

Instructor: *Professor Griffy*  
Due: *Feb., 16th 2023*  
AEEO 701

## Problem Set 2

**Problem 1: Markov Chains** We can apply Markov Chains in a variety of circumstances that involve dynamics. We will think of a simple “lake model” of employment dynamics. There are three states: employed, unemployed, and non-participant (out of the labor force). Denote these as  $E$  for employed,  $U$  for unemployed, and  $N$  for non-participant. The transition probabilities are as follows:  $E \rightarrow E$ : 0.9;  $E \rightarrow U$ : 0.1;  $E \rightarrow N$ : 0.0;  $U \rightarrow E$ : 0.5;  $U \rightarrow U$ : 0.5;  $U \rightarrow N$ : 0.0;  $N \rightarrow E$ : 0.0;  $N \rightarrow U$ : 0.0;  $N \rightarrow N$ : 1.0

- a) Write down the transition equation in the following way:  $x'_{t+1} = x'_t A$ . Define each state in  $x$  clearly and denote the transition matrix,  $A$ , correctly.
- b) Is there a unique stationary distribution? Why or why not?
- c) Use a computer programming language to find the ergodic distribution for the following initial condition:  $x'_0 = [0.55 \ 0.05 \ 0.4]$ . Given an initial condition, is this distribution stationary?
- d) **Sectoral Decline:** Suppose now that we are modeling an individual sector in the economy. Over time, this sector’s reliance on labor is declining. Unfortunately for workers in this sector, they have a great deal of sector-specific human capital and do not have enough general human capital to find jobs in other sectors. The transition probabilities for  $E$  and  $N$  remain unchanged, but now the transition probabilities for  $U$  are given by  $U \rightarrow E$ : 0.5;  $U \rightarrow U$ : 0.45;  $U \rightarrow N$ : 0.05. Write down the transition equation. Does this have a unique stationary distribution? Use the same initial condition and find the resulting distribution for large  $t$ .
- e) Suppose now that the government institutes a worker retraining program. Keeping the probabilities for transitions out of unemployment fixed at their values from part *d*, this new policy makes the transition probabilities from  $N$ :  $N \rightarrow E$ : 0.1;  $N \rightarrow U$ : 0.0;  $N \rightarrow N$ : 0.9. Use the initial distribution from part (c) and find the distribution for large  $t$ .
- f) Draw two random series uniformly distributed. To do this, draw 3 numbers from a uniform distributed between 0 and 1. The first number is the (un-normalized) measure of workers starting employed; the second unemployed; the third, non-participant. Normalize by the sum of these three numbers. Simulate the Markov Chain for this series over 1000 periods and plot this series. Repeat this a second time and plot this series again. Discuss the differences between your results.