Quantitative Macro-Labor: Frictions and Heterogeneity with the Simplicity of the Representative Agent: The Block Recursive Equilibrium

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Announcements

- Today:
  1. Discuss Menzio and Shi (2011)
  2. Block Recursive Equilibrium
  3. Why this is useful.
- Everyone should have started the first project: the "empirical regularities" project.

Motivation

- Search and matching models of the labor market can be hard to solve:
  1. Vacancy posting firm: needs to know the distribution of workers & their reservation/application strategies.
  2. Searching worker: needs to know the number of vacancies to set reservation/application strategies.
  3. Potentially very complicated fixed point problem.
- Problem becomes much harder with heterogeneity:
  1. Vacancy posting firm needs to know the type distribution of workers, as well as the reservation/application strategy by types.
  2. Workers need to know the number of vacancies, i.e., the type distribution of workers.
- Menzio and Shi: specify prices in such a way that they do not depend on the distribution.

Menzio and Shi (2011)

- Empirical goal: match business cycle "regularities" about worker flows:
  1. UE (unemployment-employment) rate: 42% monthly
  2. EU rate: 2.6% monthly
  3. EE rate (OTJ transitions): 2.9% monthly
  4. Substantial fluctuations over business cycle

x	u	v	h^{ue}	h^{eu}	h^{ee}	π
$SD(x)/SD(\pi)$	9.56	10.9	5.96	5.48	5.98	1
Autocorr(x)	.872	.909	.822	.698	.597	.760
Corr$(\cdot, x)$:						
u	1	−.902	−.916	.778	−.634	−.283
v	. . .	1	.902	−.778	.607	.423
h^{ue}	. . .	. . .	1	−.677	.669	.299
h^{eu}	. . .	. . .	. . .	1	−.301	−.528
h^{ee}	. . .	. . .	. . .	. . .	1	.208
π	. . .	. . .	. . .	. . .	. . .	1

Menzio and Shi (2011)

- To match worker flow regularities, need:
  1. On-the-job search
  2. Productivity fluctuations
  3. Match heterogeneity with endogenous separation
- Problem:
  1. Worker on-the-job reservation/application strategy impacted by current and future productivity & worker distributions.
  2. $\rightarrow$ vacancy posting impacted by expected future productivity, and worker distributions.
  3. $\rightarrow$ equilibrium hard to solve out of steady-state.
- Here:
  1. Specify equilibrium so that vacancy posting and search behavior do not depend on distribution of workers.
  2. Becomes a decision theory problem.
  3. "...is just as easy as solving the planner's problem in a representative agent model"

Environment

- Workers (risk-neutral):
  1. Infinitely-lived, can be employed or unemployed.
  2. Directed search on and off the job (otjs efficiency $\lambda_e <= 1$).
- Firms (risk-neutral):
  1. Productivity of job (match): $y + z$.
  2. y is an evolving aggregate component $y' \sim \Phi(y'|y)$.
  3. z is a fixed match-specific component (iid between matches).
- Jobs (matched worker-firm pair):
  1. Workers apply for vacancies posted by unmatched firms.
  2. Signal of match quality: $s = z$ w/ prob. α, $s \sim f(z)$ w/ $1 - \alpha$.
  3. Separation rate $d =$ exog. δ and endog. (OTJS + fired)
- Discrete time; common discount factor β.

Directed Search & Posting

- Canonical random search model (Pissarides, 1985):
  1. Workers "randomly" meet firms.
  2. Terms of employment are not settled until after matched.
  3. Some meetings not accepted.
- Directed search (Moen, 1997; Shimer 1996):
  1. Terms of employment announced prior to match.
  2. Workers "direct" their search to preferred terms.
  3. No "inefficient unemployment": all meetings accepted.
- Key terminology:
  1. Submarket "tightness": $\theta = \frac{v}{u}$.
  2. Submarket indexed by worker/firm state and terms.
  3. Contact rate of workers: $p(\theta)$.
  4. Contact rate of firms: $q(\theta) = \frac{p(\theta)}{\theta}$.

Contracts

- Firms offer promised value x, and reservation signal, r.
- Contracts are complete:
  1. Specify separation threshold $d(z, y)$ for each (z, y)
  2. Specify submarket for OTJS: (x, r)
  3. Maximize joint value of match
- Equivalent to firm setting wage as function of tenure and productivity
- & worker picking separation threshold.

Timing

1. Aggregate productivity, y realized.
2. Jobs are destroyed with probability $d \in [\delta, 1]$.
3. Workers search, firms offer contracts.
4. Workers and firms match, draw productivity, z, see signal, s.
5. Consume and produce.

Unemployed Decentralized Problem

- Submarkets are indexed by promised utility, x, and the signal required to maintain employment, r, $s >= r$.

- ψ is aggregate productivity & worker distributions.

- Bellman Equation for search sub-period:

$$D(x, r, V, \psi) = p(\theta(x, r, \psi))m(r)(x - V) \quad (1)$$

- Unemployed Bellman Equation for consumption sub-period:

$$V_u(\psi) = b + \beta E[\max_{x,r}\{V_u(\hat{\psi}) + \lambda_u D(x, r, V(\hat{\psi}), \hat{\psi})\}] \quad (2)$$

Matched Decentralized Problem

- Transferability of utility $\rightarrow$ surplus sum of worker & firm value.
- Matched Bellman Equation for consumption sub-period:

$$V_e(z,\psi) = y + z + \beta E\big[\max_{d,x,r}\{dV_u(\hat{\psi}) + (1-d)[V_e(z,\hat{\psi}) + \lambda_e D(x,r,V(z,\hat{\psi}),\hat{\psi})]\}\big] \quad (3)$$

- $d(z,y) = 1$ iff $z < r_d(y)$: unemp val. $>$ cont. val.
- Bellman Equation for search sub-period:

$$D(x,r,V,\psi) = p(\theta(x,r,\psi))m(r)(x-V) \quad (4)$$

- Unemployed Bellman Equation for consumption sub-period:

$$V_u(\psi) = b + \beta E[\max_{x,r}\{V_u(\hat{\psi}) + \lambda_u D(x,r,V(\hat{\psi}),\hat{\psi})\}] \quad (5)$$

Vacancy Creation Condition

- Unmatched firms must open vacancies at cost κ to find workers.
- Expected profits from opening a vacancy (vacancy creation):

$$V_v(x, r, \psi) = \underbrace{-\kappa}_{Cost} + q(\theta(x, r, \psi)) \sum_{s \geq r} \{ \underbrace{\alpha V_e(s, \psi)}_{Correct\ Signal} + \underbrace{(1-\alpha) E_z[V_e(z, \psi) - x]\} f(s)}_{Random\ Signal} \tag{6}$$

- If $\alpha = 1$: pure "inspection" good
- If $\alpha = 0$: pure "experience" good
- No learning: z known immediately after employment.

Free Entry Condition

- Profits competed to zero (free entry):

$$V_v(x, r, \psi) = 0$$
$$\rightarrow \kappa \geq q(\theta(x, r, \psi)) \sum_{s \geq r} \{\alpha V_e(s, \psi) + (1 - \alpha) E_z[V_e(z, \psi) - x]\} f(s) \tag{7}$$

- Note, if $q^{-1} = \theta$ exists:

$$q(\theta(x, r, \psi)) = \frac{\kappa}{\sum_{s \geq r} \{\alpha V_e(s, \psi) + (1 - \alpha) E_z[V_e(z, \psi) - x]\} f(s)} \tag{8}$$

$$\theta(x, r, \psi) = q^{-1}\left(\frac{\kappa}{\sum_{s \geq r} \{\alpha V_e(s, \psi) + (1 - \alpha) E_z[V_e(z, \psi) - x]\} f(s)}\right) \tag{9}$$

Key Points

- Free Entry in Random Search:

$$q(\theta) = \frac{\kappa}{\underbrace{[\text{Expected surplus of match}]}_{\textit{Depends on distribution}}\underbrace{[\text{Acceptance probability of match}]}_{\textit{Depends on distribution}}}$$

- Here:

$$q(\theta(x, r, \psi)) = \frac{\kappa}{\sum_{s \geq r}\{\alpha V_e(s, \psi) + (1-\alpha)E_z[V_e(z, \psi) - x]\}f(s)}$$
$$= \frac{\kappa}{\underbrace{[\text{Expected surplus of match}]}_{\textit{Does not depend on distribution}}}$$

- Submarket indexed by value x and reservation productivity r
- $\rightarrow$ expected profits and $prob(s >= r)$ known.
- Vacancy creation only depends on ψ through y.

Block Recursive Equilibrium

A block-recursive equilibrium (BRE) consists of a market tightness function θ, a value function for the unemployed worker V_u, a policy function for the unemployed worker (x_u, r_u), a joint value function for the firm-worker match V_e, and policy functions for the match d and (x_e, r_e). These functions satisfy the following:

1. $\theta(x, r, y)$ satisfies free entry in all submarkets.
2. $V_u(y)$ satisfies the unemployed problem with associated policy functions $(x_u(y), r_u(y))$.
3. $V_e(z, y)$ satisfies the joint problem with associated policy functions $d(z, y)$ and $(x_e(z, y), r_e(z, y))$

4' The evolution of the aggregate state is consistent with all policy functions, $\psi' = \Psi(\psi)$.

Block recursive means that the equilibrium solved without the last "block": 4' recovered via simulation.

How does it work?

- Simpler to see in a life-cycle model.
- Matched value in terminal period (T):

$$V_e^T(z,\psi) = y + z$$
$$V_e^T(z,y) = y + z$$

- Free entry in terminal period:

$$q(\theta^T(x,r,\psi)) = \frac{\kappa}{\sum_{s\geq r}\{\alpha V_e^T(s,\psi) + (1-\alpha)E_z[V_e^T(z,\psi) - x]\}f(s)}$$
$$q(\theta^T(x,r,y)) = \frac{\kappa}{\sum_{s\geq r}\{\alpha V_e^T(s,y) + (1-\alpha)E_z[V_e^T(z,y) - x]\}f(s)}$$

How does it work? (II)

- Easy to show that $\psi = y$ for search & unemp Bellman at T.
- Matched value in $T-1$:

$$\begin{aligned} V_e^{T-1}(z,\psi) &= y + z + \beta E\big[\max_{d,x,r}\{dV_u^T(\hat{\psi}) \\ &\quad + (1-d)[V_e^T(z,\hat{\psi}) + \lambda_e D^T(x,r,V(z,\hat{\psi}),\hat{\psi})]\}\big] \\ V_e^{T-1}(z,y) &= y + z + \beta E\big[\max_{d,x,r}\{dV_u^T(\hat{y}) \\ &\quad + (1-d)[V_e^T(z,\hat{y}) + \lambda_e D^T(x,r,V(z,\hat{y}),\hat{y})]\}\big] \end{aligned}$$

- This "...is just as easy as solving the planner's problem in a representative agent model"

“Calibration”

- Calibration two models:
  1. “Experience” good model ($\alpha = 0$)
  2. “Inspection” good model ($\alpha = 1$)
- Weibull distribution for idiosyncratic productivity ($f(z)$)
- Assume period length is 1 month.

	Description	EXP	INS	P-00	MP-94
β	Discount factor	.996	.996	.996	.996
b	Home productivity	.907	.716	.710	.739
λ_u	Off-the-job search	1	1	1	1
λ_e	On-the-job search	.735	.904	0	0
γ	Elasticity of p with respect to θ	.600	.250	.270	.270
k	Vacancy cost	1.55	2.37	1.85	1.89
δ	Exogenous destruction	.012	.026	.026	.012
μ_z	Average idiosyncratic productivity	0	0	0	0
σ_z	Scale idiosyncratic productivity	.952	.008	0	.467
α_z	Shape idiosyncratic productivity	4	10	. . .	10

Findings

- Hit model economy with 1% aggregate productivity increase.
- Compare experience and inspection goods model.
- Track:
  1. Transition Rates (EU, UE, EE)
  2. Levels (u, v, θ)
  3. Average Productivity
- Compare volatility results with data (9,000 mos. sim.)
- Find:
  1. Experience goods model better at matching data.
  2. Both more accurate than canonical random search models.
  3. Still underpredict volatility.

Experience Model: Transition Rates

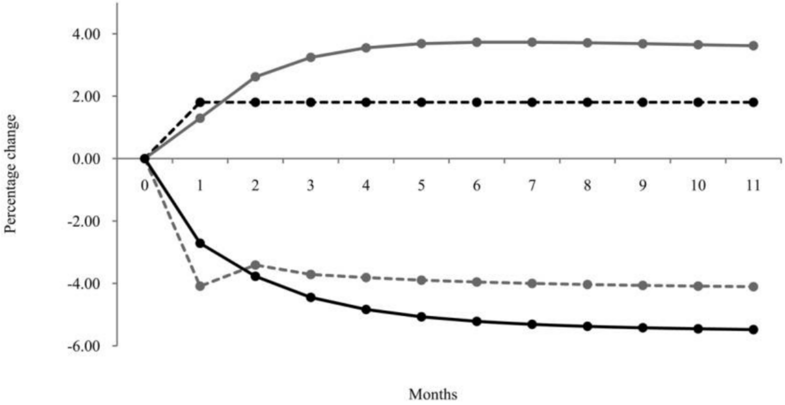

FIG. 2.—Experience model, percentage change of the UE rate (dashed black line), the EU rate (dashed grey line), the EE rate (solid grey line), and the unemployment rate (solid black line) in response to a 1 percent increase in y.

Experience Model: Levels

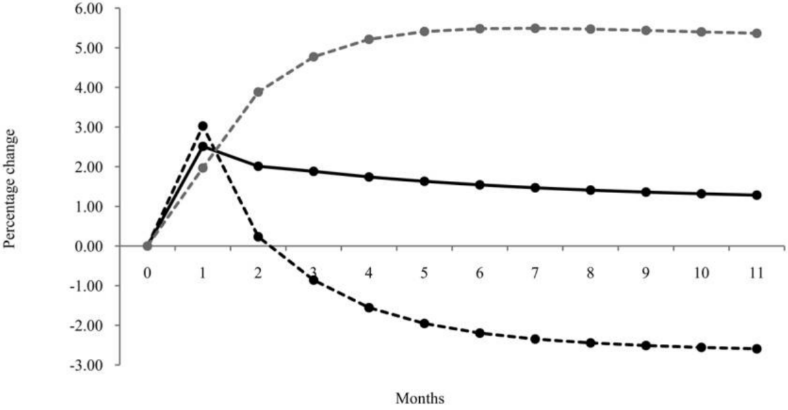

FIG. 3.—Experience model, percentage change of total vacancies (solid black line), vacancies for unemployed workers (dashed black line), and vacancies for employed workers (dashed grey line) in response to a 1 percent increase in y.

Experience Model: Productivity

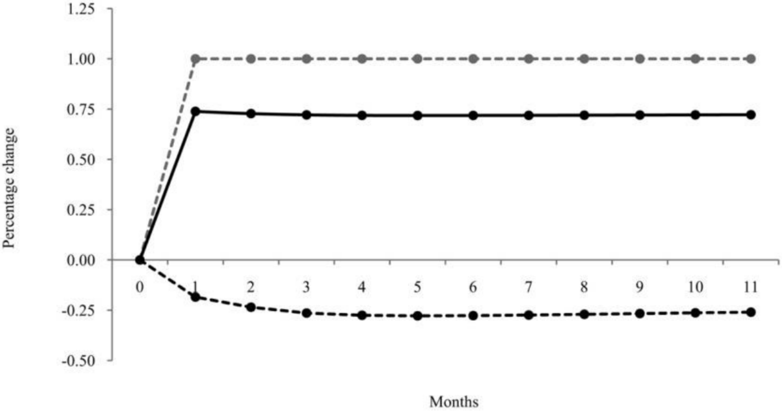

FIG. 4.—Experience model, percentage change of the aggregate component of productivity (dashed grey line), the average idiosyncratic component of productivity (dashed black line), and the average labor productivity (solid black line) in response to a 1 percent increase in y.

Experience Model: Volatility

- Data:

x	u	v	h^{ue}	h^{eu}	h^{ee}	π
$SD(x)/SD(\pi)$	9.56	10.9	5.96	5.48	5.98	1
Autocorr(x)	.872	.909	.822	.698	.597	.760
Corr$(\cdot, x)$:						
u	1	−.902	−.916	.778	−.634	−.283
v	...	1	.902	−.778	.607	.423
h^{ue}	...	...	1	−.677	.669	.299
h^{eu}	...	...	...	1	−.301	−.528
h^{ee}	...	...	...	...	1	.208
π	...	...	...	...	...	1

- Model:

x	u	v	v_u	v_e	h^{ue}	h^{eu}	h^{ee}	π
$SD(x)/SD(\pi)$	7.88	2.54	4.29	8.21	2.51	6.23	5.59	1
Autocorr(x)	.850	.637	.748	.824	.799	.772	.823	.762
Corr$(\cdot, x)$:								
u	1	−.807	.841	−.980	−.976	.972	−.979	−.977
v	...	1	−.380	.855	.897	−.898	.858	.894
π	...	...	−.729	.984	.999	−.979	.983	1

Experience Model: Comparison w/ Random Search

- Here:

x	u	v	v_u	v_e	h^{ue}	h^{eu}	h^{ee}	π
SD(x)/SD(π)	7.88	2.54	4.29	8.21	2.51	6.23	5.59	1
Autocorr(x)	.850	.637	.748	.824	.799	.772	.823	.762
Corr($\cdot$, x):								
u	1	−.807	.841	−.980	−.976	.972	−.979	−.977
v	. . .	1	−.380	.855	.897	−.898	.858	.894
π	. . .	. . .	−.729	.984	.999	−.979	.983	1

- Canonical Models:

x	u	$v = v_u$	h^{ue}	h^{eu}	π
	A. P-00 Model				
SD(x)/SD(π)	.82	2.69	.91	0	1
Autocorr(x)	.815	.677	.994	1	.745
Corr($\cdot$, x):					
u	1	−.932	−.936	0	−.972
v	. . .	1	.990	0	.990
π	. . .	. . .	.999	0	1
	B. MP-94 Model				
SD(x)/SD(π)	5.98	4.55	.83	6.61	1
Autocorr(x)	.674	.453	.740	.397	.736
Corr($\cdot$, x):					
u	1	.726	−.737	.906	−.732
v	. . .	1	−.267	.481	−.259
π	. . .	. . .	.998	−.583	1

Inspection Model

- Don't include table of results, but doesn't look good:

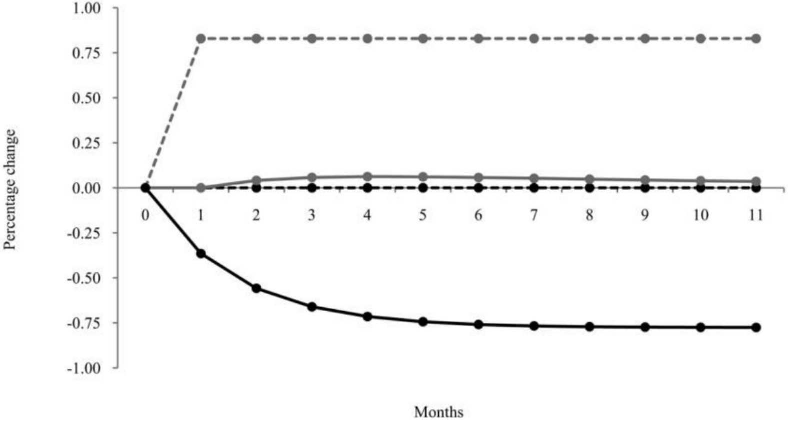

Fig. 5.—Inspection model, percentage change of the UE rate (dashed black line), the EU rate (dashed grey line), the EE rate (solid grey line), and the unemployment rate (solid black line) in response to a 1 percent increase in y.

Why is this useful?

- Aggregate shocks often intractable in random OTJ search and matching framework.
  1. Moscarini and Postel-Vinay (2009)
- Heterogeneity hard to handle in random search framework.
- Here: both much easier.
  1. Menzio, Telyukova, and Visschers (2018): Life-cycle
  2. Herkenhoff (multiple): risk aversion + housing delinquency, risk aversion + life-cycle + consumer credit & default
  3. Garriga and Hedlund (2018): risk aversion + mortgage debt
- Downsides:
  1. Do workers reject job offers?
  2. Do some job postings have excess "congestion"?
  3. What about realistic features like multiple applications?

What does this mean more generally?

- Consider a problem in which workers make the following decisions:
  1. Decision over college attendance and non-defaultable student debt;
  2. Subsequent job search decision (on and off-the-job);
  3. Within-period unsecured borrowing and default;
  4. Human capital accumulation.
- Generally very hard problem:
  - Workers: integrate over distribution across all states to determine labor market.
  - Firms: same.
- BRE: separate each market.

Next Time

- Solution techniques or Wage Dispersion.
- Or maybe my Affirmative Action paper.
- Start your projects!