infty} w v'((1-\tau)\, w)\, dG(w)\right]\end{aligned} \tag{65}$$

Substituting back into (64) and imposing $r \approx 0$ everywhere, we get

$$\frac{dU}{db} \approx u v'(b) - (1-u)\frac{d\tau}{db}\int_{w_R}^{\infty} w v'((1-\tau)\, w)\, dG(w) \tag{66}$$

To get (16), we substitute in $\frac{b}{\tau}$ using the budget constraint (14).

Derivation of (19). To derive the approximation in (19), we proceed in two steps. First, for any w, we write the Taylor expansion

$$w v'((1-\tau)\, w) \approx \overline{w} v'((1-\tau)\, \overline{w}) + (w-\overline{w})\,[(1-\tau)\, \overline{w} v''((1-\tau)\, \overline{w}) + v'((1-\tau)\, \overline{w})] \tag{67}$$

Since $\overline{w} = \int_{w_R}^{\infty} w dG(w)$ by definition, we have

$$\int_{w_R}^{\infty} w v'((1-\tau)\, w)\, dG(w) \approx \overline{w} v'((1-\tau)\, \overline{w}) \tag{68}$$

A similar Taylor expansion establishes that

$$\int_{w_R}^{\infty} v'((1-\tau)w)dG(w) \approx v'((1-\tau)\, \overline{w}) \tag{69}$$

Next, we write the Taylor expansion

$$v'(b) \approx v'\left((1-\tau)\,\overline{w}\right) - ((1-\tau)\,\overline{w} - b)v''\left((1-\tau)\,\overline{w}\right) \tag{70}$$

Dividing both sides by $v'\left((1-\tau)\,\overline{w}\right)$ and using $\int_{w_R}^{\infty} v'((1-\tau)w)dG(w) \approx v'\left((1-\tau)\,\overline{w}\right)$, we obtain

$$\begin{aligned}\frac{v'(b)}{\int_{w_R}^{\infty} v'((1-\tau)w)dG(w)} &\approx \frac{v'(b)}{v'\left((1-\tau)\,\overline{w}\right)} \\ &\approx 1 - \frac{(1-\tau)\,\overline{w} - b}{(1-\tau)\,\overline{w}} \cdot \frac{(1-\tau)\,\overline{w}v''((1-\tau)\,\overline{w})}{v'((1-\tau)\,\overline{w})} \\ &= 1 + \gamma(1-\rho)\end{aligned} \tag{71}$$

Substituting (68), (69) and (71) into (16) gives (19).

B Additional details on the calibration

In this section we detail how we parameterize κ_u and κ_e. We can obtain $\kappa_u = h_u/\delta$ directly from the job-finding rate and the job separation rate. At a monthly frequency, we find $h_u = 0.43$, $\delta = 0.03$, and so $\kappa_u = 14.3$. It remains to calibrate $\kappa_e = \lambda_e^*/\delta$, where we define $\lambda_e^* = \lambda_e\left(1 - F\left(w_R\right)\right)$. Following Nagypal (2005) and Hornstein et al. (2011), this can be obtained from the job-to-job transition rate, denoted by h_{ee}. We can calculate

$$h_{ee} = \delta\left[\frac{\delta + \lambda_e^*}{\lambda_e^*}\ln\left(\frac{\delta + \lambda_e^*}{\delta}\right) - 1\right] \tag{72}$$

To arrive at this expression, we used integration by parts on

$$\begin{aligned} h_{ee} &= \lambda_e \int_{w_R}^{\infty} \left(1 - F\left(w\right)\right) dG\left(w\right) \\ &= \lambda_e \int_{w_R}^{\infty} G\left(w\right) dF\left(w\right) \\ &= \lambda_e \int_{w_R}^{\infty} \frac{\delta}{\delta + \lambda_e\left(1 - F\left(w\right)\right)} \frac{F\left(w\right) - F\left(w_R\right)}{1 - F\left(w_R\right)} dF\left(w\right) \\ &= \delta\lambda_e^* \int_0^1 \frac{z}{\delta + \lambda_e^*\left(1 - z\right)} dz \end{aligned} \tag{73}$$

where we used the change of variables

$$z = \frac{F\left(w\right) - F\left(w_R\right)}{1 - F\left(w_R\right)}$$

If $h_{ee} = 2.2\%$, we obtain $\lambda_e^* \approx 7\%$ and $\kappa_e \approx 2.3$.