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6 Appendix A - Hansen's Model

Hansen (1985) writes a model that includes utility over consumption and the intensive margin of labor with stochastic shocks to production. Then, we can formulate his problem in the following way:

$$E\sum_{t=0}^{\infty}\beta^t[ln(C_t)-\gamma H_t] \tag{6.1}$$

subject to

$$Y_t = A_t K_t^{\theta}(\eta^t H_t)^{1-\theta} \tag{6.2}$$

$$ln(A_t) = (1-\rho)ln(A) + \rho ln(A_{t-1}) + \epsilon_t \quad \epsilon \sim N(0, \sigma_\epsilon^2) \tag{6.3}$$

$$Y_t = C_t + I_t \tag{6.4}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{6.5}$$

Taking the first order in C_t, H_t and I_t yields:

$$\frac{\partial L}{\partial H_t} = -\gamma + \lambda_t[(1-\theta)A_t(\frac{K_t}{H_t})^{\theta}\eta^{t(1-\theta)}] = 0$$

$$\frac{\partial L}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$

$$\frac{\partial L}{\partial I_t} = -\lambda_t + E[\beta\lambda_{t+1}(\theta A_t(\frac{\eta^t H_t}{K_t})^{1-\theta} + 1 - \delta)]$$

We have included η as a labor-augmenting growth factor. We can combine these first order conditions to remove the Lagrange multiplier and get a system of six equations from which we will get our solution:

$$\gamma = \frac{1}{C_t}[(1-\theta)A_t(\frac{K_t}{H_t})^{\theta}\eta^{t(1-\theta)}] \tag{6.6}$$

$$\frac{1}{C_t} = \beta E[\frac{1}{C_{t+1}}(\theta A_{t+1}(\frac{\eta^{t+1}H_{t+1}}{K_{t+1}})^{1-\theta} + 1 - \delta)] \tag{6.7}$$

Now, we detrend by taking out the growth factor η to get a system of equations from (6.2 - 6.7):

$$y_t = a_t k_t^{\theta} h_t^{1-\theta} \tag{6.8}$$

$$ln(a_t) = (1-\rho)ln(A) + \rho ln(a_{t-1}) + \epsilon_t \tag{6.9}$$

$$y_t = c_t + i_t \tag{6.10}$$

$$\eta k_{t+1} = (1-\delta)k_t + i_t \tag{6.11}$$

$$\gamma c_t h_t = (1-\theta)y_t \tag{6.12}$$

$$\frac{\eta}{c_t} = \beta E_t[\frac{1}{c_{t+1}}(\theta(\frac{y_{t+1}}{k_{t+1}}) + 1 - \delta)] \tag{6.13}$$

7 Appendix B - Steady State Values

Our goal is to log-linearize this system of equations around the steady-state; in order to accomplish this, we must first find steady-state values of each of these variables. We first find the steady state level of the shocks:

$$ln(a^*) = (1-\rho)ln(A) + \rho ln(a^*)$$

$$\Rightarrow ln(a^*) = ln(A) \tag{7.1}$$

To find the rest of the variables, we first suppose that we know some value for steady-state y; then we solve for k using (6.13):

$$\frac{\eta}{c^*} = \beta E_t[\frac{1}{c^*}(\theta(\frac{y^*}{k^*}) + 1 - \delta)]$$

$$\Rightarrow \frac{\eta}{\beta} = \theta\frac{y^*}{k^*} + 1 - \delta$$

$$\Rightarrow k^* = (\frac{\theta}{\frac{\eta}{\beta} - 1 + \delta})y^* \tag{7.2}$$

Using equation (6.11) with this above steady-state result, we can solve for steady state investment:

$$\eta k^* = (1-\delta)k^* + i^*$$

$$\Rightarrow (\eta - 1 + \delta)k^* = i^*$$

$$\Rightarrow i^* = (\frac{\theta(\eta - 1 + \delta)}{\frac{\eta}{\beta} - 1 + \delta})y^* \tag{7.3}$$

We use (6.10) to find steady-state consumption:

$$y^* = c^* + i^*$$

$$\Rightarrow y^* = c^* + (\frac{\theta(\eta - 1 + \delta)}{\frac{\eta}{\beta} - 1 + \delta})y^*$$

$$\Rightarrow c^* = [1 - (\frac{\theta(\eta - 1 + \delta)}{\frac{\eta}{\beta} - 1 + \delta})]y^* \tag{7.4}$$

We use (6.12) to find h^*:

$$\gamma c^* h^* = (1-\theta)y^*$$

$$\Rightarrow \gamma[1-(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]y^*h^* = (1-\theta)y^*$$

$$\Rightarrow h^* = (\frac{1-\theta}{\gamma})[1-(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]^{-1} \tag{7.5}$$

Finally, we use (6.8) to find steady state output:

$$y^* = a^* k^{*\theta} h^{*1-\theta}$$

$$y^* = a^*((\frac{\theta}{\frac{\eta}{\beta}-1+\delta})y^*)^{\theta}[(\frac{1-\theta}{\gamma})[1-(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]^{-1}]^{1-\theta}$$

$$y^{*1-\theta} = a^*(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})^{\theta}[(\frac{1-\theta}{\gamma})[1-(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]^{-1}]^{1-\theta}$$

$$y^* = a^{*\frac{1}{1-\theta}}(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})^{\frac{\theta}{1-\theta}}[(\frac{1-\theta}{\gamma})[1-(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]^{-1}]^{1-\theta} \tag{7.6}$$

8 Appendix C - Log-Linearizing Around the Steady-State

Using these equations, we can now log-linearize around the steady-state: we define $\tilde{x}_t = ln(x_t) - ln(x)$ and can think about this as deviations about the steady state. Because we expect the detrended economy to rarely stray far from these steady-state values, we think that this linear approximation will do a relatively good job approximating the equilibrium conditions at the benefit of much simplicity.

We first wish to rewrite $\tilde{x}_t = ln(x_t) - ln(x)$ in two convenient ways:

$$\tilde{x}_t = ln(\frac{x_t}{x})$$

Then, the first-order Taylor Approximation to this equation yields:

$$\tilde{x}_t \approxeq \tilde{x}_t(x) + \frac{\partial \tilde{x}_t}{\partial x_t}(x)(x_t - x)$$

$$\Rightarrow \tilde{x}_t \approxeq ln(1) + \frac{1}{x}(x_t - x)$$

Rearranging this yields:

$$x(\tilde{x}_t + 1) \approxeq x_t \tag{8.1}$$

We can also rewrite the equation for $\tilde{x}_t$ as

$$x_t = xe^{\tilde{x}_t} \tag{8.2}$$

Using (8.1 - 8.2), we will be able to log-linearize equations (6.8 - 6.13) around the steady-state values given by (7.1 - 7.6). Starting with equation (6.8):

$$ln(y_t) = ln(a_t) + \theta ln(k_t) + (1-\theta)ln(h_t)$$

$$ln(y) = ln(a) + \theta ln(k) + (1-\theta)ln(h)$$

$$\Rightarrow \tilde{y}_t = ln(y_t) - ln(y) = ln(a_t) + \theta ln(k_t) + (1-\theta)ln(h_t) - (ln(a) + \theta ln(k) + (1-\theta)ln(h))$$

$$\Rightarrow \tilde{y}_t = \tilde{a}_t + \theta\tilde{k}_t + (1-\theta)\tilde{h}_t \tag{8.3}$$

Using (6.9), we get the deviations for a_t:

$$ln(a_t) = (1-\rho)ln(A) + \rho ln(a_{t-1}) + \epsilon_t$$

$$ln(a) = (1-\rho)ln(A) + \rho ln(a)$$

$$\Rightarrow \tilde{a}_t = \rho\tilde{a}_{t-1} + \epsilon_t \tag{8.4}$$

From (6.10),

$$y_t = c_t + i_t$$

We use the approximation from (8.1):

$$y(\tilde{y}_t + 1) = c(\tilde{c}_t + 1) + i(\tilde{i}_t + 1)$$

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{i}{y}\tilde{i}_t$$

Here, we use the steady-state values for c, i and y to get

$$\tilde{y}_t = \frac{[1 - (\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})]y}{y}\tilde{c}_t + \frac{(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})y}{y}\tilde{i}_t$$

$$\Rightarrow (\frac{\eta}{\beta} - 1 + \delta)\tilde{y}_t = [\frac{\eta}{\beta} - 1 + \delta - \theta(\eta - 1 + \delta)]\tilde{c}_t + \theta(\eta - 1 + \delta)\tilde{i}_t \tag{8.5}$$

Now we use (6.11) to get the transition of k in log-linearized terms:

$$\eta k_{t+1} = (1-\delta)k_t + i_t$$

$$\Rightarrow \eta k(\tilde{k}_{t+1} + 1) = (1-\delta)k(\tilde{k}_t + 1) + i(\tilde{i}_t + 1)$$

$$\Rightarrow \eta\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \frac{i}{k}\tilde{i}_t + (1-\delta) + \frac{i}{k} - \eta$$

Subbing in for the steady-state values of i and k, we get:

$$\eta\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \frac{(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})y^*}{(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})y^*}\tilde{i}_t + (1-\delta) + \frac{(\frac{\theta(\eta-1+\delta)}{\frac{\eta}{\beta}-1+\delta})y^*}{(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})y^*} - \eta$$

$$\Rightarrow \eta\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + (\eta - 1 + \delta)\tilde{i}_t \tag{8.6}$$

Moving to (6.12), we get:

$$\gamma c_t h_t = (1-\theta)y_t$$

$$\Rightarrow y_t = \frac{\gamma}{1-\theta}c_t h_t$$

$$\Rightarrow ln(y_t) = ln(\gamma) - ln(1-\theta) + ln(c_t) + ln(h_t)$$

and around the steady-state,

$$\Rightarrow ln(y) = ln(\gamma) - ln(1-\theta) + ln(c) + ln(h)$$

Then, subtracting these two, we get

$$\tilde{y}_t = ln(y_t) - ln(y) = ln(\gamma) - ln(1-\theta) + ln(c_t) + ln(h_t) - ln(\gamma) - ln(1-\theta) + ln(c) + ln(h)$$

$$\Rightarrow \tilde{y}_t = \tilde{c}_t + \tilde{h}_t \tag{8.7}$$

Finally, we use the Euler equation (6.13) along with the alternative representation of the log-deviation (8.2):

$$\frac{\eta}{c_t} = \beta E_t[\frac{1}{c_{t+1}}(\theta(\frac{y_{t+1}}{k_{t+1}} + 1 - \delta))]$$

$$\Rightarrow \frac{\eta}{ce^{\tilde{c}_t}} = \beta E_t[\frac{1}{ce^{\tilde{c}_{t+1}}}(\theta(\frac{ye^{\tilde{y}_{t+1}}}{ke^{\tilde{k}_{t+1}}}) + 1 - \delta)]$$

$$\Rightarrow \frac{\eta}{\beta}e^{-\tilde{c}_t} = E[e^{-\tilde{c}_{t+1}}(\theta\frac{y}{k}e^{\tilde{y}_{t+1}}e^{-\tilde{k}_{t+1}} + 1 - \delta)]$$

Using (8.1), we know $e^{-\tilde{x}_t} \approxeq (1-\tilde{x}_t)$:

$$\frac{\eta}{\beta}(1-\tilde{c}_t) = E[(1-\tilde{c}_{t+1})(\theta\frac{y}{k}(1+\tilde{y}_{t+1})(1-\tilde{k}_{t+1}) + 1 - \delta)]$$

Since we believe that deviations from the steady-state will be relatively small once we account for the trend, terms of the type $\tilde{x}_t\tilde{z}_t$ are approximately zero:

$$\frac{\eta}{\beta}(1-\tilde{c}_t) = E[(1-\tilde{c}_{t+1})(\theta\frac{y}{k}(1+\tilde{y}_{t+1}-\tilde{k}_{t+1})+1-\delta)]$$

$$\Rightarrow \frac{\eta}{\beta}(1-\tilde{c}_t) = E[\theta\frac{y}{k}(1+\tilde{y}_{t+1}-\tilde{k}_{t+1})+1-\delta-\tilde{c}_{t+1}\theta\frac{y}{k}-\tilde{c}_{t+1}+\delta\tilde{c}_{t+1}]$$

Now, we sub in for the steady-state value of k:

$$\frac{\eta}{\beta}(1-\tilde{c}_t) = E[\theta\frac{y}{(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})y}(1+\tilde{y}_{t+1}-\tilde{k}_{t+1})+1-\delta-\tilde{c}_{t+1}\theta\frac{y}{(\frac{\theta}{\frac{\eta}{\beta}-1+\delta})y}-\tilde{c}_{t+1}+\delta\tilde{c}_{t+1}]$$

$$\Rightarrow \frac{\eta}{\beta}(1-\tilde{c}_t) = E[(\frac{\eta}{\beta}-1+\delta)(1+\tilde{y}_{t+1}-\tilde{k}_{t+1})+1-\delta-(\frac{\eta}{\beta}-1+\delta)\tilde{c}_{t+1}-\tilde{c}_{t+1}+\delta\tilde{c}_{t+1}]$$

$$0 = \frac{\eta}{\beta}\tilde{c}_t + E[(\frac{\eta}{\beta}-1+\delta)(\tilde{y}_{t+1}-\tilde{k}_{t+1})-\frac{\eta}{\beta}\tilde{c}_{t+1}] \tag{8.8}$$