Macro II

Professor Griffy

UAlbany

Spring 2023

Introduction

- ▶ Last class! Final May 9th, 10:30am-12:30pm.
- ▶ Today: the Hosios Condition
- ▶ Efficiency in search and matching models.
- ▶ (Note: largely derived from Christine Braun's lecture on the DMP model).
- ▶ Homework on my website.
- ▶ Code for Aiyagari w/ labor-leisure choice on the cluster.
- ▶ Due thursday.

Efficiency

- Is zero unemployment efficient? No

  - higher unemployment incentivizes firms to post vacancies

  - but high unemployment is costly, less production

- Is a high vacancy rate efficient?

  - vacancy creation is costly

  - but lots of vacancies reduces unemployment

- So what is the efficient level of θ?

Efficiency

- Congestion externality

    - one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off

    - one more searching worker makes hiring firms better off and makes all other searching workers worse off

- Appropriability

    - firm pays a cost κ to post vacancy but does not get to keep the entire output p

Efficiency

- What value of θ would the social planer choose to maximize total output/utility if he is constrained by the same matching frictions?

    - does not care about wage b/c it's a linear transfer from the firm to the worker

- Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?

- Can we achieve this wage with the Nash solution?

The DMP Model ("Ch. 1 of Pissarides (2000)")

- ▶ Agents:
    1. Employed workers;
    2. unemployed workers;
    3. vacant firms;
    4. matched firms.

- ▶ Linear utility ($u = b, u = w$) and production $y = p > b$.

- ▶ Matching function:
    1. Constant returns to scale (L is lab. force):

    $$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta)$$

    2. where $\theta = \frac{v}{u}$ is "labor market tightness"
    3. Match rates:

    $$\underbrace{p(\theta)}_{Worker} = \theta \underbrace{q(\theta)}_{Firm}$$

- ▶ Social planner: pick θ optimally, no need to respect free entry condition.

Social Planner's Problem

$$\int_0^\infty e^{-rt}[p(1-u) + bu - \kappa\theta u]\ dt$$

$$\text{s.t.}\quad \dot{u} = \delta(1-u) - p(\theta)u$$

▶ Social planner's problem

    ▶ $p(1-u)$: social output of employment

    ▶ bu: leisure enjoyed by unemployed workers

    ▶ $\kappa\theta u$: cost of jobs

▶ Social planner is subject to the same transition equation for unemployment

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow \quad -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow \qquad\qquad -e^{-rt}\kappa u - \mu u(q(\theta) + \theta q'(\theta)) = 0$$

- μ: marginal value of an extra unemployed worker.

Social Planner's Problem

- Optimal θ

$$H_\theta = 0 \Rightarrow \qquad -e^{-rt}\kappa u - \mu u q(\theta)(1 + \frac{\theta q'(\theta)}{q(\theta)}) = 0$$

- What is $\frac{\theta q'(\theta)}{q(\theta)}$?

$$m(u, v) = v q(\theta)$$
$$\rightarrow \frac{\partial m(u, v)}{\partial u} = v q'(\theta)\frac{-v}{u^2}$$
$$\rightarrow \frac{\partial m(u, v)}{\partial u} = -\theta^2 q'(\theta)$$
$$\rightarrow \frac{\frac{\partial m(u,v)}{\partial u}}{m(u, v)} = -\frac{\theta^2 q'(\theta)}{v q(\theta)}$$
$$\rightarrow u\frac{\frac{\partial m(u,v)}{\partial u}}{m(u, v)} = -\frac{\theta q'(\theta)}{q(\theta)}$$

- $\frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function wrt u.

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow \quad -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow \quad -e^{-rt}\kappa u - \mu u q(\theta)(1 - \eta(\theta)) = 0$$

- $\eta(\theta)$: elasticity of match fun. wrt u.

Optimal θ

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$):

$$\frac{p - b + \kappa\theta}{\delta + r + p(\theta)} = \frac{\kappa}{q(\theta)(1 - \eta(\theta))}$$

$$(p - b)(1 - \eta(\theta)) + \kappa(1 - \eta(\theta))\frac{p(\theta)}{q(\theta)} = \frac{(\delta + r + p(\theta))\kappa}{q(\theta)}$$

$$\rightarrow (1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \qquad (1)$$

- This is optimal θ

Decentralized solution

- Can the decentralized solution achieve the same level of θ?
- i.e., can the decentralized level of unemployment be *efficient*?

Decentralized θ

- Free entry $V = 0$:

$$rJ(w) = (p - w) + \delta[\cancel{V} - J(w)]$$
$$(r + \delta)J(w) = (p - w)$$

- Vacancy creation condition (i.e., free entry imposed):

$$q(\theta) = \frac{\kappa}{E[J(w)]}$$
$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$
$$\theta = q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)$$

- Thus, mapping between wages and θ. 1 equation, 2 unknowns.

- Need equation to determine wages in equilibrium.

Wage Determination

▶ Recall Nash Bargained wages:

$$w = argmax_w \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta}\frac{\partial W}{\partial w}$$

$$+ (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U)\frac{\partial J}{\partial w}$$

▶ $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\beta(\frac{J(w)}{W(w) - U})^{1-\beta} = (1 - \beta)(\frac{W(w) - U}{J(w)})^\beta$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

Wage Determination

▶ Note that $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w))$$
$$+ (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

▶ And $(1 - \beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1-\beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w))$$
$$+ (1 - \beta)(p(\theta) + \delta)\beta \frac{J(w)}{1 - \beta}$$
$$w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

▶ Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

▶ Job creation curve:

$$(r + \delta)J(w) = (p - w)$$

$$q(\theta) = \frac{\kappa}{J(w)}$$

$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$

$$p - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

▶ Now, plug in using wages we just found:

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

- Job creation curve:

$$p - ((1 - \beta)b + \beta p + \beta \theta \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- identities: $p(\theta) = \theta q(\theta) \rightarrow \theta = \frac{p(\theta)}{q(\theta)}$

$$\rightarrow p - ((1 - \beta)b + \beta p + \beta \frac{p(\theta)}{q(\theta)} \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \beta \frac{p(\theta)}{q(\theta)} \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \frac{r + \delta + \beta p(\theta)}{q(\theta)} \kappa = 0$$

- Looks familiar?

Social Planner's Problem

▶ Using $p(\theta) = \theta q(\theta)$ and solving in steady state $(\dot{\mu} = 0)$

$$(1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \qquad (2)$$

▶ From the decentralized solution, plug the wage curve into the Job creation curve

$$(1 - \beta)(p - b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)}\kappa = 0 \qquad (3)$$

Efficiency

▶ Comparing (1) and (2) we see that we have efficiency in the decentralized market if $\beta = \eta(\theta)$. The workers bargaining power is equal to the elasticity of the matching function with respect to u.

▶ This is a general result: we have efficiency when

$$\eta(\theta) = \beta$$

▶ This is called the Hosios (1990) condition

Thanks for a good semester!

- ▶ I'm buying beers at City Line after you guys finish your prelims!