

# Macro II

Professor Griffy

UAlbany

Spring 2022

# Introduction

- ▶ Today: the Hosios Condition
- ▶ Efficiency in search and matching models.
- ▶ (Note: largely derived from Christine Braun's lecture on the DMP model).
- ▶ Homework on my website.
- ▶ Code for Aiyagari w/ labor-leisure choice on the cluster.
- ▶ Due tonight.

# Efficiency

- ▶ Is zero unemployment efficient? **No**
  - ▶ higher unemployment incentivizes firms to post vacancies
  - ▶ but high unemployment is costly, less production
- ▶ Is a high vacancy rate efficient?
  - ▶ vacancy creation is costly
  - ▶ but lots of vacancies reduces unemployment
- ▶ So what is the efficient level of  $\theta$ ?

# Efficiency

- ▶ Congestion externality
  - ▶ one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off
  - ▶ one more searching worker makes hiring firms better off and makes all other searching workers worse off
- ▶ Appropriability
  - ▶ firm pays a cost  $\kappa$  to post vacancy but does not get to keep the entire output  $p$

# Efficiency

- ▶ What value of  $\theta$  would the social planner choose to maximize total output/utility if he is constrained by the same matching frictions?
  - ▶ does not care about wage b/c it's a linear transfer from the firm to the worker
- ▶ Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?
- ▶ Can we achieve this wage with the Nash solution?

# The DMP Model (“Ch. 1 of Pissarides (2000)”)

▶ Agents:

1. Employed workers;
2. unemployed workers;
3. vacant firms;
4. matched firms.

▶ Linear utility ( $u = b, u = w$ ) and production  $y = p > b$ .

▶ Matching function:

1. Constant returns to scale ( $L$  is lab. force):

$$M(uL, vL) = uL \times M\left(1, \frac{v}{u}\right) = uL \times p(\theta)$$

2. where  $\theta = \frac{v}{u}$  is “labor market tightness”
3. Match rates:

$$\underbrace{p(\theta)}_{\text{Worker}} = \theta \underbrace{q(\theta)}_{\text{Firm}}$$

▶ Social planner: pick  $\theta$  optimally, no need to respect free entry condition.

# Social Planner's Problem

$$\int_0^{\infty} e^{-rt} [p(1-u) + bu - \kappa\theta u] dt$$

$$\text{s.t. } \dot{u} = \delta(1-u) - \rho(\theta)u$$

- ▶ Social planner's problem
  - ▶  $p(1-u)$ : social output of employment
  - ▶  $bu$ : leisure enjoyed by unemployed workers
  - ▶  $\kappa\theta u$ : cost of jobs
- ▶ Social planner is subject to the same transition equation for unemployment

# Social Planner's Problem

- ▶ The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- ▶ FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow -e^{-rt}\kappa u - \mu u(q(\theta) + \theta q'(\theta)) = 0$$

- ▶  $\mu$ : marginal value of an extra unemployed worker.



# Social Planner's Problem

- ▶ Optimal  $\theta$

$$H_{\theta} = 0 \Rightarrow -e^{-rt} \kappa u - \mu u q(\theta) \left(1 + \frac{\theta q'(\theta)}{q(\theta)}\right) = 0$$

- ▶ What is  $\frac{\theta q'(\theta)}{q(\theta)}$ ?

$$\begin{aligned} m(u, v) &= vq(\theta) \\ \rightarrow \frac{\partial m(u, v)}{\partial u} &= vq'(\theta) \frac{-v}{u^2} \\ \rightarrow \frac{\partial m(u, v)}{\partial u} &= -\theta^2 q'(\theta) \\ \rightarrow \frac{\frac{\partial m(u, v)}{\partial u}}{m(u, v)} &= -\frac{\theta^2 q'(\theta)}{vq(\theta)} \\ \rightarrow u \frac{\frac{\partial m(u, v)}{\partial u}}{m(u, v)} &= -\frac{\theta q'(\theta)}{q(\theta)} \end{aligned}$$

- ▶  $\frac{\theta q'(\theta)}{q(\theta)}$  is the elasticity of the matching function wrt  $u$ .

# Social Planner's Problem

- ▶ The Hamiltonian

$$H = e^{-rt}[p(1 - u) + bu - \kappa\theta u] + \mu(t)[\delta(1 - u) - p(\theta)u]$$

- ▶ FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow -e^{-rt}\kappa u - \mu u q(\theta)(1 - \eta(\theta)) = 0$$

- ▶  $\eta(\theta)$ : elasticity of match fun. wrt  $u$ .

## Optimal $\theta$

- ▶ Using  $p(\theta) = \theta q(\theta)$  and solving in steady state ( $\dot{\mu} = 0$ ):

$$\frac{p - b + \kappa\theta}{\delta + r + p(\theta)} = \frac{\kappa}{q(\theta)(1 - \eta(\theta))}$$
$$(p - b)(1 - \eta(\theta)) + \kappa(1 - \eta(\theta))\frac{p(\theta)}{q(\theta)} = \frac{(\delta + r + p(\theta))\kappa}{q(\theta)}$$
$$\rightarrow (1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \quad (1)$$

- ▶ This is optimal  $\theta$

## Decentralized solution

- ▶ Can the decentralized solution achieve the same level of  $\theta$ ?
- ▶ i.e., can the decentralized level of unemployment be *efficient*?

## Decentralized $\theta$

- ▶ Free entry  $V = 0$ :

$$\begin{aligned}rJ(w) &= (p - w) + \delta[V - J(w)] \\(r + \delta)J(w) &= (p - w)\end{aligned}$$

- ▶ Vacancy creation condition (i.e., free entry imposed):

$$\begin{aligned}q(\theta) &= \frac{\kappa}{E[J(w)]} \\q(\theta) &= \frac{\kappa(r + \delta)}{(p - w)} \\ \theta &= q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)\end{aligned}$$

- ▶ Thus, mapping between wages and  $\theta$ . 1 equation, 2 unknowns.
- ▶ Need equation to determine wages in equilibrium.

# Wage Determination

- ▶ Recall Nash Bargained wages:

$$w = \underset{w}{\operatorname{argmax}} \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta} \frac{\partial W}{\partial w} + (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U) \frac{\partial J}{\partial w}$$

- ▶  $\frac{\partial W}{\partial w} = 1$ ,  $\frac{\partial J}{\partial w} = -1$ :

$$\beta \left( \frac{J(w)}{W(w) - U} \right)^{1-\beta} = (1 - \beta) \left( \frac{W(w) - U}{J(w)} \right)^\beta$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

## Wage Determination

- ▶ Note that  $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

- ▶ And  $(1 - \beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1 - \beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta \frac{J(w)}{1 - \beta} \\ w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

- ▶ Free entry condition:  $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

## Decentralized free entry

- ▶ Job creation curve:

$$(r + \delta)J(w) = (p - w)$$

$$q(\theta) = \frac{\kappa}{J(w)}$$

$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$

$$p - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- ▶ Now, plug in using wages we just found:

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$



## Decentralized free entry

- ▶ Job creation curve:

$$p - ((1 - \beta)b + \beta p + \beta\theta\kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- ▶ identities:  $p(\theta) = \theta q(\theta) \rightarrow \theta = \frac{p(\theta)}{q(\theta)}$

$$\rightarrow p - ((1 - \beta)b + \beta p + \beta \frac{p(\theta)}{q(\theta)} \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \beta \frac{p(\theta)}{q(\theta)} \kappa - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \frac{r + \delta + \beta p(\theta)}{q(\theta)} \kappa = 0$$

- ▶ Looks familiar?

## Social Planner's Problem

- ▶ Using  $p(\theta) = \theta q(\theta)$  and solving in steady state ( $\dot{\mu} = 0$ )

$$(1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \quad (2)$$

- ▶ From the decentralized solution, plug the wage curve into the Job creation curve

$$(1 - \beta)(p - b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)}\kappa = 0 \quad (3)$$

# Efficiency

- ▶ Comparing (1) and (2) we see that we have efficiency in the decentralized market if  $\beta = \eta(\theta)$ . The workers bargaining power is equal to the elasticity of the matching function with respect to  $u$ .
- ▶ This is a general result: we have efficiency when

$$\eta(\theta) = \beta$$

- ▶ This is called the Hosios (1990) condition

Thanks for a good semester!

- ▶ I'm buying beers at City Line after you guys finish your prelims!