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AECO 701

Short Handout Assignment 1

The assignments this week are meant to review a couple of important concepts for the prelim. They do not need to be turned in, but can hopefully shed some light on a couple sources of confusion. **Thinking about state and choice ariables** Consider the following stylized neoclassical growth model. A representative agent maximizes utility by choosing consumption, c, capital saved, k', and labor supplied, ℓ. They take prices, $r(K,L)$ and $w(K,L)$ as well as the aggregate law of motion for capital, $K' = G(K)$. They face the following problem:

$$V(k;K) = \max_{c,l,k'} u(c, 1-\ell) + \beta V(k';K') \text{s.t.} \quad c + k' \leq \quad (1 + r(K,L) - \delta)k + w(K,L)\ell \tag{1}$$

$$\ell \in (0,1), k_0 \text{ given.} \tag{2}$$

where prices are given by a profit maximizing firm:

$$\Pi = \max_{K,L} F(K,L) - wL - rK \tag{3}$$

The equilibrium in this economy is a recursive competitive equilibrium, where c, ℓ, k' solves the workers problem, prices, w, and r, are determined competitively, aggregation holds, $K = \int k' dF(k)$, $L = \int \ell dF(k)$, and the individual decision rules are consistent with the aggregate laws of motion, i.e., $K' = g(K,K)$, where $g(.)$ is the representative agents decision rule ($k' = g(k,K)$).

a) Why are k and K both state variables? Why are L and ℓ not?

Answer:
There's really two questions embedded here. I'll answer them sequentially. First, why are do we need to include both individual and aggregate capital. Well, we are interested in the decentralized (competitive) solution, meaning that agents are atomistic and even though we are using a single Representative Agent, they are taking prices as given and do not understand that they have market power. This means that they need to solve their own problem (by picking k), taking as given prices. Simultaneously, they understand that prices are determined by aggregates, and therefore their Euler Equation depends directly on the evolution of capital:

$$u_c(c_t, 1-\ell^*) = \beta(1 + r(K_{t+1}, L_{t+1}) - \delta) u_c(c_{t+1}, 1-\ell^*) \tag{4}$$

where ℓ^* is the optimal solution to the stationary static problem. Assuming that we have a way of predicting the evolution of K (more on that in a moment), we can determine our allocation of resources over time. Remember that the Euler Equation is just $\frac{MU_{c_t}}{MU_{c_{t+1}}} = p$, but applied to a dynamic setting. When utility is homothetic, you know exactly what you'll consume today and in the future once you know k.

The second part of this question is about the *lack* of inclusion of either ℓ of L. Let's start with ℓ. Why do we not need to include it as a state variable? Well the reason is very similar to the intuition behind the Envelope Theorem: once we know k (today's individual capital), we can solve a static resource optimization and solve for ℓ^*. Consider the $FOC[\ell]$, combined with $FOC[c]$

$$\frac{u_c(c, 1-\ell)}{u_l(c, 1-\ell)} = \frac{w(K, L)}{p_c} \tag{5}$$

where p_c is the price of consumption, which is generically $p_c = 1$, i.e., consumption is the numeraire (the price of all goods is quoted in consumption/real terms). In other words, any marginal utility resulting from the choice of labor/leisure is proportional to marginal utility in our Euler Equation. Now consider this Euler Equation, where we assume $u(c, 1-\ell) = \gamma ln(c) + (1-\gamma) ln(1-\ell)$

$$\frac{\frac{\gamma}{c}}{\frac{1-\gamma}{1-\ell}} = w(K, L) \tag{6}$$

$$\gamma(1-\ell) = w(K, L)c^*(1-\gamma) \tag{7}$$

a standard result. Now, we can write the labor supply function l^* directly:

$$\ell = 1 - \frac{wc^*(1-\gamma)}{\gamma} \tag{8}$$

what does this say? If we take prices as given, we uniquely pin down ℓ^* given k, because consumption is a normal good. In other words, given prices and marginal utility of consumption from the Euler Equation, we can write $\ell^*(k)$ as a function of individual capital. This means that we only need to carry k. Last, how do we calculate L? Well, we can also write $\ell = h(k, K)$, much like how we defined the evolution of individual capital. A consistency requirement is that if we assign all capital to the Representative Agent, their decision is consistent with aggregate outcomes, hence $L = m(K, K)$. You can think of h being homothetic of degree 1 (i.e., I can sum over it). This means that we know L given aggregate capital by knowledge of the underlying problem and we don't need to include it in our state variables.

b) The function $K' = g(K, K)$ appears innocuous, but it is actually a fairly strict assumption. Describe in your own words, what it means.
Answer:
This is actually the essence of rational expectations: that agents understand the structure of the problem, and thus (with a few additional assumptions about preferences and complete markets), use the same decision rule that the economy as a whole uses to determine the evolution of capital. To get some insight into why this matters, suppose that $k' = g(k, K)$, but $K' < g(K, K)$. Let's look at the Euler Equation:

$$u_c(c_t, 1-\ell^*) = \beta E[(1 + r(g(K, K), L_{t+1}) - \delta)u_c(c_{t+1}, 1-\ell^*)] \tag{9}$$

where we have use $g(K, K)$ instead of K_{t+1}. The agent is planning their resource allocation around a *lower* interest rate than will occur in equilibrium (we're thinking partial equilibrium here), and thus save less if they are a saver, and borrow more if they are a borrower.

c) Why do we "take prices as given" when using a representative agent (or any model with atomistic agents)?
Answer:
This is somewhat alluded to in part 1, but we want agents to be atomistic and focus on the competitive equilibrium. If we assumed firms were perfectly competitive, but allowed the representative agent to understand that they are the only supplier of labor, we've introduced an externality from monopsonistic competition. In essence, workers could extract rents from the firm by supplying labor sub-optimally.

Now suppose that labor productivity is determined by the human capital, h, of the representative agent. Human capital is an AR1 process that evolves according to the following: $ln(h') = \rho ln(h) + \epsilon_h, \epsilon_h \sim N(0, \sigma_h)$. The firms problem is now

$$\Pi = \max_{K,L} F(k, HL) - rK - wHL \tag{10}$$

d) What are the state variables for the workers problem?
Answer:
The question here is whether or not we need to include h and H as a state variable. The answer is: "yes." h and H evolve exogenously, and thus cannot be determined within the period. If we assume that the human capital shock is common to all workers, this is equivalent to including an aggregate shock. If not, we have a heterogeneous agents problem in which agents must use the ergodic distribution of human capital to determine H.