PhD Macro II:
What is a Macro Model?

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Announcements

- Today: Basic two-period consumption-savings model.
- Use to understand what we are doing with macro models.
- Key: macro models are
  - difference equations from a convex optimization problem
  - that are resolved by a specified equilibrium concept.
- i.e., we specify a what we think the world looks like.
- Then we show how people would figure out that world.
- Then we show how those decisions aggregate.
- Will get you access to the cluster today.
- Homework due (next) Thursday.

Basic two-period model

- A (very) basic consumption-savings model:

$$\max_{c_1, a_2, c_2} u(c_1) + \beta u(c_2) \tag{1}$$

$$\text{s.t. } c_1 + a_2 = (1+r)a_1 + w_1 \tag{2}$$

$$c_2 = (1+r)a_2 + w_2 \tag{3}$$

- What is this?:
  - concave return function (sum of concave functions is concave)
  - over convex set (budget constraint).
- Two (philosophical) ways to think about solving this problem:
  1. We are solving a decision problem of an agent, then aggregating to clear markets.
  2. We are deriving a set of difference (cont. time $\implies$ differential) equations and finding an equilibrium.
- Keep **both** in mind (will return to this later).

Euler Equation

- We solve this and get an Euler Equation:

$$u'(c_1) = \beta(1 + r)u'(c_2) \tag{4}$$

- What does this say?
  1. Agents will *allocate* their budget between two periods according to this equation.
  2. This expression tells us the growth path of consumption, given c_0.
- Euler equation: absolute, fundamental, key equation in every (dynamic) macro model.
- Note: Euler equation *need not* be over consumption.
- Budget constraint tells us path of assets/consumption for a given initial condition.

Euler Equation

- The Euler Equation tells us the evolution of consumption in an economy.
- That is, it determines the dynamics.
- The effect of taxes, the presence of frictions or wedges, adjustment costs, etc. can usually be distilled to the following:

$$u'(c_1) = (1+\Delta)\beta(1+r)u'(c_2) \tag{5}$$

- where Δ is a distortion in the economy, i.e. a friction that prevents the market from realizing the perfectly competitive equilibrium.
- These features change the marginal utility of consumption over time, and thus distort the path of consumption.

Key Insight II: Portfolio Allocation

- Let's return to the two-period model:

$$\max_{c_1, a_2, \ell, c_2} u(c_1, \ell) + \beta u(c_2, 1) \tag{6}$$

$$\text{s.t. } c_1 + a_2 = (1+r)a_1 + w_1(1-\ell) \tag{7}$$

$$c_2 = (1+r)a_2 \tag{8}$$

- Now agents are optimizing over consumption **and** leisure.
- At first blush, this looks like it could become more difficult.

Portfolio Allocation

- When we solve this model, we get

$$u_1(c_1, \ell^*) = \beta(1+r)u_1(c_2, 0) \quad (9)$$

- But also

$$\frac{\partial V}{\partial \ell} = u_2(c_1^*, \ell) - w\lambda = 0 \quad (10)$$

$$u_2(c_1^*, \ell) = wu_1(c_1^*, \ell) \quad (11)$$

- and

$$c_1 + a_2 = (1+r)a_1 + w_1(1-\ell) \quad (12)$$

- Now we have an equation that determines dynamics (Euler Equation) & one that gives corresponding change in assets.
- **And** a **static** equation that determines the allocation of resources within a period (Portfolio Allocation).
- A lot of problems boil down to these two equations (possibly more with additional static choices).

Models as Dynamic Systems

- Two (philosophical) ways one might think about solving this problem:
  1. We are solving a decision problem of an agent, then aggregating to clear markets.
  2. We are deriving a difference equation and finding an equilibrium.
- Now, we'll briefly discuss the second interpretation.

Neoclassical Growth Model

- The baseline model for most of modern macro (value function representation):

$$V(k_t) = \max_{c_t} u(c_t) + \beta V(k_{t+1}) \tag{13}$$

$$\text{s.t. } c_t + k_{t+1} = k_t^\alpha + (1-\delta)k_t \tag{14}$$

- We have a recursive formulation &
- We have a dynamic equation for capital.
- What we will solve for:
  - Euler Equation;
  - Steady state capital and consumption.

Neoclassical Growth Model

$$V_t(k_t) = \max_{c_t} u(c_t) + \beta V_{t+1}(k_{t+1}) \tag{15}$$

$$\text{s.t. } c_t + k_{t+1} = k_t^\alpha + (1-\delta)k_t \tag{16}$$

- Solving this:

$$\frac{\partial V_t}{\partial c_t} = -\lambda + u'(c_t) = 0 \tag{17}$$

$$\frac{\partial V_t}{\partial k_{t+1}} = -\lambda + \beta \frac{\partial V_{t+1}}{\partial k_{t+1}} = 0 \tag{18}$$

- Envelope condition:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial V_t}{\partial k_t} = \lambda(\alpha k_t^{\alpha-1} + (1-\delta)) \tag{19}$$

Neoclassical Growth Model

- FOCs:

$$\frac{\partial V_t}{\partial c_t} = -\lambda + u'(c_t) = 0 \tag{20}$$

$$\frac{\partial V_t}{\partial k_{t+1}} = -\lambda + \beta \frac{\partial V_{t+1}}{\partial k_{t+1}} = 0 \tag{21}$$

- Envelope condition:

$$\frac{\partial V_{t+1}}{\partial k_{t+1}} = \frac{\partial V_t}{\partial k_t} = \lambda(\alpha k_t^{\alpha-1} + (1-\delta)) \tag{22}$$

- Putting these together gives us the Euler Equation:

$$u'(c_t) = \beta(\alpha k_t^{\alpha-1} + (1-\delta))u'(c_{t+1}) \tag{23}$$

- This & BC give us dynamics of neoclassical growth model.

Steady State

- What is a steady state and why do we care?
- It is challenging in general to characterize the solution to our model:
- Even if we specify a utility function, it will have no closed form solution unless $\delta = 1$.
- But we can characterize the solution in the steady-state, i.e., where variables are constant over time:
- $c_t = c_{t+1} = c^*$, $k_t = k_{t+1} = k^*$.

Steady State

- But we can characterize the solution in the steady-state, i.e., where variables are constant over time:
- $c_t = c_{t+1} = c^*$, $k_t = k_{t+1} = k^*$.
- pick $u(c) = ln(c)$. then

$$\frac{1}{c_t} = \beta(\alpha k_t^{\alpha-1} + (1-\delta))\frac{1}{c_{t+1}} \tag{24}$$

- In steady state:

$$\frac{1}{c^*} = \beta(\alpha k^{*\alpha-1} + (1-\delta))\frac{1}{c^*} \tag{25}$$

- Why would the Euler Equation in the steady-state only be a function of capital?

Steady State

- This leaves us with capital:

$$1 = \beta(\alpha k^{*\alpha-1} + (1-\delta)) \tag{26}$$

$$k^* = (\frac{1}{\alpha\beta} - \frac{(1-\delta)}{\alpha})^{\frac{1}{\alpha-1}} \tag{27}$$

$$k^* = (\frac{\alpha\beta}{1-\beta(1-\delta)})^{\frac{1}{1-\alpha}} \tag{28}$$

- Now consumption from the budget constraint:

$$c^* + k^* = k^{*\alpha} + (1-\delta)k^* \tag{29}$$

$$c^* = k^{*\alpha} - \delta k^* \tag{30}$$

$$c^* = (\frac{\alpha\beta}{1-\beta(1-\delta)})^{\frac{\alpha}{1-\alpha}} - \delta(\frac{\alpha\beta}{1-\beta(1-\delta)})^{\frac{1}{1-\alpha}} \tag{31}$$

- Why would consumption be determined by the budget constraint, not the Euler Equation?

Dynamics

- Outside of steady-state we need to think about dynamics, i.e., how model evolves or fluctuates (in presence of shocks).
- Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1-\delta))c_t \tag{32}$$

$$k_{t+1} = k_t^{\alpha} + (1-\delta)k_t - c_t \tag{33}$$

- We have two dynamic variables: c and k.
- The behavior of this system will depend on their dynamics.

Dynamics

- Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1-\delta))c_t \tag{34}$$
$$k_{t+1} = k_t^{\alpha} + (1-\delta)k_t - c_t \tag{35}$$

- The behavior of this system will depend on their dynamics.
- At steady-state:

$$1 = \frac{c_{t+1}}{c_t} = \beta(\alpha k_t^{\alpha-1} + (1-\delta)) \tag{36}$$
$$1 = \frac{k_{t+1}}{k_t} = k_t^{\alpha-1} + (1-\delta) - \frac{c_t}{k_t} \tag{37}$$

- If both hold, we are in steady-state, if not, quantities can vary dynamically.

Dynamics

- Dynamics:

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1-\delta))c_t \tag{38}$$
$$k_{t+1} = k_t^{\alpha} + (1-\delta)k_t - c_t \tag{39}$$

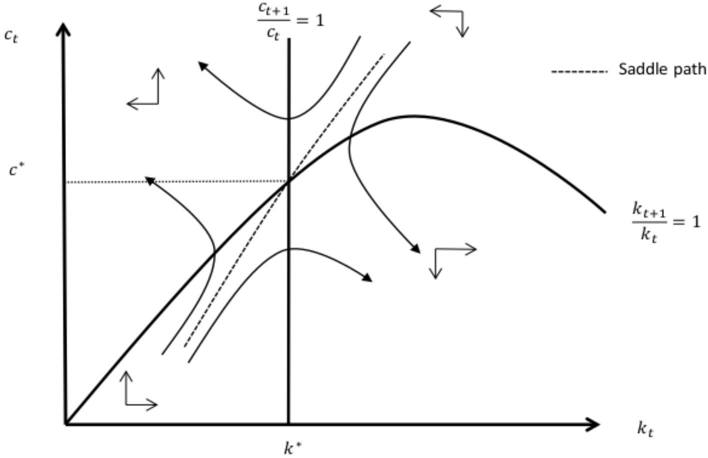
- Small value of c_t: second equation dictates that $k_t \uparrow$.
- Small value of k_t: first equation dictates that $c_t \uparrow$.
- Reverse is true.

Phase Diagram

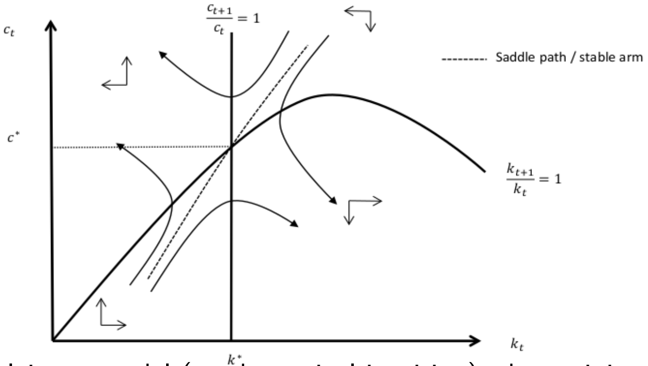
- Dynamics (figure from Eric Sim's notes):

$$c_{t+1} = \beta(\alpha k_t^{\alpha-1} + (1-\delta))c_t \quad (40)$$

$$k_{t+1} = k_t^{\alpha} + (1-\delta)k_t - c_t \quad (41)$$



Phase Diagram



- Solving a model (mathematical intuition): determining rules that put us on the saddle path (dashed line).
- Same concept for a decentralized economy.
- Seeing these models as dynamic systems expands our toolbox for solving them.
- We will discuss this later.

Next Time

- Discuss important time series preliminaries.
- Be sure to start Matlab homework.
- See online for specific assignment.