Quantitative Macro-Labor: Searching with Observable Posting Characteristics: Directed Search

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Announcements

- Today:
  1. Random vs. directed search
  2. Change primitive: workers can observe wage offers prior to match.
  3. i.e., Moen (1997)
- Uploaded all data to cluster.
- Be sure to start empirical regularities project.

Random vs. Directed Search

- How do workers find jobs?
- How much information do they have about a job before applying?
- Two extremes:
  1. Random Search: *no* information about a job prior to receiving offer.
  2. Directed Search: *all* information about a job prior to application.
- Why does this matter?
  1. Random search is generically inefficient: one worker may reject a job offer than another would accept.
  2. Directed search is generically efficient: by applying for a job, a worker signals that the job is already acceptable.
- As we will see next time, it also changes computational complexity.

Random vs. Directed Search II

- Empirically, how can we tell them apart?
- Hazard rate to wage w generically:

$$H_U(w) = \underbrace{\lambda(w)}_{Arrival\ Rate} \times \underbrace{f(w)}_{Prob.Offer = W} \tag{1}$$

- Random search:

$$H_U(w) = \underbrace{\lambda}_{Arrival\ Rate} \underbrace{[1 - F(w_R)]}_{Selectivity} \underbrace{f(w)}_{P.(Offer = W)} \tag{2}$$

- Directed search:

$$H_U(w) = \underbrace{\lambda(w)}_{Arrival} \underbrace{\cancel{[1 - F(w_R)]}}_{F(w_R)=0} \underbrace{f(w)}_{P(Offer = w)} \tag{3}$$

$$= \underbrace{\lambda(W)}_{Arrival} \underbrace{\cancel{f(w)}}_{P(w=w_j)=1} \tag{4}$$

$$= \underbrace{\lambda(w)}_{Arrival\ Rate\ \ of\ Wage\ w} \tag{5}$$

Some Evidence

- (First couple Borrowed from Shouyong Shi)
- Hall and Krueger (08):
  1. 84% had information on wage prior to first interview.
- Holzer, Katz, and Krueger (91)
  1. Firms in high-wage industries receive more applications than low-wage industries, controlling for observables.
- Braun, Engelhardt, Griffy, and Rupert: unemployment insurance changes $\lambda(w) \rightarrow$ inconsistent with random search.

Mortensen and Pissarides Model

- Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U] \tag{6}$$

- Employed flow value:

$$rW(w) = w + \delta[U - W(w)] \tag{7}$$

- Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V] \tag{8}$$

- Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)] \tag{9}$$

- Free entry equilibrium condition:

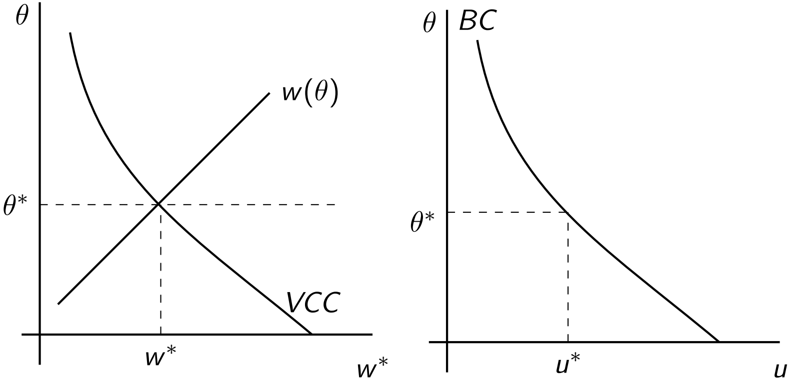
$$V = 0 \tag{10}$$

$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta) \tag{11}$$

Equilibrium

- The equilibrium we have described is a steady-state equilibrium characterized by value functions U, W, J, V, a wage function w, a market tightness function θ, and steady-state level unemployment u, such that
  1. A steady-state level of unemployment, derived from the flow unemployment equation.
  2. A wage rule that splits the surplus of a match according to a sharing rule with bargaining weight β
  3. A free entry condition that determines θ given wages and steady-state unemployment.
- What were these policy functions?
  1. $w = (1-\beta)b + \beta p + \beta\theta\kappa$
  2. $\theta = q^{-1}(\frac{\kappa(r+\delta)}{(p-w)})$
  3. $u = \frac{\delta}{\delta+p(\theta)}$

Equilibrium



Directed/Competitive Search

- In DMP, wages are negotiated/revealed after meeting.
- This can create inefficiency:
  1. Consider example with unemployed workers A and B.
  2. $w_R^A = 10$, $w_R^B = 15$
  3. Firm pays a cost κ to open a vacancy and posts a wage 12.
  4. Both worker A and B apply for the job. Firm randomly picks worker B.
  5. Worker B rejects job that would have been acceptable to worker A.
- Directed search: Worker B applies for different job with $w \geq w_R^B$.
- (Directed and competitive search generally used interchangeably).

The Competitive Search Model (Moen, 1997)

- Agents:
  1. Employed workers employed in submarket i;
  2. unemployed workers considering searching in $i \in \{1, ..., N\}$;
  3. unmatched firms indexed by productivity $y_i \in y_1, ..., y_N$;
  4. matched firms indexed by productivity $y_i \in y_1, ..., y_N$;
  5. "Market Maker": benevolent overlord who announces eqm. w_i.
- Linearity: $(u = z, u = w_i)$ and $y = y_i > z$ in open submarkets.
- Matching function:
  1. Determines *number* of meetings between firms & workers in submarket i:

$$M(u_i L_i, v_i L_i) = u_i L_i \times M(1, \frac{v_i}{u_i}) = u_i L_i \times p(\theta_i) \tag{12}$$

  2. where $\theta_i = \frac{v_i}{u_i}$ is "submarket tightness"
  3. Match rates:

$$\underbrace{p(\theta_i)}_{Worker\ wage\ i} = \theta_i \underbrace{q(\theta_i)}_{Firm\ wage\ i} \tag{13}$$

- i indexes both the productivity and wage.

Worker Value Functions

- Value functions:
  1. Employed in submarket i: W_i
  2. Unemployed and searching in submarket i: U_i.
  3. Unemployed: $U = \max\{U_1, ..., U_N\}$.
- Unemployed flow value in submarket i:

$$rU_i = z + p(\theta_i)(W_i - U_i) \tag{14}$$

- Employed flow value in submarket i:

$$rW_i = w_i + \delta(U_i - W_i) \tag{15}$$

- Both problems are stationary: optimal choice of i true $\forall\ t$.

Worker Value Functions II

- We can solve for match rates:

$$rU_i = z + p(\theta_i)(W_i - U_i) \tag{16}$$

$$(r + p(\theta_i))U_i = z + p(\theta_i)\frac{w_i + \delta U_i}{r + \delta} \tag{17}$$

$$(r + \delta)(r + p(\theta_i))U_i - p(\theta_i)\delta U_i = (r + \delta)z + p(\theta_i)w_i \tag{18}$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))} \tag{19}$$

$$p(\theta_i) = \frac{rU_i - z}{w_i - rU_i}(r + \delta) \tag{20}$$

$$\tag{21}$$

- $U = \max\{U_1, ..., U_N\}$ and ex-ante homogeneity among workers implies

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + \delta) \tag{22}$$

$$\tag{23}$$

Firm Value Functions

- Firm observes own productivity, chooses to open vacancy given submarkets (w, θ).
- Value functions:
  1. Vacant with productivity y_i: $V(y_i, w, \theta)$
  2. Filled with productivity y_i, paying wage w: $J(y_i, w)$
- Vacant flow value:

$$rV(y_i, w, \theta) = -\kappa + q(\theta)(J(y_i, w) - V(y_i, w, \theta)) \tag{24}$$

- Matched flow value:

$$rJ(y_i, w) = y_i - w + \delta(V(y_i, w, \theta) - J(y_i, w)) \tag{25}$$

Firm Value Functions II

- Value functions:
  1. Vacant with productivity y_i: $V(y_i, w, \theta)$
  2. Filled with productivity y_i, paying wage w: $J(y_i, w)$
- Moen assumes that $V(y_i, w, \theta) = 0$ in matched value only:

$$rJ(y_i, w) = y_i - w - \delta J(y_i, w) \tag{26}$$

- Asset value of vacancy in submarket (y_i, w, θ):

$$(r + q(\theta))V(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + \delta} - \kappa \tag{27}$$

Equilibrium

- We will be interested in the same equilibrium objects, but now for each submarket i:
  1. Wages w_i;
  2. unemployment u_i;
  3. $\theta_i = \frac{v_i}{u_i}$ vacancies in each submarket.
- Before, 1 & 3 were separate equilibrium conditions.
- New equilibrium objects
  1. set of open submarkets, $\mathcal{I}$;
  2. value of unemployment $\bar{V}(U)$
- Market maker sets wages according to

$$\max_{w} V(y_i, w, \theta(w; U)) \tag{28}$$

- Given $p(\theta)$ from worker's problem, find w that maximizes value of vacancy.

Free Entry

- Free entry implies that the expected value of opening a vacancy will be equal to the cost of opening it κ

- The expected value of opening a vacancy

$$\bar{V}(U) = \sum_{i=\iota(U)}^{n} f_i V(y_i, w_i^*(U), \theta_i^*(U))$$

- equilibrium:

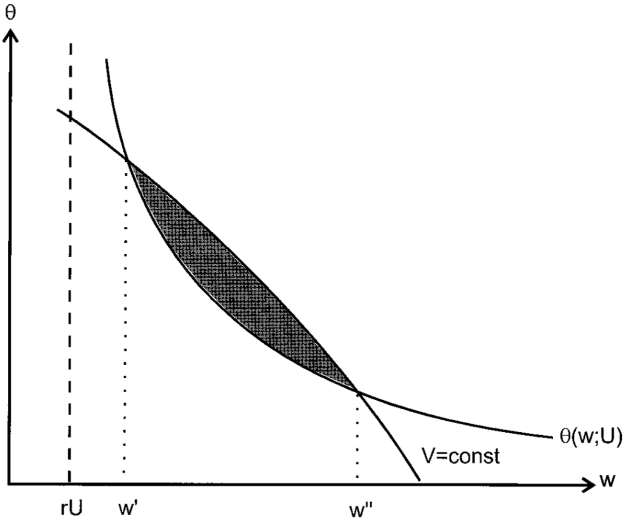
$$\bar{V}(U) = \kappa$$

Equilibrium Number of Markets

- We know that each productivity will form a separate market
- There are n productivities in the distribution
- All submarkets such that $w_i \geq rU$ will remain open
- Let ι denote the lowest submarket open

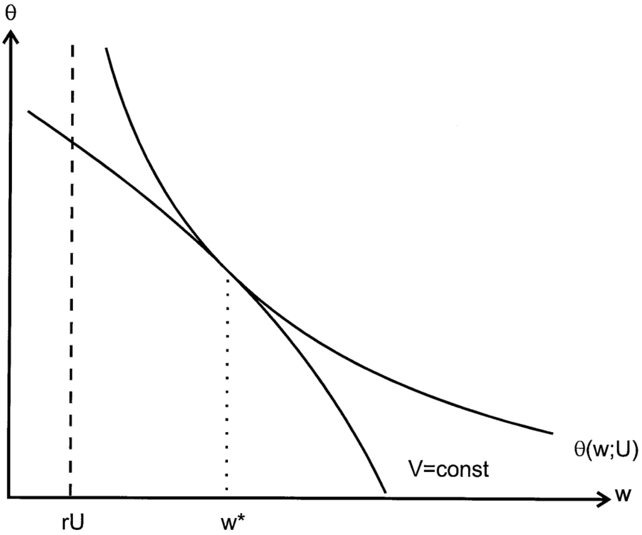
“Competitive” Search

- What is shaded region?



"Competitive" Search

- Inefficiency (rejected matches in DMP)



Equilibrium

- The resulting competitive equilibrium with frictional labor markets is characterized by the following equations

$$\bar{V}(U) = \kappa \tag{29}$$

$$w_i = \arg\max V(y_i, w, \theta(w; U)), i \geq i_R \tag{30}$$

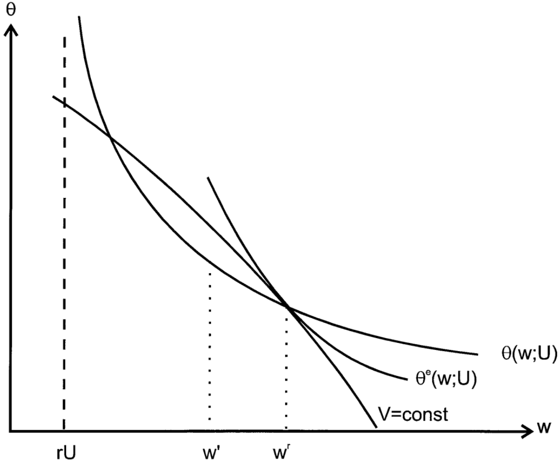
$$rU_i = \frac{(r+\delta)z + p(\theta_i)w_i}{(r+\delta+p(\theta_i))}, i \geq i_R \tag{31}$$

$$\dot{u}_i = 0; u_i p(\theta_i) = e_i \delta \tag{32}$$

$$\sum_i u_i = u \tag{33}$$

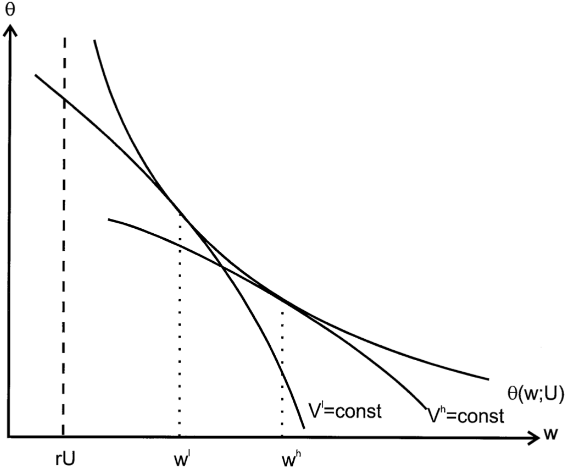
Wage Posting

- In previous description, wages were "announced in equilibrium" by a market maker.
- Would firms choose to deviate if they set their own wages?
- Suppose firms deviate and offer w':



Wage Dispersion

- Absent any ex ante heterogeneity on the worker side, is there still wage dispersion?
- Workers indifferent between open submarkets in equilibrium:



Next Time

- Directed search with heterogeneity: block recursive equilibrium.