

Macro II

Professor Griffy

UAlbany

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Introduction

- ▶ Midterm in two weeks!
- ▶ Homework Thursday after break (same day as midterm).
- ▶ Today: Real Business Cycle Model

Basic RBC Model

- ▶ Household solves

$$\begin{aligned} \max_{\{C_t, I_t, L_t, K_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t N_t \left[\ln \left(\frac{C_t}{N_t} \right) + \chi \frac{(1 - L_t/N_t)^{1-\gamma} - 1}{1 - \gamma} \right] \\ \text{s.t.} \quad & C_t + I_t = r_t K_t + W_t L_t, & \text{(BC)} \\ & K_{t+1} = (1 - \delta) K_t + I_t, & \text{(CA)} \\ & L_t \in [0, N_t], \\ & K_0 \text{ given, } C_t \geq 0. \end{aligned}$$

- ▶ Parameter restrictions: $\chi > 0, \gamma \geq 0, 0 < \beta < 1$
- ▶ $1 - L_t/N_t$ is per capita leisure
- ▶ Note that $K_t < 0$ represents borrowing

Basic RBC Model II

- ▶ Assume constant growth in population and productivity

$$N_t = N_0 N^t, \quad N_0, N > 0, \quad \beta N < 1,$$

$$A_t = A_0 A^t, \quad A_0, A > 0.$$

- ▶ The per-effective-worker problem becomes:

$$\max_{\{c_t, k_{t+1}, \ell_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta N)^t \left[\ln(A_t c_t) + \chi \frac{(1 - \ell_t)^{1-\gamma} - 1}{1 - \gamma} \right],$$

$$s.t. \quad c_t + ANk_{t+1} = R_t k_t + w_t \ell_t,$$

$$\ell_t \in [0, 1]; \quad k_0 \text{ given}, \quad c_t \geq 0,$$

$$\lim_{J \rightarrow \infty} \left(\prod_{j=1}^{J-1} R_{t+j}^{-1} \right) A_{t+J} N_{t+J} k_{t+J} = 0.$$

Solution

- ▶ The first order conditions are

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}, \quad (\text{EE})$$

$$u'(A_t c_t) A_t w_t = v'(1 - \ell_t)$$

$$\Leftrightarrow \frac{1}{c_t} w_t = \chi (1 - \ell_t)^{-\gamma}. \quad (\text{LL})$$

- ▶ Euler equation and “portfolio allocation”

Effect of interest rate changes on savings

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}$$

- ▶ Substitution effect: Increasing R_{t+1} lowers the price of future consumption, inducing substitution into the cheaper good (future consumption), inducing more saving
- ▶ Income effect
 - ▶ Positive assets: Increasing R_{t+1} raises future income and consumption, lowers future MU_C , inducing less savings
 - ▶ Negative assets: Increasing R_{t+1} reduces future income and consumption, raises future MU_C , inducing more savings

Effect of interest rate changes on savings

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}$$

- ▶ General (empirical) consensus
 - ▶ Consumers are net savers: the aggregate income effect of higher interest rates is to lower saving
 - ▶ The substitution effect weakly dominates implying that savings increases in interest rates

Labor-leisure tradeoff

$$\frac{1}{c_t} w_t = \chi (1 - \ell_t)^{-\gamma}$$

- ▶ $MU_C \times \text{wage} = MU_L$
- ▶ Wealth effects: Holding w_t constant, higher permanent income raises current consumption, lowers marginal benefit of working
 - ▶ Higher assets
 - ▶ Higher current or future non-labor income
 - ▶ Higher current or future labor income
 - ▶ Increasing non-labor component of permanent income lowers labor supply

Effects of increasing the current wage ($MU_C \times wage = MU_L$)

$$\frac{1}{c_t} w_t = \chi (1 - \ell_t)^{-\gamma}$$

- ▶ Substitution effect: holding MU_C constant, and raising w_t increases marginal benefit of working
- ▶ Income effect: raising w_t increases y_t^P , lowers MU_C and marginal benefit of working

General (empirical) consensus

$$\frac{1}{c_t} w_t = \chi (1 - \ell_t)^{-\gamma}$$

- ▶ Temporary wage increases generate more hours due to small income effect
- ▶ Permanent wage increases generate no more hours because income and substitution effects offset. Consistent with long-term data where wage rises but labor hours do not
- ▶ Our specification delivers this

Labor supply curve

- ▶ Rearrange (LL) to get

$$l_t = 1 - (c_t \chi)^{1/\gamma} w_t^{-1/\gamma}.$$

- ▶ Frisch supply curve

$$l_t = f(w_t, MU_C) = f(w_t, y_t^P).$$

- ▶ Consider effects of changing wages with MU_C held constant
- ▶ Wealth effects ignored
- ▶ Note: MU_C can depend on things besides y_t^P , although it does not here

Intertemporal elasticity of substitution of labor (IES_L or Frisch elasticity)

- ▶ Measures willingness to vary labor over time, holding MU_C (wealth) constant

$$IES_L = \left. \frac{d \ln(\ell_1/\ell_2)}{d \ln(w_1/w_2)} \right|_{MU_C} .$$

Derivation

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}, \quad (\text{EE})$$

$$\frac{1}{c_t} w_t = \chi (1 - l_t)^{-\gamma}. \quad (\text{LL})$$

- Combine (EE) and (LL)

$$\chi \frac{(1 - l_1)^{-\gamma}}{w_1} = \beta A^{-1} \chi \frac{(1 - l_2)^{-\gamma}}{w_2} R_2.$$

Portfolio Allocation

- ▶ Note that the household smooths leisure as well as consumption
- ▶ For example, interest rates affect labor supply
- ▶ Rearrange the previous equation

$$\begin{aligned}\beta A^{-1} R_2 \left(\frac{w_1}{w_2} \right) &= \frac{(1 - \ell_1)^{-\gamma}}{(1 - \ell_2)^{-\gamma}}, \\ \ln(\beta A^{-1} R_2) + \ln \left(\frac{w_1}{w_2} \right) &= -\gamma \ln(1 - \ell_1) + \gamma \ln(1 - \ell_2), \\ &= -\gamma \left[\ln(1 - \exp(\ln \ell_1)) \right. \\ &\quad \left. - \ln(1 - \exp(\ln \ell_2)) \right].\end{aligned}$$

- Implicitly differentiate:

$$d \ln \left(\frac{w_1}{w_2} \right) = \gamma \frac{\exp(\ln(l_1))}{1 - \exp(\ln(l_1))} d \ln(l_1) - \gamma \frac{\exp(\ln(l_2))}{1 - \exp(\ln(l_2))} d \ln(l_2).$$

- Now assume that $l_1 = l_2 = l$

$$\begin{aligned} d \ln \left(\frac{w_1}{w_2} \right) &= \gamma \frac{l}{1-l} d \ln(l_1) - \gamma \frac{l}{1-l} d \ln(l_2) \\ &= \gamma \frac{l}{1-l} [d \ln(l_1) - d \ln(l_2)] \\ &= \gamma \frac{l}{1-l} d \ln \left(\frac{l_1}{l_2} \right). \end{aligned}$$

- ▶ Finally, we get

$$\begin{aligned} IES_L &= \frac{d \ln(\ell_1/\ell_2)}{d \ln(w_1/w_2)} \Big|_{MU_C} \\ &= \frac{1}{\gamma} \left(\frac{1-\ell}{\ell} \right). \end{aligned}$$

- ▶ Tip: if $\gamma = 0$ such that utility is linear in leisure, then IES_L is infinite

Non-Separable Preferences (Low, 2005)

- ▶ Household solves

$$\begin{aligned} \max_{\{c_t, k_{t+1}, \ell_t\}_{t=0}^{\infty}} E_0 \left(\sum_{t=0}^{\infty} (\beta N)^t u(A_t c_t, 1 - \ell_t) \right), \\ \text{s.t. } c_t + ANk_{t+1} = R_t k_t + w_t \ell_t, \\ \ell_t \in [0, 1], \end{aligned}$$

- ▶ and the other usual constraints
- ▶ The first-order conditions are

$$\begin{aligned} u_{Ac} (A_t c_t, 1 - \ell_t) A_t &= \lambda_t, \\ u_{1-\ell} (A_t c_t, 1 - \ell_t) &= \lambda_t w_t, \\ \lambda_t &= \beta A^{-1} E_t (R_{t+1} \lambda_{t+1}). \end{aligned}$$

where λ_t is the multiplier on the budget constraint

- ▶ Benchmark utility specification is isoelastic Cobb-Douglas

$$u(A_t c_t, 1 - \ell_t) = \frac{1}{1 - \gamma} ((A_t c_t)^\chi (1 - \ell_t)^{1 - \chi})^{1 - \gamma}$$

- ▶ The derivatives of this function are

$$u_{Ac} = \chi(1 - \gamma) \frac{1}{A_t c_t} u(A_t c_t, 1 - \ell_t),$$

$$u_{1-\ell} = (1 - \chi)(1 - \gamma) \frac{1}{1 - \ell_t} u(A_t c_t, 1 - \ell_t),$$

$$u_{1-\ell, Ac} = \frac{\chi(1 - \chi)(1 - \gamma)^2}{(1 - \ell_t) A_t c_t} u(A_t c_t, 1 - \ell_t).$$

- ▶ Key issue: Is consumption at time- t a substitute or a complement for leisure at time- t ?
 - ▶ This depends on the sign of the cross-partial derivative $u_{Ac,1-\ell}(\cdot)$: $u_{Ac,1-\ell} > 0$ implies complements
 - ▶ For the benchmark specification

$$u_{1-\ell,Ac} = \chi(1-\chi)(1-\gamma) \times (A_t c_t)^{\chi(1-\gamma)-1} (1-\ell_t)^{(1-\chi)(1-\gamma)-1}.$$

- ▶ This term will be negative if $\gamma > 1$
- ▶ Baseline assumption: $\gamma = 2.2$, implying consumption and leisure are substitutes

- ▶ Combining first-order conditions yields

$$u_{1-\ell}(A_t c_t, 1 - \ell_t) = u_{Ac}(A_t c_t, 1 - \ell_t) A_t w_t.$$

- ▶ With the baseline preferences, this becomes

$$\begin{aligned} \frac{1 - \chi}{1 - \ell_t} &= \chi \frac{w_t}{c_t}, \\ \Rightarrow \ell_t &= 1 - \left(\frac{1 - \chi}{\chi} \right) \frac{c_t}{w_t}. \end{aligned}$$

- ▶ This specification produces constant hours along a balanced growth path
- ▶ King et al (1989) provide a general set of conditions

Data Puzzle 1

- ▶ Consumption tracks income over the life-cycle: Inconsistent with consumption smoothing
- ▶ If consumption and leisure are substitutes, people working more hours will consume more implying that consumption tracks income (Heckman, 1974)

Data Puzzle 2

- ▶ There is a discrete drop in consumption immediately after retirement which is inconsistent with consumption smoothing
- ▶ If consumption and leisure are substitutes, then consumption will drop at retirement (French 2005, Aguiar and Hurst 2005)

Data Puzzle 3

- ▶ Low-wage young people work many hours; high-wage old people work fewer hours: Inconsistent with the intertemporal substitution of labor
- ▶ Young people work long hours to fund precautionary saving
- ▶ This precautionary saving builds up assets and reduces the need to work when old
- ▶ This result does not require non-separable preferences
- ▶ It does require life-cycle (not infinite-horizon) framework with low initial wealth

Conclusion

- ▶ Spring break! Watch March Madness!
- ▶ Midterm in two weeks!
- ▶ Homework in two weeks!