Macro II

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Introduction

- Today: Financial frictions
- Introduce 2-period model that captures key insights about financial frictions:
  - uncertain returns on financial assets;
  - caused by information frictions;
  - can have real effects on economy.
- Will be a new homework soon (probably 2 more total).

Why study financial frictions?

- Data show a strong correlation between financial conditions and real economic activity
- What does this mean?
  - Real activity affects financial conditions
  - Financial dynamics amplify or extend the effects of real shocks: the "financial accelerator"
  - Financial shocks affect the real economy
- In an Arrow-Debreu world, financial frictions do not exist. We need
  - Incomplete markets to create frictions
  - Heterogeneity to make frictions relevant

What we need

- Incompleteness
  - Can be imposed exogenously
  - Endogenous incompleteness
    - Incomplete information
    - Limited enforcement (of repayment)
- Restricting internal financing
  - Finite life spans
  - Heterogeneous discounting
  - Tax incentives

A two-period model

- Two periods, 1 and 2: use primes to denote period-2 values
  - Period 1: Produce using capital and labor, invest in future capital
  - Period 2: Produce using capital accumulated in previous period
- Two types of agents with risk-neutrality (exogenously given)
  - Unit mass of workers who maximize

$$E\left(c - \frac{\ell^2}{2} + \beta c'\right).$$

  - Unit mass of entrepreneurs who maximize

$$E(c + \beta c').$$

- intuition: entrepreneurs take on risky projects with higher rewards.

Period 1 overview

- Entrepreneurs endowed with K units of capital and B units of debt (owed to workers)
- Entrepreneurs produce according to

$$Y = AK^{\alpha}\ell^{1-\alpha}.$$

- Capital accumulation and resource constraints

$$k' = K + \omega i, \tag{CA}$$
$$i = Y - c_w - c_e.$$

  ω is an idiosyncratic, entrepreneur-specific, shock with aggregate value of unity (b/c risk neutral)

  - distributed $\Phi(\omega)$
  - observed after the entrepreneur chooses i (investment)
  - distinction between workers and entrepreneurs

Period 2 overview

- Capital (produced by entrepreneurs in first period) is sold to entrepreneurs and worker-owned firms

$$k' = k'_w + k'_e$$

- Entrepreneurs produce according to

$$y'_e = a' k'_e,$$

- Worker-owned firms produce according to

$$y'_w = a' G(k'_w),$$

  where $G(\cdot)$ is strictly concave, with $G'(0) = 1$
- Entrepeneur capital more productive, but risky (ω unknown).
- Assume that $\beta a' > 1$, implying that can raise utility by postponing consumption through investment

Worker's problem

$$\max_{c_w,\ell,k_w',b'} \left\{ c_w - \frac{\ell^2}{2} + \beta c_w' \right\}$$

$$\begin{aligned} s.t. \quad & B + w\ell = c_w + \frac{b'}{R} + qk_w', \\ & c_w' = a'G(k_w') + b', \\ & c_w \geq 0; \quad c_w' \geq 0, \end{aligned}$$

where

- w is the real wage
- R is the gross interest rate earned on bonds
- q is the price of capital in terms of consumption/output

Solving worker's problem

- First order conditions for workers' problem (no uncertainty)

$$
\begin{aligned}
\ell &= w(1+\lambda), \\
(1+\lambda)q &= \beta a' G'(k_w'), \\
1+\lambda &= \beta R,
\end{aligned}
$$

where λ is the multiplier on $c_w \geq 0$ (Note: $\beta a' > 1 \Rightarrow c_w' > 0$ since agents will always want to transfer consumption forward.)

Entrepreneurs' problem

$$\max_{c_e,\ell,i,k_e',b'} E\left\{c_e + \beta c_e'\right\}$$

$$\begin{aligned} s.t. \quad & qk_e' = V + qK + (q\omega - 1)i + \frac{b'}{R} - B - c_e, \qquad \text{(EFBC)} \\ & V = AK^\alpha \ell^{1-\alpha} - w\ell, \\ & c_e' = a'k_e' - b', \\ & c_e \geq 0; \quad c_e' \geq 0; \quad i \geq 0. \end{aligned}$$

- c_e, b' and k_e' chosen after ω is realized
- i chosen before ω is realized
- B: initial bonds.
- V: firm profits.

Solving entrepreneur's problem

- First order conditions for entrepreneurs

$$\begin{aligned} w &= (1-\alpha)AK^{\alpha}\ell^{-\alpha} \\ qE(\omega) &\leq 1, \quad (= \text{if } i > 0), \\ (1+\gamma)q &= \beta a', \\ 1+\gamma &= \beta R, \end{aligned}$$

where γ is the multiplier on $c_e \geq 0$

Market clearing

- F.O.C. on b' for entrepreneur and worker imply $\gamma = \lambda$
  - Non-negativity constraint on period 1 consumption is either binding or not for both entrepreneur and worker
- Combine the F.O.C.'s for labor (ℓ) for worker (labor supply) and entrepreneur (labor demand) to get

$$\begin{aligned} w &= (1-\alpha)AK^{\alpha}[w(1+\lambda)]^{-\alpha} \\ &= (1-\alpha)^{\frac{1}{1+\alpha}} A^{\frac{1}{1+\alpha}} K^{\frac{\alpha}{1+\alpha}} (1+\lambda)^{\frac{-\alpha}{1+\alpha}} \\ \ell &= w(1+\lambda) \\ &= (1-\alpha)^{\frac{1}{1+\alpha}} A^{\frac{1}{1+\alpha}} K^{\frac{\alpha}{1+\alpha}} (1+\lambda)^{\frac{1}{1+\alpha}}, \end{aligned}$$

  so that labor, current output, and profits $\left(V = AK^{\alpha}\ell^{1-\alpha} - w\ell = \alpha Y\right)$ are all increasing in current productivity, A, and the multiplier λ

Frictionless market, no uncertainty ($\omega = 1$)

$$\beta a' = \beta a' G'(k_w'),$$

- so that $k_w' = 0$ and $k' = k_e'$
- $\rightarrow$ entrepreneurs use all the capital
- why? entrepreneurs use risky production tech. with higher returns
- here no risk.

Uncertainty, high returns $E\omega = 1$

- Investment occurs $(i > 0)$, and

$$\begin{aligned} q &= \frac{1}{E(\omega)} = 1 \\ \lambda &= \beta a' q^{-1} - 1 \\ &= \beta a' - 1 \geq 0, \end{aligned}$$

  so that labor, current output and profits are all increasing in future productivity, a'
- Additionally

$$c_w = c_e = 0$$

$$i = Y$$

  so that all output is optimally invested

Uncertainty, low returns $E\omega = 0$

- No investment occurs
- Consumption for both worker and entrepreneur in first period must be positive satisfying resource constraint implying that

$$\lambda = \gamma = 0$$

- Actual price of capital

$$q = \beta a'$$

- No benefits of transferring resources forward due to low expected price for capital
- Inefficient outcome because output not transferred forward due to expectations, implying that expectations have real effects

Costly state verification

- Asymmetric information between borrowers and lenders
- Sources of funds: $i = N + D$, where

$$N = qK + V - B$$

  is the entrepreneur's net worth
- $D \geq 0$ is a within-period loan (no interest)
- D is repaid immediately after ω is realized and capital is produced
- qK: value of capital
- V: profits
- B: bonds (debt).

Incomplete information

- Entrepreneur observes realization of ω
- Lender observes ω only by paying cost μi
- Solution (Townsend, 1979): standard debt contract
  - If the borrower pays $(1 + r_k)D$, lender doesn't check ω
  - If the borrower pays less than $(1 + r_k)D$, lender pays cost and verifies ω
- With incomplete information, equity requires verification every period and is therefore less efficient

Contract

- Verification ensures that borrower defaults only if $\omega < \tilde{\omega}$, where

$$\widetilde{\omega} q i = \widetilde{\omega} q(N + D) = (1 + r_k)D,$$
$$\Rightarrow \widetilde{\omega} = \widetilde{\omega}(r_k, q, N, D) = \frac{1 + r_k}{q} \left(\frac{D}{N + D} \right).$$

- i.e., only default if really bad shock.
- Competitive lenders implies a zero-profit condition

$$\int_0^{\widetilde{\omega}(r_k,q,N,D)} (\omega - \mu) q(N + D) \, d\Phi(\omega) + \int_{\widetilde{\omega}(r_k,q,N,D)}^{\infty} (1 + r_k) D \, d\Phi(\omega) = D.$$

defines $r_k(q, N, D)$ and $\widetilde{\omega}(q, N, D) = \widetilde{\omega}(r_k(q, N, D), q, N, D)$

Contract II

- Competitive lenders implies a zero-profit condition

$$\int_0^{\widetilde{\omega}(r_k,q,N,D)} (\omega - \mu) q(N + D)\; d\Phi(\omega) + \int_{\widetilde{\omega}(r_k,q,N,D)}^{\infty} (1 + r_k) D\; d\Phi(\omega) = D.$$

  defines $r_k(q, N, D)$ and $\widetilde{\omega}(q, N, D) = \widetilde{\omega}(r_k(q, N, D), q, N, D)$
- Solve for interest rate on debt

$$(1 + r_k) = \frac{1 - \int_0^{\widetilde{\omega}} (\omega - \mu) q(\frac{N+D}{D})\; d\Phi(\omega)}{\int_{\widetilde{\omega}}^{\infty} d\Phi(\omega)}$$

- Smaller $N \rightarrow$ smaller repayments in default
- $\rightarrow$ and a larger $\widetilde{\omega}$,
- $\rightarrow$ probability of default $\uparrow$.
- $\rightarrow$ Interest rate on debt increases.
- Real effects on economy.

Optimal loans

- Lenders compete to make loans, offer contracts that maximize entrepreneur's profits
- The optimal loan $D(N, q)$ is picked to solve

$$\max_{D} \int_{\widetilde{\omega}(q,N,D)}^{\infty} \Pi(\omega, q, N, D)\; d\Phi(\omega),$$
$$\Pi(\omega, q, N, D) = \omega q(N + D) - [1 + r_k(q, N, D)]D.$$

- This yields

$$D(N, q)$$
$$r_k(q, N) = r_k(q, N, D(q, N))$$
$$\widetilde{\omega}(q, N) = \widetilde{\omega}(q, N, D(q, N))$$
$$\Pi(\omega, q, N) = \max\left\{0, \Pi(\omega, q, N, D(q, N))\right\}$$

(don't enter or do)

Savings

- Now, entrepreneur can save after realizing ω.
- The entrepreneur's hiring decision is same as before, and is independent of his other decisions (given ω)
  - After ω has been realized, the entrepreneur solves

$$
\begin{aligned}
&\max_{c_e, k_e', b'} \{c_e + \beta c_e'\} \\
&s.t. \quad qk_e' = \Pi(\omega, q, N) + \frac{b'}{R} - c_e, \qquad \text{(EFBC2)} \\
&\qquad\;\; c_e' = a' k_e' - b', \\
&\qquad\;\; c_e \geq 0; \quad c_e' \geq 0.
\end{aligned}
$$

- The worker's problem is unchanged

- First-order conditions

$$\begin{aligned}(1+\lambda)q &= \beta a' G'(k'_w),\\ 1+\lambda &= \beta R,\\ (1+\gamma)q &= \beta a',\\ 1+\gamma &= \beta R,\end{aligned}$$

where λ and γ are multipliers

Savings scenarios

- Scenario 1: N is large enough to support $i = Y$. Similar to frictionless case
- Scenario 2: N is smaller so that $i < Y$
- Focus on scenario 2

Scenario 2

- With $i < Y$, there is period-1 consumption $\Rightarrow \lambda = \gamma = 0$ and

$$\begin{aligned} q &= \beta a' > 1, \\ R &= \beta^{-1}, \end{aligned}$$

- Effect of increasing A (aggregate shock period-1)
  - N increases $\Rightarrow i = N + D$ increases, implying that y' increases
  - Unless $\Delta i > \Delta Y$, increase in y' is less than in the frictionless environment

Dynamics

- Effects of increasing a' (aggregate shock period-2)
  - $q = \beta a'$ increases
  - q increases $\rightarrow$ investment increases $\rightarrow$ y' increases
  - q increases $\rightarrow$ investment more profitable $\rightarrow$ D increases $\rightarrow$ y' increases
- Quantitative performance
  - Frictions dampen initial responses to TFP shocks
  - Carlstrom and Fuerst (AER 1997): in a multi-period model, net worth accumulation generates a hump-shaped impulse response function
  - Capital adjustment costs that affect q increase propagation (Bernanke, Gertler and Gilchrist, 1999)

Conclusion

- Today: Financial frictions
- Next time: maybe more financial frictions or income fluctuation problem/heterogeneous agents.