

Macro II

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Introduction

- ▶ Today: consumption smoothing and permanent income.
- ▶ “The income fluctuation problem”
- ▶ New homework next week (no class on next Wednesday).

Thinking about Uncertainty in Macroeconomic Models

- ▶ Uncertainty makes macroeconomic models more difficult to solve.
- ▶ We make assumptions about the environment (preferences, technology, etc.) to decrease complexity of problem.
- ▶ Euler Equation:

$$u'(c_t) = \beta E[\underbrace{(1 + r_{t+1})}_{GE} \underbrace{u'(c_{t+1})}_{Non-linear}] \quad (1)$$

- ▶ Each agent chooses consumption and savings based on a
 1. general equilibrium object (given by the decision rules of all other agents)
 2. (potentially highly) non-linear marginal utility.

Today

- ▶ Today: Think about how workers insure against income risk.
- ▶ Foundation for consumption smoothing.
- ▶ Explore using different preferences:
 1. Certainty Equivalence - Quadratic Utility.
 2. Constant Absolute Risk Aversion - Exponential Utility.
- ▶ These each imply different ways in which agents respond to income shocks and uncertainty.
- ▶ We will return to this when we study heterogeneous agents.

Risk

- ▶ How do we typically think about risk in economic models?
- ▶ Absolute Risk Aversion:

$$AR = -\frac{u''(c)}{u'(c)} \quad (2)$$

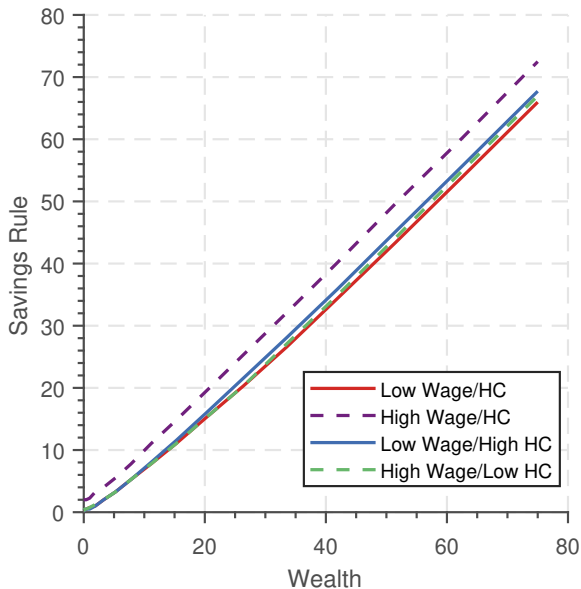
- ▶ A measure of the agent's risk aversion unconditional upon their level of wealth.
- ▶ Relative Risk Aversion:

$$RRA = -\frac{u''(c)c}{u'(c)} \quad (3)$$

- ▶ Conditioning upon an agent's wealth, how does his risk aversion change?
- ▶ Probably most reasonable are "DARA" "CRRA"
- ▶ These will have different implications for savings and consumption.

When approximations work

- ▶ For a lot of the distribution, decision rules are linear:



Introduction

- ▶ In the case of quadratic utility, we will see that agents don't change their consumption choices when faced with shocks.
- ▶ Uncertainty still decreases expected utility, but does not change choices.
- ▶ Why is this relevant? One solution technique (LQ) assumes that agents have a quadratic utility function (locally risk-neutral).
- ▶ We will see that this is sometimes not a great assumption.

Quadratic Utility

- Utility is given by the following:

$$\max E\left[\sum_{t=0}^{\infty} \beta^t (aC_t - bC_t^2)\right] \quad (4)$$

$$\text{s.t. } A_{t+1} = (1+r)A_t + Y_t - C_t \quad (5)$$

$$Y_{t+1} = \rho Y_t + \epsilon_{t+1} \quad (6)$$

Euler Equation

- ▶ Do the usual steps to find the Euler Equation:

$$V(A) = \max_{C, A'} aC_t - bC_t^2 + \beta E[V(A')] \quad (7)$$

$$\text{s.t. } A' = (1+r)A + Y - C \quad (8)$$

$$Y' = \rho Y + \epsilon' \quad (9)$$

$$\frac{\partial V}{\partial C} = a - 2bC - \lambda \quad (10)$$

$$\frac{\partial V}{\partial A'} = -\lambda + \beta E\left[\frac{\partial V}{\partial A'}\right] \quad (11)$$

$$\frac{\partial V}{\partial A} = (1+r)\lambda \quad (12)$$

$$\Rightarrow C = \beta(1+r)E[C'] \quad (13)$$

Certainty Equivalence

- ▶ Suppose that $\beta = (1 + r)$:

$$C = E[C'] \quad (14)$$

- ▶ Suppose that there were two states of the world: high and low.

$$C = P_h C_h + P_l C_l \quad (15)$$

- ▶ This is equivalent to an agent receiving the mean income between both states:

$$C = C_m \quad (16)$$

- ▶ i.e., workers make savings decisions *as though they are receiving the average consumption with certainty.*

Prudence

- ▶ Agents in this economy are not “prudential.”
- ▶ That is, they don’t change their choices based upon uncertainty about the future.
- ▶ Another way to express this is in the third derivative of the utility function:

$$U''' = 0 \tag{17}$$

- ▶ This captures the response of marginal utility (i.e., decisions) to uncertainty.
- ▶ Marginal utility changes linearly, so any convex combination is equal to the expected value.

Random Walk

- ▶ Can show for the AR(1) case:

$$C_t - C_{t-1} = \frac{r}{1 + r - \rho} \epsilon \quad (18)$$

- ▶ Now, consider the case in which income shocks are iid:

$$Y_{t+1} = Y_t + \epsilon_{t+1} \quad (19)$$

- ▶ Then the difference in consumption becomes:

$$C_t - C_{t-1} = \epsilon_t \quad (20)$$

- ▶ In other words, the agent consumes all of the shock in each period (will also happen with CRRA and autarky).
- ▶ This is a martingale!

Conclusion

- ▶ In the quadratic utility world, uncertainty does not change an agents decision when compared with an identical income stream.
- ▶ In the case of CARA utility, we will see that agents have precautionary savings that result from curvature in the utility function.
- ▶ The choices are the same as they would be under complete markets.

Introduction to CARA World

- ▶ Now, use CARA preferences to think about world in which certainty equivalence does not hold.
- ▶ Now, we will allow agents to be prudential in their savings response to future uncertainty.

Constant Absolute Risk Aversion Utility

- ▶ The maximization problem is given by

$$\max E\left[\sum_{t=0}^{\infty} -\frac{1}{\alpha} \exp(-\alpha C_t)\right] \quad (21)$$

$$\text{s.t. } A_{t+1} = A_t + Y_t - C_t \quad (22)$$

$$Y_t = \rho Y_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma^2) \quad (23)$$

- ▶ Key difference: first derivative (i.e., policy functions), no longer linear.

Euler Equation

- ▶ Bellman Equation (implicitly assume $\beta = (1 + r)$):

$$V(A) = \max_{C, A'} -\left(\frac{1}{\alpha}\right) \exp(-\alpha C) + E[V(A')] \quad (24)$$

$$\text{s.t. } A' = A + Y - C \quad (25)$$

$$Y' = \rho Y + \epsilon' \quad (26)$$

$$\frac{\partial V}{\partial C} = \exp(-\alpha C) - \lambda \quad (27)$$

$$\frac{\partial V}{\partial A'} = -\lambda + E\left[\frac{\partial V}{\partial A'}\right] \quad (28)$$

$$\frac{\partial V}{\partial A} = \lambda \quad (29)$$

$$\Rightarrow \exp(-\alpha C) = E[\exp(-\alpha C')] \quad (30)$$

Euler Equation

- ▶ Bellman Equation (implicitly assume $\beta = (1 + r)$):

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \quad (31)$$

- ▶ For normally distributed random variables, the following holds:

$$E[\exp(x)] = \exp(E[x] + \sigma_x^2/2) \quad (32)$$

- ▶ Thus, we can rewrite the Euler Equation as

$$\exp(-\alpha C) = E(\exp(-\alpha C' + \alpha^2 \sigma^2/2)) \quad (33)$$

$$\Rightarrow C' = C + \frac{\alpha \sigma^2}{2} + \nu \quad (34)$$

Policy Function

- ▶ Policy function:

$$\Rightarrow C' = C + \frac{\alpha\sigma^2}{2} + \nu \quad (35)$$

- ▶ This says that consumption is *increasing* ex-ante in response to uncertainty, measured by σ^2 .
- ▶ What does this mean about life-cycle consumption?
- ▶ We would expect it to be upward-sloping, at least initially.

Consumption in time t

- ▶ Can show:

$$C_t = \left(\frac{1}{T-t}\right)A_t + Y_t - \frac{\alpha(T-t-1)\sigma^2}{4} \quad (36)$$

- ▶ Certainty equivalence: last term is equal to zero. i.e., cake-eating problem.
- ▶ Agents consume less than they would if their income stream was certain!

Prudence

- ▶ What is different in this case?
- ▶ Agents are prudential: $U''' > 0$.
- ▶ The Euler Equation is given by:

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \quad (37)$$

- ▶ Suppose $C = C'$, then consider Jensen's Inequality:

$$\exp(-\alpha E(C)) < E[\exp(-\alpha C)] \quad (38)$$

- ▶ This needs to hold in equilibrium, thus agents must decrease current consumption.
- ▶ Agents save in excess of what they would under certainty!

CARA Utility

- ▶ When CARA agents cannot perfectly insure, they change their choices from the certainty equivalence (quadratic utility) case.
- ▶ Unfortunately, CARA has some problems: Marginal utility is finite when consumption is equal to zero.
- ▶ CRRA utility will solve this problem, but is more challenging to solve.

Permanent Income Hypothesis

- ▶ Theory developed by Milton Friedman that describes how agents allocate resources over their lifetime.
- ▶ Consumption is based on not just current income, but expectations over future income as well.
- ▶ Implies that agents want to consumption smooth, rather than consume out of transitory income shocks.

Lifetime Budget Constraint

- ▶ Solve the flow budget constraint forward

$$\begin{aligned}A_0 &= \frac{1}{1+r}A_1 - (y_0 - c_0) \\&= \frac{1}{1+r} \left(\frac{1}{1+r}A_2 - (y_1 - c_1) \right) - (y_0 - c_0) \\&= - \sum_{t=0}^T \left(\frac{1}{1+r} \right)^t (y_t - c_t) \\&\quad + \left(\frac{1}{1+r} \right)^{T+1} A_{T+1},\end{aligned}$$

- ▶ Impose No-Ponzi condition requiring, $A_{T+1} = 0$, to yield

$$A_0 = - \sum_{t=0}^T \left(\frac{1}{1+r} \right)^t (y_t - c_t)$$

Lifetime Budget Constraint

- ▶ Rearrange to derive the present value budget constraint

$$\sum_{t=0}^T \left(\frac{1}{1+r} \right)^t c_t(l_t) = A_0 + \sum_{t=0}^T \left(\frac{1}{1+r} \right)^t y_t(l_t),$$

(PVBC)

- ▶ Holds for all realized $\{y_t\}$
- ▶ Not an expectation
- ▶ Right-hand side of (PVBC) is lifetime wealth
- ▶ (PVBC) does not imply that the time path of consumption is known in advance

Derivation

- ▶ In finite-horizon case with $J \equiv T$, expected present value budget constraint (EPVBC) becomes

$$E_t \left\{ \sum_{j=0}^J \left(\frac{1}{1+r} \right)^j c_{t+j} \right\} = W_t. \quad (\text{EPVBC})$$

$$W_t \equiv A_t + E_t \left\{ \sum_{j=0}^J \left(\frac{1}{1+r} \right)^j y_{t+j} \right\}.$$

- ▶ As $J \rightarrow \infty$, (EPVBC) follows from (FBC) and (ENPG)

- ▶ Equation (EE') and the law of iterated expectations imply that

$$\begin{aligned} E_t(c_{t+2}) &= E_t(E_{t+1}(c_{t+2})) \\ &= E_t(c_{t+1}) \\ &= c_t, \end{aligned}$$

so that

$$\begin{aligned} E_t \left\{ \sum_{j=0}^J \left(\frac{1}{1+r} \right)^j c_{t+j} \right\} &= c_t \sum_{j=0}^J \left(\frac{1}{1+r} \right)^j \\ &\equiv R_J c_t. \end{aligned}$$

Permanent Income

- ▶ Define permanent income as the constant value of future income such that its present value equals the present value of actual income and assets

$$y_t^P \equiv \frac{1}{R_J} W_t$$

- ▶ (EPVBC) becomes

$$c_t = \frac{1}{R_J} W_t \equiv y_t^P$$

where

$$\sum_{j=0}^J \left(\frac{1}{1+r} \right)^j y_t^P = W_t.$$

- ▶ Therefore, consumption equals permanent income

Overview

- ▶ A temporary change in income leads to a permanent change in expected consumption: consumption smoothing extends the effects of income changes over time
- ▶ The effect of a change in current income on current consumption depends on its effect on permanent income
- ▶ Permanent changes in income have larger consumption effects than temporary changes

Empirical Implications (Friedman (1957))

- ▶ Consider the linear projection of consumption on total income

$$\hat{c}_t = \alpha_1 + \alpha_2 y_t$$

- ▶ For a cross-section of households at a point in time, $\alpha_1 > 0$, and α_2 is much less than 1
 - ▶ For a country over time, $\alpha_1 \approx 0$, and α_2 is closer to 1
- ▶ Define transitory income

$$y_t^T = y_t - y_t^P.$$

- ▶ Suppose $C(y_t^T, y_t^P) = 0$

Friedman (1957)

- ▶ The coefficient α_2 is given by

$$\begin{aligned}\alpha_2 &= \frac{C(y_t, c_t)}{V(y_t)} = \frac{C(y_t^T + y_t^P, y_t^P)}{V(y_t^T + y_t^P)} \\ &= \frac{V(y_t^P)}{V(y_t^P) + V(y_t^T)}.\end{aligned}$$

- ▶ Cross-section data: $V(y_t^T)$ is large because of wide variance of household transitory income implying small α_2
- ▶ Time-series data: $V(y_t^T)$ is small because transitory income averages out across households in the aggregate implying large α_2 close to one

Hall (1978)

- ▶ Consider an alternative equation

$$\hat{c}_t = \alpha_1 + \alpha_2 c_{t-1} + \gamma x_{t-1},$$

where x_{t-1} is some other variable

- ▶ Recall that under linear quadratic preferences

$$E_t(c_{t+1}) = \frac{a}{b} \left[1 - \frac{1}{\beta(1+r)} \right] + \frac{1}{\beta(1+r)} c_t,$$

so that $\gamma = 0$. Nothing should predict consumption except lagged consumption

- ▶ There is some evidence that $\gamma \neq 0$
- ▶ Perhaps permanent income changes over time and the change takes time for agents to realize so that c_{t-1} is not affected, but c_t is

Conclusion

- ▶ Next: Cover Asset Pricing and Lucas Free
- ▶ No class next Wednesday.
- ▶ New homework posted on next Monday to cover this and next week's lectures.