Macro II

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Introduction

There is a new homework online.

► Today:

Talk about Lucas Critique and Rational Expectations

Use simple two period model

Show intuition behind Lucas Critique.

Lecture largely based on Eric Sims' (Notre Dame) notes.

Lucas Critique Overview

- Some history:
 - Prior to the late 1970s, macroeconomists had no systematic way of modeling consumer expectations.
 - They found *empirical relationships* between *equilibrium objects* and interpreted these as causal.
 - This is a problem!
 - (Old) Phillips Curve: inverse relationship between inflation and unemployment
 - $\blacktriangleright more money \rightarrow more demand \rightarrow more employment.$
 - This led policy makers to institute persistent inflation.
 - But this broke down in the 70s: we had stagflation: inflation and unemployment.
 - The reason is that consumers came to expect an increase in prices and adjusted their demand.
- Lucas Critique (broadly):
 - Need to use "deep" (structural) parameters to inform policy.
 - Otherwise policy may affect these parameters.

A (very) basic consumption-savings model:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2)$$
(1)

s.t.
$$c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r}$$
 (2)

Simple set-up:

- Household faces an endowment w₁, w₂, which are known and fixed.
- r is fixed over time, household takes as given.
- ▶ Standard definitions for u: u' > 0, u'' < 0, $u'(0) = \infty$

A (very) basic consumption-savings model:

$$V = \max_{c_1, c_2} u(c_1) + \beta u(c_2)$$
(3)

s.t.
$$c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r}$$
 (4)

Solve by first finding the Euler Equation:

$$\frac{\partial V}{\partial c_1} = u'(c_1) - \lambda = 0 \tag{5}$$

$$\frac{\partial V}{\partial c_2} = \beta u'(c_2) - \frac{\lambda}{1+r} = 0$$
 (6)

$$\rightarrow u'(c_1) = \beta(1+r)u'(c_2) \tag{7}$$

We know dynamics, now need to pin down c_t using budget constraint (boundary condition).

Dynamics and budget:

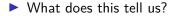
$$u'(c_1) = \beta(1+r)u'(c_2)$$
 (8)

s.t.
$$c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r}$$
 (9)

• Assume log utility: u(c) = ln(c).

This yields

$$\frac{1}{c_1} = \beta (1+r) \frac{1}{c_2}$$
(10)
$$c_1 = \frac{1}{1+\beta} (w_1 + \frac{w_2}{1+r})$$
(11)



$$c_1 = \frac{1}{1+\beta} (w_1 + \frac{w_2}{1+r}) \tag{12}$$

This tells us that consumption today is a function of

income today (not surprising)

income in the future (possibly a problem)

- Suppose there is a recession.
- A policymaker wants to implement a tax cut based on empirical evidence

$$c_1 = \frac{1}{1+\beta} (w_1 + \frac{w_2}{1+r})$$
(13)

Policymaker:

Run the following regression:

$$c_1 = \alpha + \gamma w_t + u_t \tag{14}$$



• Give people money, consumption will increase by γ !

$$c_{1} = \frac{1}{1+\beta} (w_{1} + \frac{w_{2}}{1+r})$$
(15)
$$\hat{c_{1}} = \alpha + \gamma w_{t}$$
(16)

• Assume
$$\frac{\partial w_2}{\partial w_1} = 0$$
 (i.e., uncorrelated). Then

$$\frac{\partial c_1}{\partial w_1} = \frac{1}{1+\beta}$$
(17)

$$\frac{\partial \hat{c}_1}{\partial w_1} = \alpha$$
(18)

• In this context, $\alpha = \frac{1}{1+\beta}$. We're good!

$$c_{1} = \frac{1}{1+\beta} (w_{1} + \frac{w_{2}}{1+r})$$
(19)
$$\hat{c_{1}} = \alpha + \gamma w_{t}$$
(20)

▶ Assume
$$\frac{\partial w_2}{\partial w_1}$$
=0. Then

$$\frac{\partial c_1}{\partial w_1} = \frac{1}{1+\beta} \left(1 + \frac{\frac{\partial w_2}{\partial w_1}}{1+r}\right)$$
(21)
$$\frac{\partial \hat{c}_1}{\partial w_1} = \alpha$$
(22)

If income was positively correlated (AR, etc.), we're not going to get the response we want.

Lucas critique overview

- In this context, policymakers might over predict the response of consumption.
- Why? Because consumers understand that this is a temporary increase in income.
- ▶ They won't believe that w₂ will increase.
- ▶ Therefore, they will respond less than predicted by the model.
- This is the crux of the Lucas Critique: that you need to find deep parameters that don't change with consumer behavior.

Lucas critique

Lucas made his critique in the context of monetary policy.

There had been multiple decades of inflation, aimed at reducing unemployment.

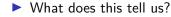
Consumers eventually built in the expectation of inflation and this empirical relationship no longer held.

Let's use a simple monetary policy model to understand what happened.

Phillips Curve

Suppose that inflation is characterized by the following difference equation:

$$\pi_t = \theta(u_t - u^*) + \beta \mathbb{E}(\pi_{t+1})$$
(23)



If we hold expectations fixed,

> an increase in current inflation, π_t ,

leads to a θ reduction in unemployment (in percentage points).

 θ was observed to be negative, ie inflation reduced unemployment.

Policymaker

$$\pi_t = \theta(u_t - u^*) + \beta \mathbb{E}(\pi_{t+1})$$
(24)

Suppose that an econometrician ran the following specification:

$$\pi_t = \gamma(u_t - u^*) + \epsilon_t \tag{25}$$

• They conclude that $\gamma < 0$.

They tell the policymaker to raise inflation to reduce unemployment.

What happens?

$$\pi_t = \theta(u_t - u^*) + \beta \mathbb{E}(\pi_{t+1})$$
(26)
$$\pi_t = \gamma(u_t - u^*) + \epsilon_t$$
(27)

- Well, as long as expectations don't change, the empirical specification will appear to hold.
- But if they change, consequences!
- An increase in inflation can lead to one of two things:
 - 1. a decrease in unemployment (good!) or
 - 2. an increase in expected future inflation
- and the equation will still hold.
- ▶ This is what we saw in the 1970s/1980s.

Log linearization

- Non-linear difference equations are tricky to solve.
- Macroeconomists often log-linearize these difference equations to make them easier to solve.

Basic idea:

- In some area around the steady state, deviations are small.
- Can approximate using a log-linearized version of the model.
- Will be "wrong," but close as long as deviations stay small.
- Today, short refresher.

Log linearization II

► Take a generic difference equation with a single variable *x*:

$$x_{t+1} = A x_t \tag{28}$$

Suppose that A = 1 + g:

$$x_{t+1} = (1+g)x_t$$
 (29)

Taking logs of both sides:

$$ln(x_{t+1}) = ln(1+g) + ln(x_t)$$
(30)

Taylor Series Approximation

A first-order taylor approximation of a function f(x) around a point x* is given by

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*)$$
 (31)

• If $(x - x^*)$ is small and f'' is not too large, this approximation is reasonable.

Idea: we know the value of a function at a particular point

We can also find the derivative at that point.

Applying this to log-linear approximation

• Taylor series approximation of growth rate (1 + g) at g = 0:

$$ln(1+g) \approx ln(1+0) + rac{1}{1+0}(1+g-1)$$
 (32)
 $\approx g$ (33)

 This means that we can approximate our difference equation as

$$ln(x_{t+1}) = ln(1+g) + ln(x_t)$$
(34)

$$\approx \ln(x_t) + g$$
 (35)

This insight will prove very useful in solving macro models.

Next Time

Start dynamic programming.

Homework due next week, one due the week after.