Macro II

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Introduction

- Today: Continue the New Keynsian Model
- RBC model: nominal variables are neutral.
- New Keynesian model:
    1. Nominal rigidities: prices are sticky, i.e. slow to adjust.
    2. This leads to a role for stabilization policies.
- Here: Linearize and show uniqueness of equilibrium

Assumptions

- Simplifying assumption:
  - ignore variation in capital or investment.
- Prices are determined by "Calvo (1983) Pricing":
  - prices are allowed to change with fixed probability
- Wages are not sticky
- Monetary policy is a choice of the nominal interest rate

Basic Model

- New Keynesian Phillips Curve

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

- Output gap $x_t = \hat{y}_t - \hat{y}_t^f$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left(\hat{\imath}_t - E_t \pi_{t+1} \right) + u_t$$

  where $u_t = E_t \hat{y}_{t+1}^f - \hat{y}_t^f$
- Taylor Rule for nominal interest rate

$$\hat{\imath}_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

Matrix form

- Substitute for the interest rate to get a two-equation system in two unknowns

$$E_t\pi_{t+1} = \frac{1}{\beta}\left[\pi_t - \kappa x_t\right]$$

$$E_t x_{t+1} = x_t + \frac{1}{\sigma}\left[\delta_\pi \pi_t + \delta_x x_t + v_t\right] - u_t - \frac{1}{\sigma\beta}\left[\pi_t - \kappa x_t\right]$$

- In matrix form

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & \frac{-\kappa}{\beta} \\ \frac{\beta\delta_\pi - 1}{\sigma\beta} & 1 + \frac{\beta\delta_x + \kappa}{\beta\sigma} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v_t}{\sigma} - u_t \end{bmatrix}$$

Uniqueness of equilibrium

- For uniqueness of equilibrium, since there are two forward-looking variables, the roots of the characteristic equation must both be greater than one in absolute value
  - equivalently, the system must be unstable.
  - Why? Single steady-state w/ unique equilibrium path.
  - More stable roots $\rightarrow$ equilibrium path not unique.
- letting roots be θ
  - determinant is product of roots

$$\begin{aligned}\theta_1\theta_2 &= \frac{1}{\beta}\left(1+\frac{\beta\delta_x+\kappa}{\beta\sigma}\right)+\frac{\kappa}{\beta}\left(\frac{\beta\delta_\pi-1}{\sigma\beta}\right)\\ &= \frac{1}{\beta}\left(1+\frac{\delta_x}{\sigma}+\frac{\kappa\delta_\pi}{\sigma}\right)\end{aligned}$$

  - Stability requirement is

$$\left(\theta_1-1\right)\left(\theta_2-1\right)>0$$

Some algebra later...

- We get two conditions

$$\delta_x \left(\frac{1}{\beta} - 1 \right) + \frac{\kappa}{\beta} (\delta_\pi - 1) > 0$$

$$\delta_x (1 - \beta) + \kappa (\delta_\pi - 1) > 0$$

- Implying that the responses to the output gap and inflation must be large enough

Stability

- Set expected changes to zero (strictly correct if shocks permanent)

$$E_t\pi_{t+1} - \pi_t = \frac{1-\beta}{\beta}\pi_t - \frac{\kappa}{\beta}x_t = 0$$

$$E_t x_{t+1} - x_t = \frac{\beta\delta_\pi - 1}{\sigma\beta}\pi_t + \frac{\beta\delta_x + \kappa}{\beta\sigma}x_t + \frac{v_t}{\sigma} - u_t = 0$$

$$\text{along } E_t\pi_{t+1} - \pi_t = 0 \qquad \pi_t = \frac{\kappa}{1-\beta}x_t$$

- slope of $\Delta\pi = 0$ is positive

$$\text{along } E_t x_{t+1} - x_t = 0 \qquad \pi_t = \frac{\beta\delta_x + \kappa}{1-\beta\delta_\pi}x_t + \frac{\beta}{1-\beta\delta_\pi}(v_t - \sigma u_t)$$

- slope of $\Delta x = 0$ depends on sign of $1 - \beta\delta_\pi$

Case 1: $1 - \beta\delta_\pi < 0$

- slope of $\Delta x = 0$ is negative, implying that slope of $\Delta \pi = 0 >$ slope of $\Delta x = 0$

$$\frac{\kappa}{1-\beta} > \frac{\beta\delta_x + \kappa}{1 - \beta\delta_\pi}$$

$$\kappa \left(1 - \beta\delta_\pi\right) < \left(\beta\delta_x + \kappa\right)\left(1 - \beta\right)$$

$$-\kappa\beta \left(\delta_\pi - 1\right) - \beta\delta_x \left(1 - \beta\right) < 0$$

$$\kappa \left(\delta_\pi - 1\right) + \delta_x \left(1 - \beta\right) > 0 \tag{1}$$

- model is globally unstable, equivalently both roots are outside the unit circle
- unique equilibrium at intersection and otherwise explosion

Cases 2 and 3

- Case 2: $1 - \beta\delta_\pi > 0$ and slope of $\Delta\pi = 0 >$ slope of $\Delta x = 0$
  - sign of (1) is reversed
  - model is saddlepath stable
  - can begin anywhere along the saddlepath and model approaches long-run equilibrium
- Case 3: $1 - \beta\delta_\pi > 0$ and slope of $\Delta\pi = 0 <$ slope of $\Delta x = 0$
  - equation (1) holds (begin with sign reversed and do not flip on second line)
  - model is globally unstable
  - unique equilibrium at intersection

Necessary and sufficient conditions for uniqueness

- Equation (1) is necessary and sufficient for global instability and unique equilibrium
  - uniqueness of equilibrium requires a strong enough response of the interest rate to deviations of inflation and output from their steady-state values

Positive interest rate shock v_t increases

- Assume $1 - \beta\delta_\pi < 0$
- $\Delta x = 0$ shifts down and inflation and output gap fall
- as the shock slowly disappears, $\Delta x = 0$ shifts back up and output and inflation return to long-run values
- if shock were permanent, output and inflation would never return to steady-state values
- note output and inflation move together – no policy tradeoffs between keeping inflation on target and output on target if no shocks to supply

- Monetary authority wants to offset positive demand shock
- Old Keynesian dynamics
  - $M \downarrow \rightarrow i \uparrow \rightarrow$ demand $\downarrow \rightarrow$with sticky prices $Y \downarrow$
  - over time $P \downarrow$ due to low demand, implying $\frac{M}{P}$ back up
  - model is stable, and economy returns to equilibrium over time

- New Keynesian dynamics
  - $i \uparrow \rightarrow Y$ and π both jump down
  - once shock goes away, everything returns to equilibrium
  - no dynamics other than dynamics of shock
  - model is unstable, so economy always in equilibrium
  - if output and inflation do not jump correct amount, off on explosive path

Taylor Rule

$$i_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

- When v_t increases, both π_t and x_t must jump to rule out explosions
  - Therefore error and rhs variables are correlated
  - There are no good instruments as need i_t to respond to the variables caused by the error
- δ_π and δ_x do not show up in the dynamics of the model
- They appear in the roots
- They need to be large enough for both roots to be unstable (requiring the jump)

- Write IS curve as a function of real interest rate gap

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t \pi_{t+1}\right) + u_t$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\hat{r}_t - \tilde{r}_t\right) \qquad \text{where } \tilde{r}_t = \sigma u_t$$

- $\tilde{r}_t$ is the Wicksellian real interest rate
- solve forward

$$x_t = -\frac{1}{\sigma}\sum_{i=0}^{\infty} E_t\left(\hat{r}_{t+i} - \tilde{r}_{t+i}\right)$$

- output depends negatively on expected future real interest rate deviations
- if interest rates are always expected to equal the Wicksellian real rate, then output gaps are zero
- Where is money in the model?

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)(\sigma \hat{y}_t - \hat{\imath}_t)$$

- money adjusts to get the desired interest rate

Demand

- Taste
  - Amend utility function to have a taste shock (ψ)

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{\psi_{t+i}^{1-\sigma} C_{t+i}^{1-\sigma}}{1-\sigma} \right) + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+_i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

  - Euler equation becomes

$$\psi_t^{1-\sigma} C_t^{-\sigma} = \beta E_t \psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma} \frac{(1+i_t)}{1+\pi_{t+1}}$$

  - Linearized around the zero inflation steady state

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} \left(\hat{\imath}_t - E_t \pi_{t+1} \right) + \left(\frac{\sigma - 1}{\sigma} \right) \left(E_t \psi_{t+1} - \psi_t \right)$$

Government spending

- Resource constraint changes

$$\hat{y}_t = \left(\frac{C}{Y}\right)^{ss} \hat{c}_t + \left(\frac{G}{Y}\right)^{ss} \hat{g}_t$$

- IS curve becomes

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\tilde{\sigma}} \left(\hat{\imath}_t - E_t \pi_{t+1}\right) + \xi_t$$

$$\begin{aligned} \xi_t \quad = \quad & \left(\frac{\sigma - 1}{\sigma}\right) \left(\frac{C}{Y}\right)^{ss} \left(E_t \psi_{t+1} - \psi_t\right) - \left(\frac{G}{Y}\right)^{ss} E_t \left(\hat{g}_{t+1} - \hat{g}_t\right) \\ & + \left(E_t \hat{y}^f_{t+1} - \hat{y}^f_t\right) \end{aligned}$$

$$\frac{1}{\tilde{\sigma}} = \frac{1}{\sigma} \left(\frac{C}{Y}\right)^{ss}$$

- expected changes matter

Supply

- If no shocks to supply, then stabilization of inflation stabilizes output gap
- Solving New Keyneisan Phillips Curve forward yields

$$\pi_t = \kappa \sum_{t=0}^{\infty} \beta^i E_t x_{t+i}$$

- Implies that if expected future output gaps are zero, then inflation is at its target of zero
- Supply shocks change this, raising inflation for each level of output and producing policy tradeoffs
- What might supply shocks be?

- Equation determining inflation before linearization, contained tastes, marginal costs, and productivity
- Shocks to these variables can add shocks to the Phillips curve
  - However, these shocks constitute shocks to the output gap evaluated at changing values of full-employment output
  - Remember the output gap is evaluated relative to a fixed steady state, so the equilibrium output gap responds to supply shocks
  - Monetary authority should be stabilizing output around the fluctuating output gap, yielding no policy tradeoffs

Sticky wages and prices

- If only prices are sticky, optimal policy adjusts to keep prices from ever having to adjust (zero inflation)
- If wages are sticky, and there are real shocks, then a policy which keeps prices from ever having to adjust could increase the output gap
  - Negative productivity shock reduces the marginal product of labor
  - Rigid nominal wages and a monetary policy to keep prices from adjusting would imply $\text{MPL}(N_0) < \frac{W}{P}$. Firms would reduce employment and output.
  - Now, there is a tradeoff between stabilizing inflation and the output gap.

Caplin and Spulber (1987)

- The firms most likely to adjust prices are the ones with the most suboptimal prices
- The distribution of prices (relative to optimal) is stationary
- If the initial distribution of prices is uniform, so is the stationary distribution
- A monetary shock changes the rate of adjustment, but not the distribution or prices implying that aggregate prices are not sticky

Conclusion

- New Keynesian model:
  1. Nominal rigidities: prices are sticky, i.e. slow to adjust.
  2. This leads to a role for stabilization policies.
- Next time: financial frictions.

- trace is sum of roots

$$\theta_1 + \theta_2 = \frac{1}{\beta} + 1 + \frac{\beta\delta_x + \kappa}{\beta\sigma}$$
$$= 1 + \frac{\delta_x}{\sigma} + \frac{1}{\beta}\left(1 + \frac{\kappa}{\sigma}\right)$$

- characteristic equation

$$\left[\frac{1}{\beta} - \theta\right]\left[1 + \frac{\beta\delta_x + \kappa}{\beta\sigma} - \theta\right] + \frac{\kappa}{\beta}\left[\frac{\beta\delta_\pi - 1}{\sigma\beta}\right] = 0$$

Stability requirements

- Since determinant and trace are positive, eigenvalues must be positive
- Therefore, stability requirement is

$$(\theta_1 - 1)(\theta_2 - 1) > 0$$

- Multiplying out yields

$$\theta_1\theta_2 - (\theta_1 + \theta_2) + 1 > 0 \tag{SR}$$

- Determinant is

$$\theta_1\theta_2 = \frac{1}{\beta}\left(1 + \frac{\delta_x}{\sigma} + \frac{\kappa\delta_\pi}{\sigma}\right)$$

- Trace is

$$\theta_1 + \theta_2 = 1 + \frac{\delta_x}{\sigma} + \frac{1}{\beta}\left(1 + \frac{\kappa}{\sigma}\right)$$

- Substituting into stability requirements (SR)

$$\frac{1}{\beta}\left(1 + \frac{\delta_x}{\sigma} + \frac{\kappa\delta_\pi}{\sigma}\right) - \left[1 + \frac{\delta_x}{\sigma} + \frac{1}{\beta}\left(1 + \frac{\kappa}{\sigma}\right)\right] + 1 > 0$$

- Simplifying

$$\delta_x\left(\frac{1}{\beta} - 1\right) + \frac{\kappa}{\beta}(\delta_\pi - 1) > 0$$

$$\delta_x(1-\beta) + \kappa(\delta_\pi - 1) > 0$$

Implying that the responses to the output gap and inflation must be large enough