Instructor: *Professor Griffy*
Due: *May 4th, 2023*
AECO 701

Problem Set 6: The Heterogeneous Agent Model

Problem 1. Huggett Model. On the campus cluster, you will find code to solve the Aiyagari model with a labor-leisure choice. Please start from that code (email me if you cannot access the cluster). The Huggett (1993) Model is given by

$$V(a, \epsilon; \psi) = u(c) + \beta E[V(a', \epsilon'; \psi')] \tag{1}$$

subject to

$$c + a' \leq (1 + r(\psi))a + \epsilon \tag{2}$$

$$\epsilon \sim \mathit{Markov}, \Pi(\epsilon'|\epsilon) \tag{3}$$

$$\psi' = \Psi(\psi) \tag{4}$$

Assume the following calibration:

Parameter	Value
$u(c)$	$\frac{c^{1-\sigma}}{1-\sigma}$
β	0.993
σ	1.5
a'	$\geq$ -2
a grid	[-2, 12]
a nodes	100

$$\pi_t = \begin{bmatrix} 0.925 & 0.075 \\ 0.5 & 0.5 \end{bmatrix} \tag{5}$$

$$\epsilon = \begin{bmatrix} 1.0 \\ 0.1 \end{bmatrix} \tag{6}$$

Note that market clearing is given by

$$\int_{a \times \epsilon} a' d\psi = 0 \tag{7}$$

1. Solve the model. Plot the decision rules for savings across the a grid for an agent in employment state 1 and employment state 2.
2. Plot the stationary distribution of wealth.

Problem 2. Aiyagari Model. Now we will extend the problem to include a firm, as in Aiyagari (1994). In this economy, the household's problem is given by

$$V(k,l;\psi) = u(c) + \beta E[V(k',l';\psi')] \tag{8}$$

subject to

$$
\begin{aligned}
c + k' &\leq (1 + r(K,L) - \delta)k + w(K,L)l && (9)\\
k' &\geq 0 && (10)\\
ln(l') &= \rho ln(l) + \sigma(1-\rho^2)^{\frac{1}{2}}\epsilon',\ \epsilon \sim N(0,1) && (11)\\
\psi' &= \Psi(\psi) && (12)
\end{aligned}
$$

and the firm's problem is given by

$$\Pi = \max_{K,L} F(K,L) - r(K,L)K - w(K,L)L \tag{13}$$

Assume the following calibration:

Parameter	Value
$u(c)$	$\frac{c^{1-\mu}}{1-\mu}$
$F(K,L)$	$K^{\alpha}L^{1-\alpha}$
β	0.96
δ	0.08
α	0.36
k Grid	[0, 18]
k nodes	100

Note that market clearing is given by

$$\int_{k\times l} l d\psi = L \tag{14}$$

$$\int_{k\times l} k' d\psi = K \tag{15}$$

and that μ, σ, and ρ will be given in the following parts.

1. Use Tauchen's method to approximate the AR(1) process for $\rho = 0.6$ and $\sigma = 0.2$. Write out the resulting grid and transition matrix.

2. Find the ergodic distribution of employment using this grid and transition matrix. Call this L. Note that you will need to calculate this for each ρ, σ pair.

3. Solve the model. Plot the decision rules for savings across the k grid for $\rho = 0.6$, $\sigma = 0.2$, and $\mu = 3$.

4. Plot the stationary distribution of wealth for $\rho = 0.6$, $\sigma = 0.2$, and $\mu = 3$.

5. Now pick $\sigma = 0.4$ and plot the net return to capital for $\rho = \{0, 0.3, 0.6, 0.9\}$ and $\mu = \{1, 3, 5\}$.