Quantitative Macro-Labor:
General Equilibrium Search and Matching

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Announcements

- ▶ Today: the Mortensen and Pissarides model (canonical equilibrium search)
- ▶ Everyone should have started the data project.
- ▶ Next Tuesday: Research proposal and presentations.

Arrival Rates of Job Offers

▶ So far, we have assumed that the arrival rate of job offers is *exogenous*: regardless of equilibrium, the frequency with which you receive an offer is the same.

▶ Consider an example:
  1. There is a productivity downturn:
  2. How does a firm respond?
  3. BM, BC, PVR: the quality of the offer distribution deteriorates, but searchers receive offers at the same rate.

▶ Essentially, slackness in the labor market is due to worker selectivity, not due to decisions made by the firm.

▶ Obviously, firms do respond.

The Beveridge Curve

▶ Another implication: there is no relationship between unemployment and vacancy creation.
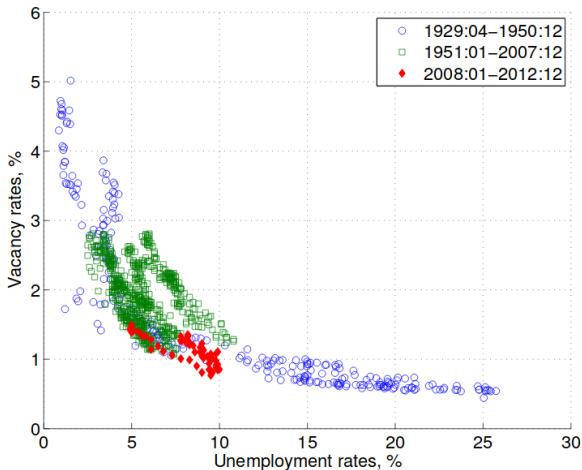
(source: Petrosky-Nadeau and Zhang, 2017)

Hazard vs. Arrival Rate

- ▶ Unmatched firm problem in Burdett-Mortensen:

$$\pi = \max_{w}(p - w)l(w|w_R, F)$$

- ▶ The hazard rate is an equilibrium object; the arrival rate is not.
- ▶ But what if firms can adjust along the extensive margin in addition to the intensive margin?
- ▶ "Vacant" firm's problem:

$$\pi^V(w) = -\kappa + q(.)J(w)$$

- ▶ q is the worker-finding rate. Now an equilibrium object.

The DMP Model ("Ch. 1 of Pissarides (2000)")

▶ Agents:
  1. Employed workers;
  2. unemployed workers;
  3. vacant firms;
  4. matched firms.

▶ Linear utility ($u = b, u = w$) and production $y = p > b$.

▶ Matching function:
  1. Determines *number* of meetings between firms & workers.
  2. Args: levels searchers & vacancies ($U = u \times L, V = v \times L$)
  3. Constant returns to scale (L is lab. force):

$$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta)$$

  4. where $\theta = \frac{v}{u}$ is "labor market tightness"
  5. Match rates:

$$\underbrace{p(\theta)}_{Worker} = \theta \underbrace{q(\theta)}_{Firm}$$

Worker Value Functions

▶ Value functions:
  1. Employed at wage w: $W(w)$
  2. Unemployed: U.

▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

Firm Value Functions

▶ Value functions:
1. Filled, paying wage w: $J(w)$
2. Vacant V.

▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V]$$

▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

▶ Free entry equilibrium condition:

$$rV = 0$$
$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta)$$

▶ This is just a market clearing condition!

Equilibrium Objects

▶ Three key equilibrium objects:
   1. Wages;
   2. unemployment;
   3. $\theta = \frac{v}{u}$ (vacancies).

▶ How we determine each of these is largely a modeling decision.

▶ Steady-state: pin down unemployment via flow equation.

▶ Free-entry: Assume that firms always post vacancies so that free entry binds.

▶ Wages: Assume that wages are determined by a surplus-(profit) sharing rule.

Steady-State Unemployment

▶ Flow of unemployment:

$$\dot{u} = \delta(1 - u) - p(\theta)u$$

▶ Steady-state:

$$0 = \delta(1 - u) - p(\theta)u$$
$$p(\theta)u = \delta(1 - u)$$
$$u = \frac{\delta}{\delta + p(\theta)}$$

▶ Same as McCall with $\alpha = p(\theta)$.

▶ (Note: no heterogeneity & $p > b \rightarrow$ all wages accepted.)

Free Entry

▶ Free entry $V = 0$:

$$rJ(w) = (p - w) + \delta[\cancel{V} - J(w)]$$
$$(r + \delta)J(w) = (p - w)$$

▶ Vacancy creation condition (i.e., free entry imposed):

$$q(\theta) = \frac{\kappa}{E[J(w)]}$$
$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$
$$\theta = q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)$$

▶ Thus, mapping between wages and θ. 1 equation, 2 unknowns.

▶ Need equation to determine wages in equilibrium.

Wage Determination

- ▶ Previous wage determination approaches:
    1. Burdett-Mortensen: wage-posting game, contracts non-negotiable.
    2. Postel-Vinay & Robin: wage-posting game, contracts set wages to equalize value of outside offer.
- ▶ Here: workers and firms bargain over the surplus of a match.
- ▶ Surplus of a match:

$$S(w) = W(w) + J(w) - U - \cancel{V}$$
$$S(w) = W(w) + J(w) - U$$

- ▶ Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^{\beta}}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

Wage Determination

▶ Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^{\beta}}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta}\frac{\partial W}{\partial w}$$

$$+ (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U)\frac{\partial J}{\partial w}$$

▶ $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\beta(\frac{J(w)}{W(w) - U})^{1-\beta} = (1 - \beta)(\frac{W(w) - U}{J(w)})^{\beta}$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

Wage Determination

▶ Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

w solves $(W(w) - U) = \beta(W(w) + J(w) - U) = \beta S(w)$

▶ Plug in for each of these:

$$(1 - \beta)[W(w) - U] = \beta J(w)$$
$$\beta J(w) = (1 - \beta)[w - \delta(U - V(w))$$
$$- b - p(\theta)(W(w) - U)]$$
$$(1 - \beta)(w - b) = \beta J(w) + (1 - \beta)(p(\theta) + \delta)[W(w) - U]$$
$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w))$$
$$+ (1 - \beta)(p(\theta) + \delta)[W(w) - U]$$

Wage Determination

- ▶ Note that $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w))$$
$$+ (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

- ▶ And $(1 - \beta)S(w) = J(w) \to S(w) = \frac{J(w)}{1-\beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w))$$
$$+ (1 - \beta)(p(\theta) + \delta)\beta\frac{J(w)}{1 - \beta}$$
$$w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

- ▶ Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \to p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Computation

- ▶ How would we solve this model?
- ▶ Need way to compute three equilibrium objects:
  1. Wages;
  2. unemployment;
  3. $\theta = \frac{v}{u}$ (vacancies).
- ▶ How we determine each of these is largely a modeling decision.
- ▶ Steady-state: pin down unemployment via flow equation.
- ▶ Free-entry: Assume that firms always post vacancies so that free entry binds.
- ▶ Wages: Assume that wages are determined by a surplus- (profit) sharing rule.
- ▶ Computation:
  - ▶ Wages, vacancies: depend on surplus.
  - ▶ Unemployment: law of motion.
- ▶ Here: add aggregate shocks.

Worker Value Functions

- Value functions:
  1. Employed at wage w: $W(w)$
  2. Unemployed: U.
- Unemployed flow value:

$$rU(z) = b + p(\theta)E[W(w, z) - U(z)] + \gamma E[U(z') - U(z)]$$

- Employed flow value:

$$rW(w, z) = w(z) + \delta[U(z) - W(w, z)]$$
$$+ \gamma E[W(w', z') - W(w, z)]$$

Firm Value Functions

▶ Value functions:
  1. Filled, paying wage w: $J(w)$
  2. Vacant V.

▶ Vacant flow value:

$$rV(z) = -\kappa + q(\theta(z))E[J(w,z) - V(z)] + \gamma[V(z') - V(w,z)]$$

▶ Matched flow value:

$$rJ(w,z) = (z + p - w) + \delta[V(z) - J(w,z)]$$
$$+ \gamma[J(w',z') - J(w,z)]$$

▶ Free entry equilibrium condition:

$$rV = 0$$
$$\rightarrow \frac{\kappa}{E[J(w,z)]} = q(\theta)$$

Computation

▶ Surplus of a match:

$$S(w, z) = W(w, z) + J(w, z) - U(z) - \cancel{V(z)}$$
$$S(w, z) = W(w, z) + J(w, z) - U(z)$$

▶ Plugging in and using $\beta S(w, z)$ is workers surplus and $(1 - \beta)S(w, z)$ is firm surplus:

$$S(z) = \frac{p + z}{r + \delta + \gamma} - \frac{b + \theta\kappa\frac{\beta}{1-\beta}}{r + \delta + \gamma} + \frac{\gamma}{r + \delta + \gamma} \int_{z'} S(x)dF(x)$$

▶ This is just a contraction: $\frac{\gamma}{r+\delta+\gamma} < 1$.

▶ Pick $S_0(z_i) = 0, \ \forall \ i$ and iterate.

▶ Yields vacancies $q(\theta) = \frac{\kappa}{(1-\beta)S(z)}$ and wages ($w = \beta S(z)$).

▶ Solve discrete time version for homework due Thursday after next (10/6).

▶ Then simulate unemployment dynamics for 1000 periods.

Conclusion

- Next Tuesday: Research proposal and presentations.
- Thursday: Start endogenous separations (Mortensen and Pissarides, 1994).
- Start your data projects.