Macro II

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Introduction

- Today: Asset pricing
- The "Lucas Tree Model."
- Homework due tonight.
- New homework sometime soon...

Rational Expectations Competitive Equilibrium

Definition: Given the set of exogenous stochastic processes $\{x_t\}$, and initial conditions a_0, a rational expectations equilibrium is a set of stochastic processes for prices $\{p_t\}$ and quantities $\{q_t\}$ such that:

- Given $\{p_t\}$, $\{q_t\}$ is consistent with optimal behavior on the part of consumers, producers and (if relevant) the government.
- Given $\{p_t\}$, $\{q_t\}$ satisfies the government's budget constraints and borrowing restrictions.
- $\{p_t\}$ satisfies any market-clearing conditions.

Lucas Tree Overview

- We are given equilibrium quantities and equilibrium demand functions—back out equilibrium prices.
- We know the function D(p) and the quantity q_0: now find p_0.

Compare with Literature on Consumption

- Consumption: Take rates of return as given, solve for consumption.
- Asset Pricing: Take consumption as given, solve for rates of return.

Model Structure

- Preferences: n identical consumers, maximizing

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right),$$
$$\beta\in(0,1), \quad u'(\cdot)>0, \quad u''(\cdot)\leq 0.$$

- Endowment: one durable "tree" per individual. Each period, the tree yields some "fruit" ($d_t \equiv$ dividends).
- Technology: Fruit cannot be stored. Dividends are exogenous and follow a time-invariant Markov process

$$\Pr\left(d_{t+1}\leq y \mid d_t = x\right) = F\left(y,x\right), \forall t,$$

with density $f\left(y,x\right)$.

Solution strategy

- Find the competitive equilibrium allocation via the social planner's problem. Assume equal weights on each person's utility (parallel to equal endowments).
- i.e., use welfare theorems.
- Calculate the FOC for individuals with the opportunity to buy/sell specific assets.
- Evaluate FOC at the competitive equilibrium allocation.

Step 1: Social planner's problem

- Use a representative agent
- Social planner solves

$$\max_{\{c_t\}_{t=0}^{\infty}} E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right)$$
$$s.t. \quad c_t \leq d_t.$$

- Solution: $c_t = d_t, \forall t$ (*non-storable good!*).
- What does this mean?

Definitions

$$
\begin{aligned}
c_t &= \text{consumption,} \\
p_t &= \text{price of a tree} = \text{price of stock,} \\
x_t &= \text{total resources} \\
s_{t+1} &= \text{number of trees/shares of stock,} \\
R_t &= \text{gross return on one-period risk-free bond,} \\
R_t^{-1} &= \text{price of a one-period, risk-free } \underline{\text{discount bond}}\text{,} \\
b_{t+1} &= \text{risk-free discount bonds.}
\end{aligned}
$$

Step 2: Representative consumer's problem

$$
\begin{aligned}
\max_{\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}} & E\left(\sum\nolimits_{t=0}^{\infty} \beta^t u\left(c_t\right) \middle| I_0 \right) \\
s.t. \quad & c_t + p_t s_{t+1} + R_t^{-1} b_{t+1} = x_t, \\
& x_t = \left(p_t + d_t\right) s_t + b_t, \\
& \lim_{J \to \infty} \beta^J E_t \left(u'\left(c_{t+J}\right) p_{t+J} s_{t+J+1} \right) = 0 \\
& \lim_{J \to \infty} \beta^J E_t \left(u'\left(c_{t+J}\right) b_{t+J+1} \right) = 0, \\
& s_0, \ b_0 \text{ given}
\end{aligned}
$$

- where I_0 is the information set at time 0.

Consumer's problem

- Consumer i picks c_t^i, b_{t+1}^i and s_{t+1}^i on the basis of

$$I_t^i = \left\{ \begin{array}{l} \{d_{t-m}, p_{t-m}, R_{t-m}\}_{m=0}^{t}, \\ \left\{s_{t+1-m}^j, b_{t+1-m}^j\right\}_{m=0}^{t+1}, \ \forall j \neq i, \\ \left\{c_{t-m}^j, x_{t-m}^j\right\}_{m=0}^{t}, \ \forall j \neq i, \\ \{s_{t-m}^i, b_{t-m}^i, x_{t-m}^i\}_{m=0}^{t}, \{c_{t-m}^i\}_{m=1}^{t}, \end{array} \right\},$$

- It turns out that d_t summarizes the state of the aggregate economy, with $p_t = p(d_t)$ and $R_t = R(d_t)$.
- It is the only stochastic variable, and aggregate resources equal d_t.
- Because d_t is a time-invariant Markov process, the consumer's problem is time-invariant

Recursive formulation

Bellman's functional equation:

$$
\begin{aligned}
V(x_t, d_t) = \\
& \min_{\lambda_t \geq 0} \max_{c_t \geq 0,\ s_{t+1},\ b_{t+1}} u(c_t) + \lambda_t \left(x_t - c_t - p_t s_{t+1} - R_t^{-1} b_{t+1}\right) \\
& + \beta \int V\big((p(d_{t+1}) + d_{t+1}) s_{t+1} + b_{t+1}, d_{t+1}\big) dF(d_{t+1}, d_t).
\end{aligned}
$$

- The FOC for an interior solution are:

$$
\begin{aligned}
u'(c_t) &= \lambda_t, \\
\lambda_t p_t &= \beta \int \frac{\partial V[t+1]}{\partial x_{t+1}} (p(d_{t+1}) + d_{t+1}) \, dF(d_{t+1}, d_t), \\
\lambda_t R_t^{-1} &= \beta \int \frac{\partial V[t+1]}{\partial x_{t+1}} dF(d_{t+1}, d_t).
\end{aligned}
$$

Euler Equations

- Note that (by Benveniste-Scheinkman)

$$\frac{\partial V[t]}{\partial x_t} = \lambda_t,$$

so that

$$\begin{aligned} p_t &= \beta E_t \left(\frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + d_{t+1}) \right), \\ R_t^{-1} &= \beta E_t \left(\frac{u'(c_{t+1})}{u'(c_t)} \right). \end{aligned}$$

Step 3: Equilibrium

- Intuition
- Agents allocate resources based on beliefs about future prices and consumption
- These decision rules determine processes for market clearing prices and quantities.
- In a rational expectations equilibrium, the actual processes must be consistent with the beliefs.

- **Sequential** definition: Given the stochastic process $\{d_t\}_{t=0}^{\infty}$ and the initial endowments $s_0 = 1$ and $b_0 = 0$, a rational expectations equilibrium consists of the stochastic processes $\{c_t, s_{t+1}, b_{t+1}, p_t, R_t\}_{t=0}^{\infty}$ such that:
    - Given the process for prices $\{p_t, R_t\}$, $\{c_t, s_{t+1}, b_{t+1}\}$ solves the consumer's problem.
    - All markets clear: $c_t = d_t$, $s_{t+1} = 1$, and $b_{t+1} = 0$, $\forall t$.

- **Recursive definition**: given the random variable d_0, the conditional distribution $F(d_{t+1}, d_t)$, and the initial endowments $s_0 = 1$ and $b_0 = 0$, a recursive rational expectations equilibrium consists of pricing functions $p(d)$ and $R(d)$, a value function $V(x, d)$, and decision functions $c(x, d)$, $s(x, d)$, and $b(x, d)$ such that:
  - Given the pricing functions $p(d)$ and $R(d)$, the value and policy functions $V(x, d)$, $c(x, d)$, $s(x, d)$, and $b(x, d)$ solve the consumer's problem.
  - Markets clear: for $x = p(d) + d$, $c(x, d) = d$, $s(x, d) = 1$, and $b(x, d) = 0$.

Backing out prices

- Find $R(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$\begin{aligned} R_t^{-1} &= \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} \right) \\ &= \beta \frac{1}{u'(d_t)} E_t \left(u'(d_{t+1}) \right). \end{aligned} \tag{EE}$$

- Find $p(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$p_t = \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \tag{EE$'$}$$

Bond Price

- Recall equation (EE):

$$R_t^{-1} = \beta \frac{1}{u'(d_t)} E_t \left(u'(d_{t+1}) \right), \qquad \text{(EE)}$$

$$R_t = \frac{u'(d_t)}{\beta E_t \left(u'(d_{t+1}) \right)}.$$

- The price of a discount bond increases (return falls) in β.
- The price increases (return falls) in expected future marginal utility, and decreases in current marginal utility: reflects consumption smoothing motive.
- Recall from last time: implies that more uncertainty raises bond price if convex preferences.

Stock prices

- Recall equation (EE′):

$$p_t = \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \qquad \text{(EE}'\text{)}$$

- Define the expected rate of return on stocks as

$$E_t(R_t^s) = E_t \left(\frac{p_{t+1} + d_{t+1}}{p_t} \right).$$

Equity premium

- The expected rate of return on stocks is

$$E_t\left(R_t^s\right) = E_t\left(\frac{p_{t+1} + d_{t+1}}{p_t}\right).$$

- Recall that

$$E_t\left(XY\right) = E_t\left(X\right)E_t\left(Y\right) + C_t\left(X, Y\right).$$

- Rewrite (EE′):

$$\begin{aligned} 1 &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}\left(\frac{p_{t+1} + d_{t+1}}{p_t}\right)\right) \\ &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}R_t^s\right) \\ &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}\right)E_t\left(R_t^s\right) + C_t\left(\beta\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right) \end{aligned}$$

Risk premium

- Insert (EE) and rearrange:

$$1 = R_t^{-1} E_t\left(R_t^s\right) + C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right),$$

$$\begin{aligned} E_t\left(R_t^s\right) &= R_t - R_t C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right), \\ &= R_t - \frac{u'\left(d_t\right)}{\beta E_t\left(u'\left(d_{t+1}\right)\right)} C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right), \\ &= R_t - \frac{C_t\left(u'\left(d_{t+1}\right), R_t^s\right)}{E_t\left(u'\left(d_{t+1}\right)\right)}. \end{aligned}$$

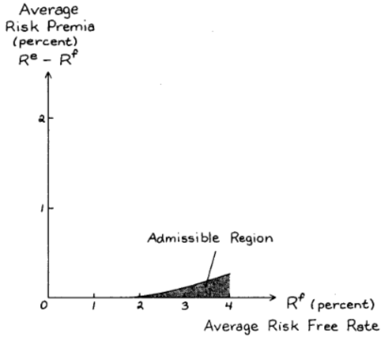
- The expected return on stocks equals the return on the risk-free bond plus the risk-premium, which is $-\frac{C_t(\cdot,\cdot)}{E_t(\cdot)}$.

Equity premium: a puzzle?

- If the covariance $C_t(\cdot, \cdot)$ is negative, which we normally expect, there is an equity premium.
- Interpretation
  - The most desirable assets yield well when marginal utility is high ($C_t(\cdot, \cdot) > 0$). Risk-aversion means that agents prefer assets that act like insurance.
  - Investors are willing to sacrifice return if $C_t(\cdot, \cdot) > 0$, and they will demand higher returns if $C_t(\cdot, \cdot) < 0$.
- Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.

Equity premium: a puzzle?

- Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.



Conclusion

- Homework due tonight.
- New homework posted soon, due a week after I post it.