# Macro II

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# Announcements

- Today: Continue heterogeneous agent models.
- First: Huggett and Aiyagari.
- See my website for homework.

# Thinking about Uncertainty in Macroeconomic Models

- Typical assumptions in macroeconomics are a convex combination of
  1. certainty equivalence:

$$u'(\bar{c}_{i,t}) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(\bar{c}_{i,t+1})}_{Closer\ to\ Linear}] \tag{1}$$

  2. linearized decision rules:

$$\sum_{i=1}^{N}((1 + r_{t+1})a_{i,t+1} + w_{i,t+1} - c_{i,t+1} - a_{i,t+2}) = 0 \tag{2}$$

$$\sum_{i=1}^{N}((1 + r_{t+1})\beta_a \hat{S}_{i,t+1} + \beta_w(\hat{S}_{i,t+1}) - \beta_c \hat{S}_{i,t+1} - \beta_a \hat{S}_{i,t+2}) = 0 \tag{3}$$

- Trick in Krusell-Smith: assume that workers make a linear prediction about prices in the future.
- i.e., workers use OLS to predict future prices.

# Heterogeneous Agent Models

- Workers change their behavior in response to uncertainty.
- First wave of heterogeneous agent models: how do aggregates change when *individual idiosyncratic* uncertainty is uninsurable.
- In other words: when agents must accumulate *precautionary savings* to insure against income shocks.
- Key "first wave" papers (no particular order):
    - Huggett (1993): Incomplete markets exchange economy with GE interest rate.
    - Imrohoroglu (1989): Individual and aggregate uncertainty with fixed interest rate.
    - Aiyagari (1994): Incomplete markets production economy with GE interest rate.
    - Bewley (1986): Individual uncertainty with fixed interest rate.
- Krusell and Smith (1998): individual and aggregate uncertainty with GE interest rate.
- Do this using an approximation to the aggregate evolution of capital.

# Heterogeneous Agent Models

- We can write a generic worker's problem as

$$\max_{\{c_t, i_t, l_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

$$\text{s.t. } c_t + i_t \leq r_t a_t + w_t l_t \tag{5}$$

$$a_{t+1} = (1-\delta) a_t + i_t \tag{6}$$

$$a_{t+1} \geq \underline{a}_t \tag{7}$$

$$w_t \sim F \tag{8}$$

$$c_t \geq 0, l_t \geq 0, a_0 \text{ given} \tag{9}$$

- How we deal with prices $r_t, w_t$ and choices $c_t, i_t, l_t$ is central to equilibrium.

# Recursive Formulation

- Can be written as

$$
\begin{aligned}
V(a) &= u(c) + \beta E[V(a')] && (10)\\
\text{s.t. } c + i &\leq ra + wl && (11)\\
a' &= (1-\delta)a + i && (12)\\
a' &\geq \underline{a} && (13)\\
w &\sim F && (14)\\
c \geq 0, l &\geq 0, a_0 \text{ given} && (15)
\end{aligned}
$$

- Under fairly general conditions, this inherits same properties as non-stochastic version.

# Huggett (1993)

- Endowment economy, no aggregate risk.
- Setup:
  - Discrete time;
  - Continuum of heterogeneous agents;
  - Idiosyncratic endowment risk (labor income stochastic).
- Single bond, $a$, can be borrowed or saved.
- Borrowling limit, $\underline{a} \leq 0 \leq a_{it}$

# Idiosyncratic Markov Income Uncertainty

- Suppose $wl = e$, $F[e'] = \pi(e'|e)$
- Two states: $e_l, e_h$
- Can be written as

$$V(a,e) = u(c) + \beta \sum_{e'} \pi(e'|e) V(a',e') \tag{16}$$

$$\text{s.t. } c + a' \leq (1+r)a + e \tag{17}$$

$$a' \geq \underline{a} \tag{18}$$

$$c \geq 0, a_0 \text{ given} \tag{19}$$

- Agents want to build precautionary savings again idiosyncratic risk.

# Equilibrium

- Define a distribution of agents over assets *as* and endowments $e$, $\psi$.
- Stationary equilibrium: aggregate state $(\psi)$ is unchanging.
- Agents move around distribution, but LLN $\rightarrow \psi' = \psi$
- Define $\psi(B)$ such that given transition function $P$:

$$\psi(B) = \int_S P(x, B) d\psi \tag{20}$$

- $P(x, B)$ the probability that an agent with state $x$ will have state $B \in \beta_S$ next period.
- $B$ is a subset of the state space.

# Stationary Equilibrium

- Roughly summarizing Huggett, 1993: A stationary equilibrium for this economy is a tuple $(c, a', r, \psi)$ that satisfy
  1. $c$ and $a'$ solve the workers problem taking prices as given.
  2. Markets clear:
     2.1 consumption = production: $\int c(x) d\psi = \int e d\psi$
     2.2 no net savings: $\int a(x) d\psi = 0$
  3. $\psi$ is stationary:

$$\psi(B) = \int_S P(x, B) d\psi \tag{21}$$

for all $B \in \beta_S$

# Aiyagari (1994)

- Production economy, no aggregate risk.
- Firms employ capital, households save using capital (really assets loaned/borrowed from firm).
- Setup:
  - Discrete time;
  - Continuum of heterogeneous agents;
  - Idiosyncratic hours shocks (labor supply stochastic).
- Capital, $k$, can be borrowed or saved.
- Borrowling limit, $\underline{k} \leq 0 \leq k_{it}$

# Heterogeneous Agent Production Economy

- In a production economy, the agent's problem is given by

$$
\begin{aligned}
V(k,\epsilon;\psi) &= u(c) + \beta E[V(k'\epsilon';\psi')] && (22)\\
\text{s.t. } c + k' &\leq (1 + r(K,L) - \delta)k + w(K,L)\epsilon && (23)\\
k' &\geq \underline{k} && (24)\\
\epsilon &\sim \text{ Markov} P(\epsilon'|\epsilon) && (25)\\
\psi' &= \Psi(\psi) && (26)\\
c \geq 0, k &\geq 0, k_0 \text{ given} && (27)
\end{aligned}
$$

- $\epsilon$ is a markov process that yields hours worked.
- $\Psi$ is an unspecified evolution of the aggregate state $(k,\epsilon)$.
- Prices are determined from the firm's problem

# Prices - The Firm's Problem

- How we handle prices determines the difficulty of this problem.
- In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} F(K, L) - wL - rK \tag{28}$$

- This yields standard competitive prices for the rental rates.

# Information

- What information do workers need in order to be able to solve this problem?
- Current period:
  - interest rate, $r(K, L)$. This is known from being told the aggregates at the beginning of the period.
  - wage rate, $w(K, L)$. This is known from being told the aggregates at the beginning of the period.
- Future:
  - interest rate and wage rate next period.
  - These depend on capital and labor next period.
  - Thus, workers need to predict capital and labor in future.
- Rep. Agent model: just need to know their own decision rule.
- Here: need to know distribution across workers, and their decision rules.

# Stationary Recursive Competitive Equilibrium

- A stationary RCE is given by pricing functions $r, w$, a worker value function $V(k, \epsilon; \psi)$, worker decision rules $k', c$, a type-distribution $\psi(k, \epsilon)$, and aggregates $K$ and $L$ that satisfy
  1. $k'$ and $c$ are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules: $\psi$ is the stationary distribution implied by worker decision rules.
  4. Aggregates are consistent with individual policy rules: $K = \int k d\psi$, $L = \int \epsilon d\psi$

# Return to Capital

- How does return to capital vary by
  - serial corr. ($\rho$) in labor income (think AR1 process)
  - and CRRA ($\mu$)?

TABLE II

A. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.2$)

| $\rho \backslash \mu$ | 1 | 3 | 5 |
|---|---|---|---|
| 0 | 4.1666/23.67 | 4.1456/23.71 | 4.0858/23.83 |
| 0.3 | 4.1365/23.73 | 4.0432/23.91 | 3.9054/24.19 |
| 0.6 | 4.0912/23.82 | 3.8767/24.25 | 3.5857/24.86 |
| 0.9 | 3.9305/24.14 | 3.2903/25.51 | 2.5260/27.36 |

B. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.4$)

| $\rho \backslash \mu$ | 1 | 3 | 5 |
|---|---|---|---|
| 0 | 4.0649/23.87 | 3.7816/24.44 | 3.4177/25.22 |
| 0.3 | 3.9554/24.09 | 3.4188/25.22 | 2.8032/26.66 |
| 0.6 | 3.7567/24.50 | 2.7835/26.71 | 1.8070/29.37 |
| 0.9 | 3.3054/25.47 | 1.2894/31.00 | −0.3456/37.63 |

- Higher $\rho$ or $\mu$, more saving, lower return.

# Krussell-Smith (1998)

- In the previous model, we relied on the aggregate *certainty* of $\psi(k, \epsilon)$ for a solution by appealing to the law of large numbers.
- i.e., individuals move around the distribution, but those shocks offset and in the aggregate the distribution is unchanged.
- But what happens if there is aggregate *uncertainty*?
- Now the distribution changes in the equilibrium, and we need a way to incorporate this into worker decision rules.
- Krussell-Smith: Aiyagari + aggregate shocks.
- Some details:
  - Idiosyncratic labor shock $\{0,1\}$ markov.
  - Aggregate shocks.
  - Idiosyncratic shock prob. changes with agg. shocks.

# Aggregate Uncertainty

- In a production economy, the agent's problem is given by

$$
\begin{aligned}
V(k, \epsilon, z; \psi) &= u(c) + \beta E[V(k'\epsilon', z'; \psi')] && (29)\\
\text{s.t. } c + k' &\leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon && (30)\\
k' &\geq \underline{k} && (31)\\
z' &= \text{ Markov} P(z'|z) && (32)\\
\epsilon &\sim \text{ Markov} P(\epsilon'|\epsilon, z') && (33)\\
\psi' &= \Psi(\psi, z, z') && (34)\\
c \geq 0, k \geq 0, k_0 &\text{ given}, z_0 \text{ given} && (35)
\end{aligned}
$$

- $\epsilon$ is a markov process for employment $\epsilon \in \{0, 1\}$
- $\Psi$ is an unspecified evolution of the aggregate state.
- $z$ *also* evolves as a markov process.
- Prices are determined from the firm's problem.

# Prices - The Firm's Problem

- How we handle prices determines the difficulty of this problem.
- In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} zF(K, L) - wL - rK \tag{36}$$

- This yields standard competitive prices for the rental rates.

# Laws of Motion

- The future aggregate state enters the probability of employment.
- This means that it impacts **all** of the laws of motion:

$$z' = \text{Markov} P(z'|z) \tag{37}$$

$$\epsilon \sim \text{Markov} P(\epsilon'|\epsilon, z') \tag{38}$$

$$k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon - c \tag{39}$$

$$\psi' = \Psi(\psi, z, z') \tag{40}$$

- Because shocks to $z$ change employment status and prices.

# Recursive Competitive Equilibrium

- An RCE is given by pricing functions $r, w$, a worker value function $V(k, \epsilon, z; \psi)$, worker decision rules $k', c$, a type-distribution $\psi(k, \epsilon)$, and aggregates $K$ and $L$ that satisfy
  1. $k'$ and $c$ are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules: $\psi$ is the distribution implied by worker decision rules given the aggregate state.
  4. Aggregates are consistent with individual policy rules: $K = \int k d\psi$, $L = \int \epsilon d\psi$

# Type Distribution

- The type distribution is a problem.
- Each policy function and transition depends on the type distribution.
- But the type distribution is time-varying in response to aggregate shocks.
- Alternative: use a smaller number of moments that can be calculated quickly to characterize the type distribution.
- Like a "sufficient statistic" for the type distribution.
- Discuss the solution to this next time.

# Business Cycle Effects

- This model is built to handle stochastic shocks.
- How do heterogeneous agents respond over a business cycle?

TABLE 2

AGGREGATE TIME SERIES

| Model | Mean($k_t$) | Corr($c_t$, $y_t$) | Standard Deviation ($i_t$) | Corr($y_t$, $y_{t-4}$) |
|---|---|---|---|---|
| Benchmark: | | | | |
| Complete markets | 11.54 | .691 | .031 | .486 |
| Incomplete markets | 11.61 | .701 | .030 | .481 |
| $\sigma = 5$: | | | | |
| Complete markets | 11.55 | .725 | .034 | .551 |
| Incomplete markets | 12.32 | .741 | .033 | .524 |
| Real business cycle: | | | | |
| Complete markets | 11.56 | .639 | .027 | .342 |
| Incomplete markets | 11.58 | .669 | .027 | .339 |
| Stochastic-$\beta$: | | | | |
| Incomplete markets | 11.78 | .825 | .027 | .459 |

# Conclusion

- Next time: Solving heterogeneous agent models.