# Macro II

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# Introduction

- Today: Asset pricing
- The "Lucas Tree Model."
- Homework due Thursday.
- Test next Thursday (I think...)
- I will be out of town on Thursday.
- New homework sometime soon...

# Rational Expectations Competitive Equilibrium

Definition: Given the set of exogenous stochastic processes $\{x_t\}$, and initial conditions $a_0$, a rational expectations equilibrium is a set of stochastic processes for prices $\{p_t\}$ and quantities $\{q_t\}$ such that:

- Given $\{p_t\}$, $\{q_t\}$ is consistent with optimal behavior on the part of consumers, producers and (if relevant) the government.
- Given $\{p_t\}$, $\{q_t\}$ satisfies the government's budget constraints and borrowing restrictions.
- $\{p_t\}$ satisfies any market-clearing conditions.

# Lucas Tree Overview

- We are given equilibrium quantities and equilibrium demand functions—back out equilibrium prices.
- We know the function D(p) and the quantity $q_0$: now find $p_0$.

# Compare with Literature on Consumption

- Consumption: Take rates of return as given, solve for consumption.
- Asset Pricing: Take consumption as given, solve for rates of return.

# Model Structure

- Preferences: $n$ identical consumers, maximizing

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right),$$
$$\beta \in (0,1), \quad u'(\cdot) > 0, \quad u''(\cdot) \leq 0.$$

- Endowment: one durable "tree" per individual. Each period, the tree yields some "fruit" ($d_t \equiv$ dividends).

- Technology: Fruit cannot be stored. Dividends are exogenous and follow a time-invariant Markov process

$$\Pr\left(d_{t+1} \leq y \mid d_t = x\right) = F(y, x), \forall t,$$

with density $f(y, x)$.

# Solution strategy

- Find the competitive equilibrium allocation via the social planner's problem. Assume equal weights on each person's utility (parallel to equal endowments).
- i.e., use welfare theorems.
- Calculate the FOC for individuals with the opportunity to buy/sell specific assets.
- Evaluate FOC at the competitive equilibrium allocation.

# Step 1: Social planner's problem

- Use a representative agent
- Social planner solves

$$\max_{\{c_t\}_{t=0}^{\infty}} E_0 \left( \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right)$$
$$s.t. \quad c_t \leq d_t.$$

- Solution: $c_t = d_t, \forall t$ (*non-storable good!*).
- What does this mean?

# Definitions

$$\begin{aligned}
c_t &= \text{consumption}, \\
p_t &= \text{price of a tree} = \text{price of stock}, \\
x_t &= \text{total resources} \\
s_{t+1} &= \text{number of trees/shares of stock}, \\
R_t &= \text{gross return on one-period risk-free bond}, \\
R_t^{-1} &= \text{price of a one-period, risk-free } \underline{\text{discount bond}}, \\
b_{t+1} &= \text{risk-free discount bonds.}
\end{aligned}$$

# Step 2: Representative consumer's problem

$$\max_{\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}} E\left(\sum_{t=0}^{\infty} \beta^t u(c_t) \middle| I_0\right)$$

$$\begin{aligned}
s.t. \quad & c_t + p_t s_{t+1} + R_t^{-1} b_{t+1} = x_t, \\
& x_t = (p_t + d_t) s_t + b_t, \\
& \lim_{J \to \infty} \beta^J E_t \left(u'(c_{t+J}) p_{t+J} s_{t+J+1}\right) = 0 \\
& \lim_{J \to \infty} \beta^J E_t \left(u'(c_{t+J}) b_{t+J+1}\right) = 0, \\
& s_0, \; b_0 \text{ given}
\end{aligned}$$

- where $I_0$ is the information set at time 0.

# Consumer's problem

- Consumer $i$ picks $c_t^i$, $b_{t+1}^i$ and $s_{t+1}^i$ on the basis of

$$I_t^i = \left\{ \begin{array}{l} \{d_{t-m}, p_{t-m}, R_{t-m}\}_{m=0}^{t}, \\ \left\{s_{t+1-m}^j, b_{t+1-m}^j\right\}_{m=0}^{t+1}, \ \forall j \neq i, \\ \left\{c_{t-m}^j, x_{t-m}^j\right\}_{m=0}^{t}, \ \forall j \neq i, \\ \left\{s_{t-m}^i, b_{t-m}^i, x_{t-m}^i\right\}_{m=0}^{t}, \left\{c_{t-m}^i\right\}_{m=1}^{t}, \end{array} \right\},$$

- It turns out that $d_t$ summarizes the state of the aggregate economy, with $p_t = p(d_t)$ and $R_t = R(d_t)$.

- It is the only stochastic variable, and aggregate resources equal $d_t$.

- Because $d_t$ is a time-invariant Markov process, the consumer's problem is time-invariant

# Recursive formulation

Bellman's functional equation:

$$V(x_t, d_t) =$$

$$\min_{\lambda_t \geq 0} \max_{c_t \geq 0,\ s_{t+1},\ b_{t+1}} u\left(c_t\right) + \lambda_t \left(x_t - c_t - p_t s_{t+1} - R_t^{-1} b_{t+1}\right)$$

$$+ \beta \int V\Big( \left(p\left(d_{t+1}\right) + d_{t+1}\right) s_{t+1} + b_{t+1}, d_{t+1}\Big) dF\left(d_{t+1}, d_t\right).$$

- The FOC for an interior solution are:

$$\begin{aligned} u'\left(c_t\right) &= \lambda_t, \\ \lambda_t p_t &= \beta \int \frac{\partial V\left[t+1\right]}{\partial x_{t+1}} \left(p\left(d_{t+1}\right) + d_{t+1}\right) dF\left(d_{t+1}, d_t\right), \\ \lambda_t R_t^{-1} &= \beta \int \frac{\partial V\left[t+1\right]}{\partial x_{t+1}} dF\left(d_{t+1}, d_t\right). \end{aligned}$$

# Euler Equations

- Note that (by Benveniste-Scheinkman)

$$\frac{\partial V[t]}{\partial x_t} = \lambda_t,$$

so that

$$\begin{aligned} p_t &= \beta E_t \left( \frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + d_{t+1}) \right), \\ R_t^{-1} &= \beta E_t \left( \frac{u'(c_{t+1})}{u'(c_t)} \right). \end{aligned}$$

# Step 3: Equilibrium

- Intuition
- Agents allocate resources based on beliefs about future prices and consumption
- These decision rules determine processes for market clearing prices and quantities.
- In a rational expectations equilibrium, the actual processes must be consistent with the beliefs.

- **Sequential** definition: Given the stochastic process $\{d_t\}_{t=0}^{\infty}$ and the initial endowments $s_0 = 1$ and $b_0 = 0$, a rational expectations equilibrium consists of the stochastic processes $\{c_t, s_{t+1}, b_{t+1}, p_t, R_t\}_{t=0}^{\infty}$ such that:
    - Given the process for prices $\{p_t, R_t\}$, $\{c_t, s_{t+1}, b_{t+1}\}$ solves the consumer's problem.
    - All markets clear: $c_t = d_t$, $s_{t+1} = 1$, and $b_{t+1} = 0$, $\forall t$.

- **Recursive definition**: given the random variable $d_0$, the conditional distribution $F(d_{t+1}, d_t)$, and the initial endowments $s_0 = 1$ and $b_0 = 0$, a recursive rational expectations equilibrium consists of pricing functions $p(d)$ and $R(d)$, a value function $V(x, d)$, and decision functions $c(x, d)$, $s(x, d)$, and $b(x, d)$ such that:

  - Given the pricing functions $p(d)$ and $R(d)$, the value and policy functions $V(x, d)$, $c(x, d)$, $s(x, d)$, and $b(x, d)$ solve the consumer's problem.

  - Markets clear: for $x = p(d) + d$, $c(x, d) = d$, $s(x, d) = 1$, and $b(x, d) = 0$.

# Backing out prices

- Find $R(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$
\begin{aligned}
R_t^{-1} &= \beta E_t \left( \frac{u'(d_{t+1})}{u'(d_t)} \right) \\
&= \beta \frac{1}{u'(d_t)} E_t \left( u'(d_{t+1}) \right). \qquad \text{(EE)}
\end{aligned}
$$

- Find $p(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$
p_t = \beta E_t \left( \frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \qquad \text{(EE}'\text{)}
$$

# Bond Price

- Recall equation (EE):

$$R_t^{-1} = \beta \frac{1}{u'(d_t)} E_t\left(u'(d_{t+1})\right), \tag{EE}$$

$$R_t = \frac{u'(d_t)}{\beta E_t\left(u'(d_{t+1})\right)}.$$

- The price of a discount bond increases (return falls) in $\beta$.
- The price increases (return falls) in expected future marginal utility, and decreases in current marginal utility: reflects consumption smoothing motive.
- Recall from last time: implies that more uncertainty raises bond price if convex preferences.

# Stock prices

- Recall equation (EE′):

$$p_t = \beta E_t \left( \frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \qquad \text{(EE}'\text{)}$$

- Define the expected rate of return on stocks as

$$E_t(R_t^s) = E_t \left( \frac{p_{t+1} + d_{t+1}}{p_t} \right).$$

# Equity premium

- The expected rate of return on stocks is

$$E_t\left(R_t^s\right) = E_t\left(\frac{p_{t+1}+d_{t+1}}{p_t}\right).$$

- Recall that

$$E_t\left(XY\right) = E_t\left(X\right)E_t\left(Y\right) + C_t\left(X,Y\right).$$

- Rewrite (EE′):

$$\begin{aligned} 1 &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}\left(\frac{p_{t+1}+d_{t+1}}{p_t}\right)\right) \\ &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}R_t^s\right) \\ &= \beta E_t\left(\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}\right)E_t\left(R_t^s\right) + C_t\left(\beta\frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right) \end{aligned}$$

# Risk premium

- Insert (EE) and rearrange:

$$1 = R_t^{-1} E_t\left(R_t^s\right) + C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right),$$

$$\begin{aligned}
E_t\left(R_t^s\right) &= R_t - R_t C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right), \\
&= R_t - \frac{u'\left(d_t\right)}{\beta E_t\left(u'\left(d_{t+1}\right)\right)} C_t\left(\beta \frac{u'\left(d_{t+1}\right)}{u'\left(d_t\right)}, R_t^s\right), \\
&= R_t - \frac{C_t\left(u'\left(d_{t+1}\right), R_t^s\right)}{E_t\left(u'\left(d_{t+1}\right)\right)}.
\end{aligned}$$

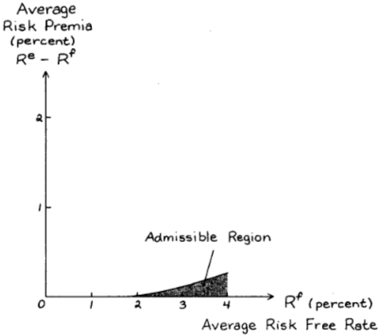
- The expected return on stocks equals the return on the risk-free bond plus the risk-premium, which is $-\frac{C_t(\cdot,\cdot)}{E_t(\cdot)}$.

# Equity premium: a puzzle?

- If the covariance $C_t(\cdot, \cdot)$ is negative, which we normally expect, there is an equity premium.
- Interpretation
  - The most desirable assets yield well when marginal utility is high ($C_t(\cdot, \cdot) > 0$). Risk-aversion means that agents prefer assets that act like insurance.
  - Investors are willing to sacrifice return if $C_t(\cdot, \cdot) > 0$, and they will demand higher returns if $C_t(\cdot, \cdot) < 0$.
- Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.

# Equity premium: a puzzle?

- Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.



# Conclusion

- Homework due tonight.
- Midterm next week.
- New homework posted soon, due a week after I post it.