

Macro II

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Spring 2022

Announcements

- ▶ Today: Solving heterogeneous agent models.
- ▶ Idea:
 - ▶ Solving these models is non-trivial.
 - ▶ Must consider the state of every agent in economy.
- ▶ One more homework on solving heterogeneous agent models.
- ▶ Start from the code on the cluster.

Heterogeneous Agent Production Economy

- ▶ In a production economy, the agent's problem is given by

$$V(k, \epsilon; \psi) = u(c) + \beta E[V(k' \epsilon'; \psi')] \quad (1)$$

$$\text{s.t. } c + k' \leq (1 + r(K, L) - \delta)k + w(K, L)\epsilon \quad (2)$$

$$k' \geq \underline{k} \quad (3)$$

$$\epsilon \sim \text{Markov } P(\epsilon' | \epsilon) \quad (4)$$

$$\psi' = \Psi(\psi) \quad (5)$$

$$c \geq 0, k \geq 0, k_0 \text{ given} \quad (6)$$

- ▶ ϵ is a markov process that yields hours worked.
- ▶ Ψ is an unspecified evolution of the aggregate state (k, ϵ) .
- ▶ Prices are determined from the firm's problem

Prices - The Firm's Problem

- ▶ How we handle prices determines the difficulty of this problem.
- ▶ In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} F(K, L) - wL - rK \quad (7)$$

- ▶ This yields standard competitive prices for the rental rates.

Stationary Recursive Competitive Equilibrium

- A stationary RCE is given by pricing functions r, w , a worker value function $V(k, \epsilon; \psi)$, worker decision rules k', c , a type-distribution $\psi(k, \epsilon)$, and aggregates K and L that satisfy
1. k' and c are the optimal solutions to the worker's problem given prices.
 2. Prices are formed competitively from the firm's problem.
 3. Consistency between aggregate evolution and individual decision rules: ψ is the stationary distribution implied by worker decision rules.
 4. Aggregates are consistent with individual policy rules:
$$K = \int k d\psi, L = \int \epsilon d\psi$$

Calibration

- ▶ Functions:
 - ▶ Utility: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$
 - ▶ Production: $F(K, L) = K^\alpha L^{1-\alpha}$
- ▶ Borrowing constraint: $\underline{k} = 0$
- ▶ $\alpha = 0.36$.

Solving the Model: Market Clearing

- ▶ In equilibrium

$$K = \sum_k \sum_{\epsilon} k_s(k, \epsilon) \psi(k, \epsilon) \quad (8)$$

- ▶ where k_s is the supply of savings.
- ▶ What must the equilibrium prices satisfy?

$$r = F_K(K_D, L) \quad (9)$$

$$K_D(r) = K_S(r) \quad (10)$$

- ▶ Fixing K_D or r yields the other variable.
- ▶ Thus, one approach is to “guess” the equilibrium and iterate until we guess correctly.

A Solution Technique: The Shooting Algorithm

- ▶ Guess r . Yields K_D and w from $r = F_K(K_D, L)$ and $w = F_L$.
- ▶ Now, given this price, calculate the *individual* savings rule.
- ▶ Simulate the economy far enough into future to reach a steady-state distribution of capital.
- ▶ Check and see if $K_D = K_S$.
- ▶ If not, adjust guess of interest rate according to following:

$$r' = r + \lambda(K_D - K_S) \quad (11)$$

- ▶ where $\lambda < 1$

A Solution Technique: The Shooting Algorithm

- ▶ Adjusting interest rates:

$$r' = r + \lambda(K_D - K_S) \quad (12)$$

- ▶ If $K_S > K_D$: too much savings.
- ▶ Interest rate must fall to be in equilibrium.

First iteration

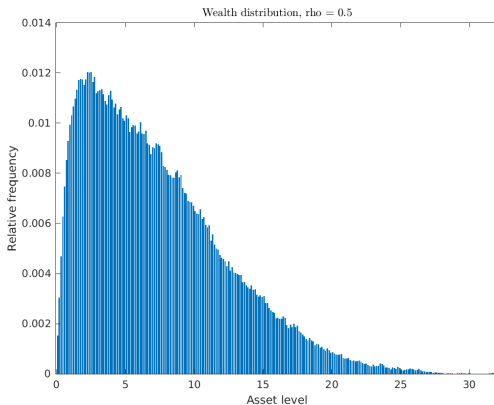
- ▶ Initial guess:
 - ▶ $r_0 = 0.03093$
- ▶ Three aggregates:
 1. $K = 8.8342$
 2. $L = 0.8582$
 3. $\rightarrow r = F_K = 0.0204$
- ▶ $r - r_0 < \text{errtol}$? $0.0309 - 0.0204$ too large.
- ▶ Algorithm: `fzero` \rightarrow pick local r_1 and try again.

Second iteration

- ▶ Initial guess:
 - ▶ $r_0 = 0.0308$
- ▶ Three aggregates:
 1. $K = 1.4531$
 2. $L = 0.9351$
 3. $\rightarrow r = F_K = 0.1985$
- ▶ $r - r_0 < \text{errtol}$? $0.0309 - 0.1935$ too large.
- ▶ Very sensitive to r_0 !

Converged Wealth Dist.

- ▶ Final wealth distribution after convergence:



Another Solution technique: Root-Finding and Excess Demand

- ▶ Functionally, this is the same as what we just did.
- ▶ Suppose we solve household decision rules k , and r .
- ▶ Then, the excess demand function is

$$\Delta(r) = K_D(r) - K_S(r) \quad (13)$$

- ▶ Where we have solved K_D for many values of r and have an expression for $K_S(r)$ (static firm optimization).
- ▶ Do one-dimensional root finding, i.e., find r^* such that

$$0 = \Delta(r^*) = K_D(r^*) - K_S(r^*) \quad (14)$$

Aggregate Uncertainty

- ▶ In a production economy, the agent's problem is given by

$$V(k, \epsilon; z, \psi) = u(c) + \beta E[V(k', \epsilon'; z', \psi')] \quad (15)$$

$$\text{s.t. } c + k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon \quad (16)$$

$$k' \geq \underline{k} \quad (17)$$

$$z' = \text{Markov}P(z'|z) \quad (18)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \quad (19)$$

$$\psi' = \Psi(\psi, z, z') \quad (20)$$

$$c \geq 0, k \geq 0, k_0 \text{ given}, z_0 \text{ given} \quad (21)$$

- ▶ ϵ is a markov process for employment $\epsilon \in \{0, 1\}$
- ▶ Ψ is an unspecified evolution of the aggregate state.
- ▶ z *also* evolves as a markov process.
- ▶ Prices are determined from the firm's problem.

Prices - The Firm's Problem

- ▶ How we handle prices determines the difficulty of this problem.
- ▶ In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} zF(K, L) - wL - rK \quad (22)$$

- ▶ This yields standard competitive prices for the rental rates.

Laws of Motion

- ▶ The future aggregate state enters the probability of employment.
- ▶ This means that it impacts **all** of the laws of motion:

$$z' = \text{Markov}P(z'|z) \quad (23)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \quad (24)$$

$$k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon - c \quad (25)$$

$$\psi' = \Psi(\psi, z, z') \quad (26)$$

- ▶ Because shocks to z change employment status and prices.

Recursive Competitive Equilibrium

- ▶ An RCE is given by pricing functions r, w , a worker value function $V(k, \epsilon, z; \psi)$, worker decision rules k', c , a type-distribution $\psi(k, \epsilon)$, and aggregates K and L that satisfy
 1. k' and c are the optimal solutions to the worker's problem given prices.
 2. Prices are formed competitively from the firm's problem.
 3. Consistency between aggregate evolution and individual decision rules: ψ is the distribution implied by worker decision rules given the aggregate state.
 4. Aggregates are consistent with individual policy rules:
$$K = \int k d\psi, L = \int \epsilon d\psi$$

Type Distribution

- ▶ The type distribution is a problem.
- ▶ Each policy function and transition depends on the type distribution.
- ▶ But the type distribution is time-varying in response to aggregate shocks.
- ▶ Alternative: use a smaller number of moments that can be calculated quickly to characterize the type distribution.
- ▶ Like a “sufficient statistic” for the type distribution.

Krusell and Smith (1998)

- ▶ Specify moments from the type distribution γ that approximate the type distribution.
- ▶ Then: $\gamma' = \Gamma(\gamma, z, z')$.
- ▶ Household predicts prices using Γ instead of Ψ
- ▶ As long as this law of motion is reasonably accurate, this approximation will work.
- ▶ Krusell and Smith:
 - ▶ Pick first j moments of distribution over k, ϵ
 - ▶ i.e., mean, standard deviation,...
 - ▶ Use this as the law of motion.
- ▶ Use means: $\ln(K') = \phi_0^z + \phi_1^z \ln(K)$

Approximate problem

- ▶ In a production economy, the agent's problem is given by

$$V(k, \epsilon; z, K) = u(c) + \beta E[V(k', \epsilon'; z', K')] \quad (27)$$

$$\text{s.t. } c + k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon \quad (28)$$

$$k' \geq \underline{k} \quad (29)$$

$$z' = \text{Markov}P(z'|z) \quad (30)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \quad (31)$$

$$\ln(K') = \phi_0^z + \phi_1^z \ln(K) \quad (32)$$

$$c \geq 0, k \geq 0, k_0 \text{ given}, z_0 \text{ given} \quad (33)$$

- ▶ LLN \rightarrow N known given z .
- ▶ Now: need aggregate capital and ϕ_0^z, ϕ_1^z .
- ▶ Note: ϕ_0^z, ϕ_1^z for each z

KS Solution Technique

► Algorithm:

1. Specify an initial forecasting function for K :
 $\ln(K') = \phi_0^z + \phi_1^z \ln(K)$. Pick initial values for ϕ_0^z, ϕ_1^z
2. Tell household that the evolution of the aggregate state is given by $\ln(K') = \phi_0^z + \phi_1^z \ln(K)$. i.e., replace the previous constraint.
3. Use value function iteration on this problem to solve for optimal policy rules.
4. Simulate model forward to obtain K, z series. Drop first X number of observations.
5. Use OLS on K, z series to see if forecasting was correct
 $||[\phi_0^z, \phi_1^z]' - \phi_0^z, \phi_1^z|| < \text{errtol}$
6. If not, update ϕ_0^z, ϕ_1^z between initial and estimates.

- Another way to think about this: You estimated the slope and intercept of K' on some series $\{K_j, z_j\}_{j=1}^{j=t}$ and you are assessing its out of sample fit on $\{K_j, z_j\}_{j=t+1}^T$

KS Solution Technique

- ▶ Why does mean work?
- ▶ Linearity:

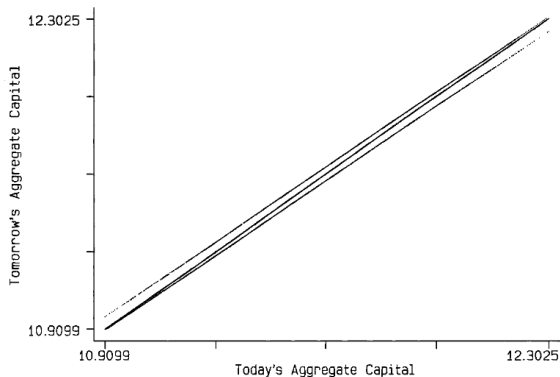


FIG. 1.—Tomorrow's vs. today's aggregate capital (benchmark model)

What do they find?

- ▶ With β heterogeneity, can hit wealth dist.

TABLE 1
DISTRIBUTION OF WEALTH: MODELS AND DATA

MODEL	PERCENTAGE OF WEALTH HELD BY TOP					FRACTION WITH WEALTH < 0	GINI COEFFICIENT
	1%	5%	10%	20%	30%		
Benchmark model	3	11	19	35	46	0	.25
Stochastic- β model	24	55	73	88	92	11	.82
Data	30	51	64	79	88	11	.79

- ▶ What is heterogeneity in β a reduced-form for?

Business Cycle Effects

- ▶ This model is built to handle stochastic shocks.
- ▶ How do heterogeneous agents respond over a business cycle?

TABLE 2
AGGREGATE TIME SERIES

Model	Mean (k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic- β :				
Incomplete markets	11.78	.825	.027	.459

Conclusion

- ▶ Today: solving heterogeneous agent models.
- ▶ Code to do this on the cluster.
- ▶ Start labor market frictions.