

Instructor: *Professor Griffy*

Due: *May 5th, 2022*

AECO 701

Problem Set 6: The Heterogeneous Agent Model

Problem 1. Huggett Model. On the campus cluster, you will find code to solve the Aiyagari model with a labor-leisure choice. Please start from that code (email me if you cannot access the cluster). The Huggett (1993) Model is given by

$$V(a, \epsilon; \psi) = u(c) + \beta E[V(a', \epsilon'; \psi')] \quad (1)$$

subject to

$$c + a' \leq (1 + r(\psi))a + \epsilon \quad (2)$$

$$\epsilon \sim \text{Markov}, \Pi(\epsilon' | \epsilon) \quad (3)$$

$$\psi' = \Psi(\psi) \quad (4)$$

Assume the following calibration:

Parameter	Value
$u(c)$	$\frac{c^{1-\sigma}}{1-\sigma}$
β	0.993
σ	1.5
a'	≥ -2
a grid	[-2, 12]
a nodes	100

$$\pi_t = \begin{bmatrix} 0.925 & 0.075 \\ 0.5 & 0.5 \end{bmatrix} \quad (5)$$

$$\epsilon = \begin{bmatrix} 1.0 \\ 0.1 \end{bmatrix} \quad (6)$$

Note that market clearing is given by

$$\int_{a \times \epsilon} a' d\psi = 0 \quad (7)$$

1. Solve the model. Plot the decision rules for savings across the a grid for an agent in employment state 1 and employment state 2.
2. Plot the stationary distribution of wealth.

Problem 2. Aiyagari Model. Now we will extend the problem to include a firm, as in Aiyagari (1994). In this economy, the household's problem is given by

$$V(k, l; \psi) = u(c) + \beta E[V(k', l'; \psi')] \quad (8)$$

subject to

$$c + k' \leq (1 + r(K, L) - \delta)k + w(K, L)l \quad (9)$$

$$k' \geq 0 \quad (10)$$

$$\ln(l') = \rho \ln(l) + \sigma(1 - \rho^2)^{\frac{1}{2}} \epsilon', \quad \epsilon' \sim N(0, 1) \quad (11)$$

$$\psi' = \Psi(\psi) \quad (12)$$

and the firm's problem is given by

$$\Pi = \max_{K, L} F(K, L) - r(K, L)K - w(K, L)L \quad (13)$$

Assume the following calibration:

Parameter	Value
$u(c)$	$\frac{c^{1-\mu}}{1-\mu}$
$F(K, L)$	$K^\alpha L^{1-\alpha}$
β	0.96
δ	0.08
α	0.36
k Grid	[0, 18]
k nodes	100

Note that market clearing is given by

$$\int_{k \times l} l d\psi = L \quad (14)$$

$$\int_{k \times l} k' d\psi = K \quad (15)$$

and that μ , σ , and ρ will be given in the following parts.

1. Use Tauchen's method to approximate the AR(1) process for $\rho = 0.6$ and $\sigma = 0.2$. Write out the resulting grid and transition matrix.
2. Find the ergodic distribution of employment using this grid and transition matrix. Call this L . Note that you will need to calculate this for each ρ, σ pair.
3. Solve the model. Plot the decision rules for savings across the k grid for $\rho = 0.6$, $\sigma = 0.2$, and $\mu = 3$.
4. Plot the stationary distribution of wealth for $\rho = 0.6$, $\sigma = 0.2$, and $\mu = 3$.
5. Now pick $\sigma = 0.4$ and plot the net return to capital for $\rho = \{0, 0.3, 0.6, 0.9\}$ and $\mu = \{1, 3, 5\}$.