Macro II

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Introduction

- Today: New Keynsian Model
- RBC model: nominal variables are neutral.
- New Keynesian model:
  1. Nominal rigidities: prices are sticky, i.e. slow to adjust.
  2. This leads to a role for stabilization policies.
- Here: Go through basic NK model.

Assumptions

- Simplifying assumption:
  - ignore variation in capital or investment.
- Prices are determined by "Calvo (1983) Pricing":
  - prices are allowed to change with fixed probability
- Wages are not sticky
- Monetary policy is a choice of the nominal interest rate

Environment

- Three agents:
  - Household: Consume, work, save, hold money.
  - Final goods producer: takes intermediate goods and produces consumable.
  - Intermediate goods producer: Uses capital and tech, sells to final goods producer.
- Preferences and technology:
  - Money in the utility function;
  - Cobb-Douglas intermeidate production;
  - CES aggregator for final good;
  - Price rigidities generate market power for intermediate goods producer.

Household's Problem

- Maximize expected utility which depends on a composite consumption good, real money, and leisure

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{C_{t+i}^{1-\sigma}}{1-\sigma} \right) + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+_i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

s.t.

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{(1+i_{t-1}) B_{t-1}}{P_t} + \Pi_t$$

  - Upper case: nominal; lower case: real.
  - $\frac{M}{P}$: real money balances;
  - $\frac{B}{P}$: real bonds;
  - Π: firm profits (rigidities yield profits);
  - i_t: nominal interest rate.

Household optimization problem

- define

$$\omega_t = \frac{\left(m_{t-1} + \left(1 + i_{t-1}\right) b_{t-1}\right)}{1 + \pi_t} = C_t + m_t + b_t - w_t N_t - \Pi_t$$

- ω: real cash in hand.
- π: rate of inflation.
- substitute b_t out by using

$$b_t = \omega_t - C_t - m_t + w_t N_t + \Pi_t$$

- value function

$$V\left(\omega_t\right) = \max \left[\left(\frac{C_t^{1-\sigma}}{1-\sigma} \right) + \frac{\gamma}{1-b} \left(\frac{M_t}{P_t} \right)^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] + \beta E_t V\left(\omega_{t+1}\right) \quad (1)$$

$$\omega_{t+1} = \frac{\left(m_t + \left(1 + i_t\right) \left(\omega_t - C_t - m_t + w_t N_t + \Pi_t\right)\right)}{1 + \pi_{t+1}}$$

- first order conditions

$$C_t^{-\sigma} - \beta E_t V_\omega\left(\omega_{t+1}\right) \frac{(1+i_t)}{1+\pi_{t+1}} = 0$$

$$\gamma m_t^{-b} - \beta E_t V_\omega\left(\omega_{t+1}\right) \frac{i_t}{1+\pi_{t+1}} = 0$$

$$-\chi N_t^{\eta} + \beta E_t V_\omega\left(\omega_{t+1}\right) w_t \frac{1+i_t}{1+\pi_{t+1}} = 0$$

- using envelope condition

$$V_\omega\left(\omega_t\right) = \beta E_t V_\omega\left(\omega_{t+1}\right) \frac{1+i_t}{1+\pi_{t+1}} = C_t^{-\sigma}$$

- FO conditions become
  - Euler:

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{(1+i_t)}{1+\pi_{t+1}}$$

  - Money holdings:

$$\gamma m_t^{-b} = \beta E_t C_{t+1}^{-\sigma} \frac{i_t}{1+\pi_{t+1}} = C_t^{-\sigma} \frac{i_t}{1+i_t}$$

  - Labor:

$$\chi N_t^{\eta} = \beta E_t C_{t+1}^{-\sigma} \frac{w_t (1+i_t)}{1+\pi_{t+1}} = C_t^{-\sigma} w_t$$

Final Goods Producer

- Composite consumption good is CES

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1$$

- Choose c_{jt} to minimize the cost of buying C_t

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t$$

- set up a Lagrangian

$$L = \int_0^1 p_{jt} c_{jt} dj + \Psi_t \left[C_t - \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \right]$$

- First order condition with respect to c_{jt}

$$p_{jt} - \Psi_t \frac{\theta}{\theta-1} \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta} c_{jt}^{\frac{-1}{\theta}} = 0$$

$$p_{jt} - \Psi_t \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{1}{\theta-1}} c_{jt}^{\frac{-1}{\theta}} = 0$$

$$p_{jt} - \Psi_t C_t^{\frac{1}{\theta}} c_{jt}^{\frac{-1}{\theta}} = 0$$

$$c_{jt} = \left(\frac{p_{jt}}{\Psi_t} \right)^{-\theta} C_t$$

- Integrate over j to solve for C_t and eliminate the multiplier

$$C_t = \left[\int_0^1 \left(\left(\frac{p_{jt}}{\Psi_t} \right)^{-\theta} C_t \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \Psi_t^{\theta} C_t \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}}$$

- Solve for Ψ_t

$$1 = \Psi_t \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{\theta-1}}$$

$$\Psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t$$

- We have an expression for the aggregate price.

Substituting into demand yields demand as a function of relative price and of composite consumption

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t$$

the price elasticity of demand is $\theta > 1$

- Important: downward-sloping demand for each good j $\rightarrow$ intermediate good producers have pricing power.

Firm's problem

- Firms maximize profits subject to constraints
  - Constraints
    - production function

$$c_{jt} = Z_t N_{jt} \qquad\qquad EZ_t = 1$$

    - demand curve

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t$$

    - with probability $1-\omega$ Calvo fairy arrives and firm can adjust price.
  - choose labor to minimize costs taking the real wage as given

$$\min w_t N_{jt} + \varphi_t \left(c_{jt} - Z_t N_{jt}\right)$$

- first order condition

$$\varphi_t = \frac{w_t}{Z_t} = \frac{w_t}{MPN} = \text{real marginal cost. common to all.}$$

- Next: choose price (p_{jt}) to maximize real discounted profits

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\frac{p_{jt}}{P_{t+i}} c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]$$

where discount factor is

$$\Delta_{i,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma}$$

- since c_{jt} depends on price through demand, substitute demand curve for $c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t$

$$\max_{p_{jt}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}$$

- optimal $p_{jt} = p_t^*$ since firms are identical in all ways except date at which last changed price: FOC with respect to p_{jt}

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} + \varphi_{t+i} \theta \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta-1} \right] \frac{C_{t+i}}{P_{t+i}} = 0$$

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \varphi_{t+i} \theta \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0$$

- first term

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} (1-\theta) \frac{p_t^*}{P_{t+i}} P_{t+i}^{\theta} C_{t+i}$$

$$= \frac{p_t^*}{P_t} (1-\theta) E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} P_t P_{t+i}^{\theta-1} C_{t+i}$$

- second term

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \varphi_{t+i} \theta P_{t+i}^{\theta} C_{t+i} = \theta E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \varphi_{t+i} P_{t+i}^{\theta} C_{t+i}$$

- solve for relative price

$$\frac{p_t^*}{P_t} = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\varphi_{t+i}P_{t+i}^{\theta}C_{t+i}}{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}P_tP_{t+i}^{\theta-1}C_{t+i}}$$

$$= \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}C_{t+i}}{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}C_{t+i}}$$

  where last equality multiplies numerator and denominator by $P_t^{-\theta}$
- φ: real marginal cost (of labor).
- use

$$\Delta_{i,t+i}C_{t+i} = \beta^iC_{t+i}^{1-\sigma}C_t^{\sigma}$$

$$\frac{p_t^*}{P_t} = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\beta^i\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}C_{t+i}^{1-\sigma}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^i\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}C_{t+i}^{1-\sigma}}$$

- Price setting equation for intermediate goods producers.

Flexible price equilibrium

- every firm adjusts every period, so $\omega = 0$; lose all but first term
$$\frac{p_t^*}{P_t} = \left(\frac{\theta}{\theta - 1}\right)\varphi_t = \mu\varphi_t$$
  - implying that price is a markup over marginal costs since $\theta > 1$
  - since price exceeds marginal cost, output is inefficiently low
  - since all firms charge the same price
$$\varphi = \frac{1}{\mu}$$
  - firms choose labor such that
$$\frac{Z_t}{\mu} = w_t$$

- households choose labor such that

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = w$$

- in flexible price equilibrium

$$C_t = Y_t = Z_t N_t = \left(\frac{1}{\chi\mu}\right)^{\frac{1}{\sigma+\eta}} Z_t^{\frac{1+\eta}{\sigma+\eta}}$$

  - Full employment output is potentially affected by shocks to
    - productivity (Z_t)
    - tastes (χ)
    - demand elasticity (markup) (μ)

Thinking about sticky prices

- Return to sticky prices
  - recall

$$\left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t$$

$$\left[\int_0^1 p_{jt}^{1-\theta} dj\right] = P_t^{1-\theta}$$

  - Aggregate price level

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

    - $1-\omega$ firms adjust this period and charge the optimal price
    - ω do not adjust, and since the adjusting firms are drawn randomly, the price level for non-adjusters is unchanged

New Keynesian Phillips Curve

- Phillips Curve: rel. btwn. inflation (expected) and unemployment (output gap).
- Intermediate price setting:

$$\frac{p_t^*}{P_t} = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} C_{t+i}^{1-\sigma}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1} C_{t+i}^{1-\sigma}}$$

- Agg. price setting:

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

- Let the relative price the firm chooses when he adjusts be

$$Q_t = \frac{p_t^*}{P_t}$$

- $Q = 1$ in steady state and when all firms can adjust every period

Tedious algebra later...

- Vars w/o time subscript are steady-state (C)
- Hats: deveiation from steady-state.
- We can derive the price that would be set by the firm:

$$\rightarrow \hat{p}_t^* = \hat{q}_t + \hat{p}_t = (1 - \omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i \left[E_t \left(\hat{\varphi}_{t+i} + \hat{p}_{t+i} \right) \right]$$

  the optimal nominal price equals the expected discounted value of future nominal (ω) marginal costs
- marginal costs: real marginal cost of labor + price.
- can write equation as

$$\omega\beta E_t \left(\hat{q}_{t+1} + \hat{p}_{t+1} \right) = \hat{q}_t + \hat{p}_t - (1 - \omega\beta) \left(\hat{\varphi}_t + \hat{p}_t \right)$$

- $\hat{q}$: deviation from SS p^*/P^*. $\hat{p}$: deviation from SS p^*

Solving for $\hat{q}_t$

$$\hat{q}_t = (1 - \omega\beta)\,\hat{\varphi}_t + \omega\beta\left[E_t\left(\hat{q}_{t+1} + \hat{p}_{t+1}\right) - \hat{p}_t\right]$$

$$\hat{q}_t = (1 - \omega\beta)\,\hat{\varphi}_t + \omega\beta E_t\left(\hat{q}_{t+1} + \pi_{t+1}\right)$$

using $\hat{q}_t = \frac{\omega}{1-\omega}\hat{\pi}_t$ to eliminate $\hat{q}_t$

$$\frac{\omega}{1-\omega}\pi_t = (1 - \omega\beta)\,\hat{\varphi}_t + \omega\beta E_t\left(\frac{\omega}{1-\omega}\pi_{t+1} + \pi_{t+1}\right)$$

$$\pi_t = \frac{(1-\omega\beta)(1-\omega)\,\hat{\varphi}_t}{\omega} + \beta E_t\pi_{t+1}$$

$$\pi_t = \tilde{\kappa}\hat{\varphi}_t + \beta E_t\pi_{t+1}$$

Recall: φ: roughly measure of the output gap (proportional to price ratio).

- no backward-looking terms, expected future inflation matters, not lagged inflation (recall Lucas Critique)
- marginal cost instead of output gap – under some restrictions the same
  - from household's labor supply decision, real wage must equal marginal rate of substitution between leisure and consumption

$$\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$$

  - using

$$\hat{c}_t = \hat{y}_t = \hat{n}_t + \hat{z}_t$$

- marginal costs equals real wage divided by marginal product of labor (Z_t)

$$\begin{aligned} \hat{\varphi}_t &= \hat{w}_t - \hat{p}_t - \hat{z}_t = \hat{w}_t - \hat{p}_t - (\hat{y}_t - \hat{n}_t) \\ &= \eta \hat{n}_t + \sigma \hat{y}_t - \hat{z}_t = \eta (\hat{y}_t - \hat{z}_t) + \sigma \hat{y}_t - \hat{z}_t \\ &= (\eta + \sigma) \left[\hat{y}_t - \frac{1+\eta}{(\eta+\sigma)} \hat{z}_t \right] \end{aligned}$$

  where

$$\hat{y}_t^f = \frac{1+\eta}{(\eta+\sigma)} \hat{z}_t$$

  implying that

$$\hat{\varphi}_t = (\eta + \sigma) \left[\hat{y}_t - \hat{y}_t^f \right] = \gamma \left[\hat{y}_t - \hat{y}_t^f \right]$$

- New Keynesian Phillips Curve becomes

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

- more complicated when do not have constant returns to scale, but principle is same

IS curve

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} \left(\hat{\imath}_t - E_t \pi_{t+1} \right)$$

- expressed in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left(\hat{\imath}_t - E_t \pi_{t+1} \right) + u_t$$

  where $u_t = E_t \hat{y}_{t+1}^f - \hat{y}_t^f$
- Taylor Rule for nominal interest rate

$$\hat{\imath}_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

- Start from this point next time.

Conclusion

- Today: New Keynsian Model
- New Keynesian model:
  1. Nominal rigidities: prices are sticky, i.e. slow to adjust.
  2. This leads to a role for stabilization policies.
- Next time: More New Keynesian Models.

Log-linearizing

- dividing second equation by $P_t^{1-\theta}$

$$1 = (1-\omega) Q_t^{1-\theta} + \omega \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta}$$

- expressed in percent deviations about steady state with $\frac{P_{t-1}}{P_t} = 1$

$$1 = (1-\omega)(1 + (1-\theta) \hat{q}_t) + \omega (1 - (1-\theta) \hat{\pi}_t)$$

$$\hat{q}_t = \frac{\omega}{1-\omega} \hat{\pi}_t$$

- rewrite first equation as (multiply through denominator)

$$Q_t E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i}^{1-\sigma} = \mu E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i}^{1-\sigma}$$

- $Q = \frac{p_t^*}{P_t} = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} C_{t+i}^{1-\sigma}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1} C_{t+i}^{1-\sigma}}$
- Approximate the left hand side as

$$\frac{C^{1-\sigma}}{1-\omega\beta} + \left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\hat{q}_t$$
$$+C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)E_t\hat{C}_{t+i} + (\theta-1)(E_t\hat{p}_{t+i} - \hat{p}_t)\right]$$

- Vars w/o time subscript are steady-state (C)
- Hats: deveiation from steady-state.
- Approximate the right hand side as

$$\mu\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\varphi$$
$$+\mu\varphi C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)E_t\hat{C}_{t+i} + \theta(E_t\hat{p}_{t+i} - \hat{p}_t) + E_t\hat{\varphi}_{t+i}\right]$$

- Equating and noting that $\mu\varphi = 1$

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t + \sum_{i=0}^{\infty}\omega^i\beta^i\left[(-1)\left(E_t\hat{p}_{t+i} - \hat{p}_t\right)\right] = \sum_{i=0}^{\infty}\omega^i\beta^i\left[E_t\hat{\varphi}_{t+i}\right]$$

$$\rightarrow \left(\frac{1}{1-\omega\beta}\right)\hat{q}_t = \sum_{i=0}^{\infty}\omega^i\beta^i\left[E_t\left(\hat{\varphi}_{t+i} + \hat{p}_{t+i}\right)\right] - \left(\frac{1}{1-\omega\beta}\right)\hat{p}_t$$

$$\rightarrow \hat{p}_t^* = \hat{q}_t + \hat{p}_t = (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left[E_t\left(\hat{\varphi}_{t+i} + \hat{p}_{t+i}\right)\right]$$

  the optimal nominal price equals the expected discounted value of future nominal marginal costs

- can write equation as

$$\omega\beta E_t\left(\hat{q}_{t+1} + \hat{p}_{t+1}\right) = \hat{q}_t + \hat{p}_t - (1-\omega\beta)\left(\hat{\varphi}_t + \hat{p}_t\right)$$