Instructor: *Professor Griffy*
Due: *Mar., 3rd 2022*
AECO 701

Problem Set 3

Problem 1: Linear difference equation economy (Sargent and Ljundqvist exercise 2.13 slightly modified) A household has preferences over consumption processes $\{c_t\}_{t=0}^{\infty}$ that are ordered by

$$-0.5\sum_{t=0}^{\infty}\beta^t[(c_t-30)^2+0.000001b_t^2] \tag{1}$$

where $\beta = 0.95$. The household chooses a consumption, borrowing plan to maximize (1) subject to the sequence of budget constraints

$$c_t + b_t = \beta b_{t+1} + y_t \tag{2}$$

for $t \geq 0$, where b_0 is an initial condition, β^{-1} is the one-period gross risk-free interest rate, b_t is the household's one-period debt that is due in period t, and y_t is its labor income, which obeys the second-order autorefressive process

$$(1-\rho_1 L-\rho_2 L^2)y_{t+1} = (1-\rho_1-\rho_2)5 + 0.05w_{t+1} \tag{3}$$

where $\rho_1 = 1.3$ and $\rho_2 = -0.4$.

a) Define the state of the household at t as $x_t = [1\ b_t\ y_t\ y_{t-1}]'$ and the control as $u_t = (c_t - 30)$. Then express the transition law facing the household in the form (2.4.22 in your book).

b) Use a computer to compute the eigenvalues of A, the transition matrix.

Problem 2: Contraction Mapping Theorem. Blackwell's Sufficiency Conditions for a contraction are given by the following:

T is monotone if for $f(x) \leq g(x)\ \forall x \in X$, then

$$Tf(x) \leq Tg(x) \quad \forall x \in X \tag{4}$$

T discounts if for some $\beta \in (0,1)$ and any $a \in \mathcal{R}_+$

$$T(f+a)(x) \leq Tf(x) + \beta a \quad \forall x \in X \tag{5}$$

Now, consider a neoclassical growth model where a social planner maximizes

$$V(k,z) = \max_{k'} u(c) + \beta E[V(k',z)] \tag{6}$$

$$\text{s.t. } c + k' = zk^{\alpha} \tag{7}$$

$$z \sim iid \tag{8}$$

a) Prove that the Bellman equation specified above satisfies the monotone requirement.

b) Prove that the Bellman equation specified above satisfies the discounting requirement.