Macro II

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Introduction

- There is a new homework online.
- Today:
  - Talk about Lucas Critique and Rational Expectations
  - Use simple two period model
  - Show intuition behind Lucas Critique.
- Lecture largely based on Eric Sims' (Notre Dame) notes.

Lucas Critique Overview

- Some history:
  - Prior to the late 1970s, macroeconomists had no systematic way of modeling consumer expectations.
  - They found *empirical relationships* between *equilibrium objects* and interpreted these as causal.
  - This is a problem!
  - (Old) Phillips Curve: inverse relationship between inflation and unemployment
    - more money $\rightarrow$ more demand $\rightarrow$ more employment.
  - This led policy makers to institute persistent inflation.
  - But this broke down in the 70s: we had stagflation: inflation and unemployment.
  - The reason is that consumers came to expect an increase in prices and adjusted their demand.
- Lucas Critique (broadly):
  - Need to use "deep" (structural) parameters to inform policy.
  - Otherwise policy may affect these parameters.

Basic two-period model

- A (very) basic consumption-savings model:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \tag{1}$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r} \tag{2}$$

- Simple set-up:
  - Household faces an endowment w_1, w_2, which are known and fixed.
  - r is fixed over time, household takes as given.
  - Standard definitions for u: $u' > 0$, $u'' < 0$, $u'(0) = \infty$

Basic two-period model

- A (very) basic consumption-savings model:

$$V = \max_{c_1, c_2} u(c_1) + \beta u(c_2) \tag{3}$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r} \tag{4}$$

- Solve by first finding the Euler Equation:

$$\frac{\partial V}{\partial c_1} = u'(c_1) - \lambda = 0 \tag{5}$$

$$\frac{\partial V}{\partial c_2} = \beta u'(c_2) - \frac{\lambda}{1+r} = 0 \tag{6}$$

$$\rightarrow u'(c_1) = \beta(1+r)u'(c_2) \tag{7}$$

- We know dynamics, now need to pin down c_t using budget constraint (boundary condition).

Basic two-period model

- Dynamics and budget:

$$u'(c_1) = \beta(1+r)u'(c_2) \tag{8}$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r} \tag{9}$$

- Assume log utility: $u(c) = ln(c)$.

- This yields

$$\frac{1}{c_1} = \beta(1+r)\frac{1}{c_2} \tag{10}$$

$$c_1 = \frac{1}{1+\beta}(w_1 + \frac{w_2}{1+r}) \tag{11}$$

- What does this tell us?

Basic two-period model

$$c_1 = \frac{1}{1+\beta}(w_1 + \frac{w_2}{1+r}) \tag{12}$$

- This tells us that consumption today is a function of
  - income today (not surprising)
  - income in the future (possibly a problem)
- Suppose there is a recession.
- A policymaker wants to implement a tax cut based on empirical evidence

Basic two-period model

$$c_1 = \frac{1}{1+\beta}(w_1 + \frac{w_2}{1+r}) \tag{13}$$

- Policymaker:
  - Run the following regression:

    $$c_1 = \alpha + \gamma w_t + u_t \tag{14}$$

  - Want to stimulate the economy.
  - Give people money, consumption will increase by γ!

Basic two-period model

$$c_1 = \frac{1}{1+\beta}(w_1 + \frac{w_2}{1+r}) \quad (15)$$

$$\hat{c}_1 = \alpha + \gamma w_t \quad (16)$$

- Assume $\frac{\partial w_2}{\partial w_1} = 0$ (i.e., uncorrelated). Then

$$\frac{\partial c_1}{\partial w_1} = \frac{1}{1+\beta} \quad (17)$$

$$\frac{\partial \hat{c}_1}{\partial w_1} = \alpha \quad (18)$$

- In this context, $\alpha = \frac{1}{1+\beta}$. We're good!

Basic two-period model

$$c_1 = \frac{1}{1+\beta}(w_1 + \frac{w_2}{1+r}) \tag{19}$$

$$\hat{c}_1 = \alpha + \gamma w_t \tag{20}$$

- Assume $\frac{\partial w_2}{\partial w_1}$=0. Then

$$\frac{\partial c_1}{\partial w_1} = \frac{1}{1+\beta}(1 + \frac{\frac{\partial w_2}{\partial w_1}}{1+r}) \tag{21}$$

$$\frac{\partial \hat{c}_1}{\partial w_1} = \alpha \tag{22}$$

- If income was positively correlated (AR, etc.), we're not going to get the response we want.

Lucas critique overview

- In this context, policymakers might over predict the response of consumption.
- Why? Because consumers understand that this is a temporary increase in income.
- They won't believe that w_2 will increase.
- Therefore, they will respond less than predicted by the model.
- This is the crux of the Lucas Critique: that you need to find deep parameters that don't change with consumer behavior.

Lucas critique

- Lucas made his critique in the context of monetary policy.
- There had been multiple decades of inflation, aimed at reducing unemployment.
- Consumers eventually built in the expectation of inflation and this empirical relationship no longer held.
- Let's use a simple monetary policy model to understand what happened.

Phillips Curve

- Suppose that inflation is characterized by the following difference equation:

$$\pi_t = \theta(u_t - u^*) + \beta\mathbb{E}(\pi_{t+1}) \tag{23}$$

- What does this tell us?
  - If we hold expectations fixed,
  - an increase in current inflation, π_t,
  - leads to a θ reduction in unemployment (in percentage points).
- θ was observed to be negative, ie inflation reduced unemployment.

Policymaker

$$\pi_t = \theta(u_t - u^*) + \beta\mathbb{E}(\pi_{t+1}) \tag{24}$$

- Suppose that an econometrician ran the following specification:

$$\pi_t = \gamma(u_t - u^*) + \epsilon_t \tag{25}$$

- They conclude that $\gamma < 0$.
- They tell the policymaker to raise inflation to reduce unemployment.

What happens?

$$\pi_t = \theta(u_t - u^*) + \beta \mathbb{E}(\pi_{t+1}) \tag{26}$$

$$\pi_t = \gamma(u_t - u^*) + \epsilon_t \tag{27}$$

- Well, as long as expectations don't change, the empirical specification will appear to hold.
- But if they change, consequences!
- An increase in inflation can lead to one of two things:
  1. a decrease in unemployment (good!) or
  2. an increase in expected future inflation
- and the equation will still hold.
- This is what we saw in the 1970s/1980s.

Log linearization

- Non-linear difference equations are tricky to solve.
- Macroeconomists often log-linearize these difference equations to make them easier to solve.
- Basic idea:
  - In some area around the steady state, deviations are small.
  - Can approximate using a log-linearized version of the model.
  - Will be "wrong," but close as long as deviations stay small.
- Today, short refresher.

Log linearization II

- Take a generic difference equation with a single variable x:

$$x_{t+1} = Ax_t \tag{28}$$

- Suppose that $A = 1 + g$:

$$x_{t+1} = (1 + g)x_t \tag{29}$$

- Taking logs of both sides:

$$ln(x_{t+1}) = ln(1 + g) + ln(x_t) \tag{30}$$

Taylor Series Approximation

- A first-order taylor approximation of a function $f(x)$ around a point x^* is given by

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*) \tag{31}$$

- If $(x - x^*)$ is small and f'' is not too large, this approximation is reasonable.

- Idea: we know the value of a function at a particular point

- We can also find the derivative at that point.

Applying this to log-linear approximation

- Taylor series approximation of growth rate $(1+g)$ at $g=0$:

$$ln(1+g) \approx ln(1+0) + \frac{1}{1+0}(1+g-1) \tag{32}$$
$$\approx g \tag{33}$$

- This means that we can approximate our difference equation as

$$ln(x_{t+1}) = ln(1+g) + ln(x_t) \tag{34}$$
$$\approx ln(x_t) + g \tag{35}$$

- This insight will prove very useful in solving macro models.

Next Time

- Start dynamic programming.
- Homework due next week, one due the week after.