Quantitative Macro-Labor: Disciplining Wage Contracts with Convex Preferences

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Announcements

- Today: Extend Burdett-Mortensen to environment where firms post *wage-tenure contracts* instead of wages.
- Research Proposal/Introduction: Due next Thursday.
- Short presentations the following Tuesday.
- I may have to cancel class at some point next week due to jury duty.

The Burdett-Mortensen Model

- What is an (one of many) important and realistic feature of the labor market missing in the standard McCall model?
- The ability to search while employed.
- Some statistics:
  1. 50% of all hires are job-to-job hires (Census)
  2. Movement up job ladder accounts for 50% of wage growth for young workers (Topel and Ward, 1992)
  3. 70% of fall in hires during Great Recession was J2J.
- But, wages also increase with *tenure*.
- BM model can't account for this.

The Burdett-Mortensen OTJS Model

- Basic idea:
  1. Workers can be in one of two states: employed or unemployed, with value functions V, U.
  2. Firms post wages, i.e., a given distribution of wages, $w \in [\underline{w}, \bar{w}], w \sim F(.)$.
  3. Unemployed receive job offers at exogenous rate α, no prior info.
  4. Employed job offers at exogenous rate λ, no prior info.
  5. Separate two ways: exogenously (rate δ) and via thru OTJS (rate $\lambda[1 - F(w)]$)
  6. Linear utility: $u(c) = b$ or $u(c) = w$.
- **Firms cannot respond to outside offers.**
- Are these contracts optimal?

The Burdett-Mortensen Model

- Flow value of unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V(x) - U, 0\} dF(x) \tag{1}$$

- Employment:

$$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w)) \tag{2}$$

- Thus, the reservation wage is

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} \frac{[1 - F(x)]}{r + \delta + \lambda[1 - F(w_R)]} dx \tag{3}$$

The Firm

- Assume equilibrium conditions & $\lambda = \alpha$.
- Define π^V as the profits of a vacant firm.
- Firm profit function:

$$\pi^V = \max_w (p - w) l(w|w_R, F) \tag{4}$$

$$\pi^V(w|w_R, F) = (p - w) l(w|w_R, F) \tag{5}$$

$$l(w|w_R, F) = \frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2} \tag{6}$$

- What is l? It is the probability of meeting a workers *and* the expected duration a worker employed at wage w will be employed with a firm.

Matched Firm Value

- Matched firm profit function:

$$r\pi^F(w|w_R, F) = (p - w) + \alpha[1 - F(w)](\pi^V - \pi^F(w|w_R, F)) + \delta(\pi^V - \pi^F(w|w_R, F)) \quad (7)$$

$$(8)$$

- Would a matched firm prefer to retain a worker?

$$r\pi^F(w_R|w_R, F) = (p - w_R) + \alpha[1 - \cancel{F(w_R)}](\pi^V - \pi^F(w_R|w_R, F)) + \delta(\pi^V - \pi^F(w_R|w_R, F)) \quad (9)$$

$$r\pi^F(w_R|w_R, F) = (p - w_R) + (\alpha + \delta)(\pi^V - \pi^F(w_R|w_R, F)) \quad (10)$$

- $\pi^F \geq \pi^V$. Why is this?
- The firm would prefer to retain a worker.
- Maybe there is room for better contracts?

Burdett and Coles (2003)

- Another way of thinking about on-the-job search: moral hazard.
- Firm can't contract on worker behavior: worker can leave anytime it finds a higher wage.
- How could we allow a firm to handle this?
  1. Respond to outside offers. (doesn't make sense in model with identical productivity).
  2. Write contracts that change wages to mitigate moral hazard.
- What does option 2 mean? They will increase wages the longer a worker remains employed.
- Backloaded contracts: option value larger so worker wants to stay.

Outline (Burdett-Coles, 2003)

- Preferences and Technology:
  1. Workers can be in one of two states: employed or unemployed, with value functions V_E, V_U.
  2. Firms post wage-*contracts*: distribution of *promised values*, $V \sim F(.)$.
  3. Workers receive job offers at exogenous rate λ.
  4. Separate two ways: "exit the model" (rate δ) and via thru OTJS (rate $\lambda[1 - F(V)]$)
  5. Risk-aversion: $u(c) = u(b)$ or $u(c) = u(w)$.
  6. Rate of time preference: $r = 0$ (simplification for analytical results).
- **Firms cannot respond to outside offers.**
- Instead, they increase wages over time to ensure that a worker does not leave early.

Contracts

- Rather than wages, a firm offers a promised value, and can distribute it in any way over the life of the contract.
- Optimally, they want to backload the contract.
- Why? Because workers are less likely to leave if continuation value is high.
- Wage-tenure contracts:
  1. Wage $w(.) \geq 0$.
  2. Where tenure, t, is its only argument.
  3. By assumption, equilibrium is symmetric, i.e., all contracts are the same given t.

Worker Value Functions

- Unemployed flow value (bc $r = 0$):

$$\delta V_U = u(b) + \lambda \int_{V_U}^{\bar{V}} [x - V_U] dF(x) \tag{11}$$

- Employed value function

$$\delta V_E(t|w(.)) - \frac{dV_E(t|w(.))}{dt} = u(w(t)) + \lambda \int_{V_E(t|w(.))}^{\bar{V}} [V_x - V_E(t|w(.))] dF(V_x) \tag{12}$$

- Currently, optimal contract unspecified.
- What is $\frac{dV_E(t|w(.))}{dt}$?

Firm Value Functions

- Define a worker survival probability (i.e., doesn't leave or die.) through tenure t.

$$\psi(t|w(.)) = e^{-\int_0^t [\delta+\lambda(1-F(V(s|w(.))))]ds} \tag{13}$$

- Define $G(V)$ the steady state number of workers with lifetime utility less than V.
- Then,

$$\Omega = \underbrace{[\lambda G(V_0)]}_{\textit{Finding Rate}} \int_0^\infty \underbrace{\psi(t|w(.))}_{\textit{Survival}} \underbrace{[p - w(t)]}_{\textit{Flow profits}} dt \tag{14}$$

- Where Ω is the equilibrium profits of a firm posting contract $w(.)$ s.t., $E[u(w(.))] = V_0$.

Optimal Contract

- Fundamental question: Why do we write contracts?
- Firm dynamic programming problem

$$\max_{w(.)\geq 0} \int_0^{\infty} \psi(t|w(.))[p - w(t)]dt \tag{15}$$

- What are the two terms here?

$$\max_{w(.)\geq 0} \int_0^{\infty} (\underbrace{\psi(t|w(.))p}_{Gains} - \underbrace{\psi(t|w(.))w(t)}_{Losses})dt \tag{16}$$

- Subject to $V(0|w(.)) = V_0$.
- "Free Entry":

$$\Omega^*(V_0) = \lambda G(V_0)\pi^*(0|V_0) \tag{17}$$

Equilibrium

- Assumptions:
  1. Convex preferences: $u, > 0$, $u'' < 0$ and exists, and $\lim_{c \leftarrow 0} u(c) = -\infty$
  2. $\forall V_0 \in (\underline{V}, \bar{V})$, F is continuously diff'ble & $F'(V_0) > 0$.
- From the paper:
  A market equilibrium is:
  1. a distribution of starting payoffs F.
  2. Optimal wage-tenure contracts $w^*(.|V_0)$, $V_0 \geq V_U$.
  3. Optimal stopping-time solutions to both worker problems.
  4. Worker value distribution G consistent with worker flows.
  5. Optimal posting game:

$$\Omega^*(V_0) = \bar{\Omega} \forall V_0 \in [\underline{V}, \bar{V}] \tag{18}$$

$$\Omega^*(V_0) \leq \bar{\Omega} \text{ otherwise} \tag{19}$$

Optimal Contract

- Fundamental question: Why do we write contracts?
- Firm dynamic programming problem

$$\max_{w(.)\geq 0} \int_0^\infty \psi(t|w(.))[p - w(t)]dt \tag{20}$$

$$\text{s.t. } w(.) \geq 0 \tag{21}$$

$$V(0|w(.)) = V_0 \tag{22}$$

- Continuous-time optimization (Hamiltonian) where

$$\dot{\psi} = -[\delta + \lambda(1 - F(V))]\psi \tag{23}$$

$$\dot{V} = \delta V - u(w) - \lambda \int_V^{\bar{V}} [x - V]dF(x) \tag{24}$$

- Hamiltonian:

$$H = \psi[p - w] + x_\psi \dot{\psi} + x_V \dot{V} \tag{25}$$

- Optimal path of the wage would solve this problem.

Optimal Contract

- Solving the contracting problem yields the following contract:

$$\frac{u'(w^*(0|.))}{u'(w^*(\tau|.))} = 1 + u'(w^*(0|.)) \int_0^{\tau} \lambda F'(V^*(t|.))\pi^*(t|.)dt \tag{26}$$

- Strictly increasing and concave utility:

$$\frac{u'(w^*(0|.))}{u'(w^*(\tau|.))} > 1 \rightarrow w^*(\tau) > w^*(0) \tag{27}$$

- Increasing wage at time τ decreases quit rate over $[0, \tau]$:

$$\underbrace{u'(w^*(0|.))}_{Slope} \int_0^{\tau} \underbrace{\lambda F'(V^*(t|.))}_{\Delta\ Quit\ Rate} \underbrace{\pi^*(t|.)}_{PDV\ of\ Profits} dt \tag{28}$$

- Moral hazard: reward those who do not quit.

Market Equilibrium

- Similar to the BM model (but with more algebra), can show that there exists a unique equilibrium, with the following:

$$(\frac{\delta}{\lambda+\delta})^2 = \frac{p-\bar{w}}{p-\underline{w}} \tag{29}$$

$$u(\underline{w}) = u(b) - \frac{\sqrt{p-\underline{w}}}{2}\int_{\underline{w}}^{\bar{w}} \frac{u'(x)dx}{\sqrt{p-x}} \tag{30}$$

- With a wage-tenure contract given by

$$\frac{dw}{dt} = \frac{\delta}{\sqrt{p-\bar{w}}}\frac{p-w}{u'(w)}\int_{w}^{\bar{w}} \frac{u'(x)dx}{\sqrt{p-x}} \tag{31}$$

Market Equilibrium

- Pins down initial wage & thus contract. (Burdett and Coles, 2003)

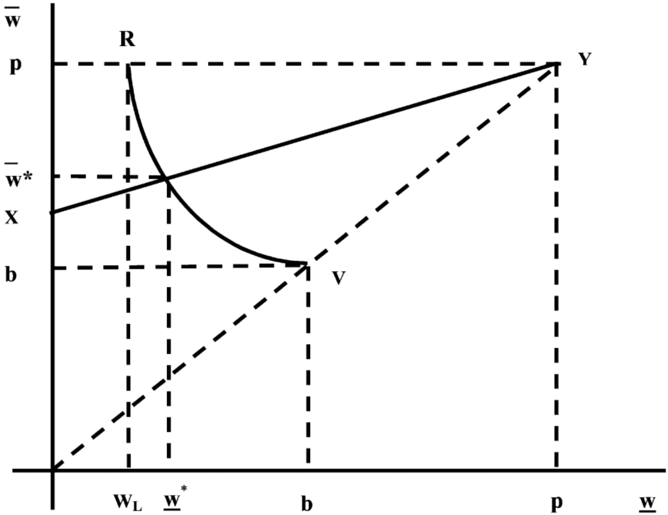

FIGURE 1.

Risk Aversion

- We breezed through preferences earlier.
- What role does consumption risk play here?
  1. Way to give workers a preference over when they receive consumption.
  2. Opportunity for firm to provide insurance against shocks.
- They use different CRRA thresholds: (Burdett and Coles, 2003)

TABLE I

σ	$\underline{w}$	$\overline{w}$	$F_w(\underline{w})$
0.0	0.600	4.964	1.000
0.1	1.333	4.970	0.911
0.2	1.751	4.973	0.864
0.4	2.223	4.977	0.803
0.8	2.728	4.981	0.727
1.4	3.126	4.985	0.657
∞	4.600	4.997	0.000

Consumption Risk

- What is strange about this table? (Burdett and Coles, 2003)

TABLE I

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- There was no precautionary savings here.
- i.e., firms attracted workers by offering *insurance* against low consumption via a guaranteed contract.
- But, transitory consumption risk already mitigated by autarky and UI.

Next Time

- Postel-Vinay and Robin (2002): Sequential auctions (i.e., firm *can* respond to outside offers)
- Next Wednesday, introduction/research proposal due.
- Presentations the following Monday.
- Start your data projects soon!