AECO 701 Final

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1 [50] **Training and Frictions**. Consider a discrete time environment in which firms may optionally invest in their worker's human capital. Workers live for two periods, have linear utility $u(c) = c$ in both periods, and do not have access to savings technology (although it doesn't matter with linear utility). They start period 1 employed at a piece-rate wage $w \times h$, where w is the piece-rate earned out of total productivity and h is a worker's human capital. The piece-rate is distributed $F(\mu_w, \sigma_w)$, and $w \in [0, 1]$. Workers may differ in either h or w. Firms employ workers in a one-worker-one-firm match and receive $h - wh - c(\tau)$, where wh is the wage and $c(\tau)$ is a convex function that takes into account the possibility that the firm may train their worker. For simplicity, let $c(\tau) = \tau^2$. Prior to production in the second period, workers are able to search for a new job and find one with probability λ_E. They accept this job if it offers $w_R > w$. If they find and accept a job, the match dissolves. Otherwise, they produce at their original firm and the match dissolves at the end of the period. Workers and firms both discount the future at a common rate $\beta \in (0, 1)$. For the worker, this yields the following value function:

$$V_1(h, w) = wh + \beta\{\lambda_E \int_{\underline{w}}^{\bar{w}} \max\{V_2(h', w'), V_2(h', w)dF(w') + (1 - \lambda_E)V_2(h', w)\} \tag{1}$$

$$h' = g(w, h) \tag{2}$$

$$V_2(h, w) = wh \tag{3}$$

The firm's problem is given by

$$J_1(h, w) = \max_{\tau} h - wh - \tau^2 + \beta\{(1 - \lambda_E(1 - \int_{w}^{\bar{w}} dF(w')))J_2(h', w)\} \tag{4}$$

$$h' = \begin{cases} h + 1 & : with\ \tau\ probability \\ h & : with\ (1 - \tau)\ probability \end{cases} \tag{5}$$

$$J_2(h, w) = h - wh \tag{6}$$

note that w in the lower limit of integration is the wage at the worker's current firm.

a) (15) Write out the firm's optimization problem by substituting their period 2 problem into their period 1 problem. Now, write out the FOC for the firm in τ. Either explicitly or intuitively explain how this decision depends on the worker's current wage, and the availability of wages higher than their current wage (i.e., $(1 - F(W))$).

Answer:

The firm's optimization problem is given by

$$J_1(h, w) = \max_{\tau} h - wh - \tau^2 + \beta\{(1 - \lambda_E(1 - \int_{w}^{\bar{w}} dF(w')))[\tau(h + 1)(1 - w) + (1 - \tau)h(1 - w)]\}$$

Then their first-order condition is given by

$$
\begin{aligned}
FOC[\tau] &= -2\tau + \beta\{(1-\lambda_E(1-\int_w^{\bar{w}} dF(w')))(h+1)(1-w) - h(1-w)\} = 0 \\
&= -2\tau + \beta\{(1-\lambda_E(1-\int_w^{\bar{w}} dF(w')))(1-w)\} = 0 \\
\rightarrow \tau &= \frac{\beta\{(1-\lambda_E(1-\int_w^{\bar{w}} dF(w')))(1-w)\}}{2}
\end{aligned}
$$

We can write this out in the following way to gain more intuition:

$$
\begin{aligned}
\tau &= \frac{\beta\{(1-\lambda_E(1-F(w)))(1-w)\}}{2} \\
2\tau &= \beta\{\underbrace{(1-\lambda_E(1-F(w)))}_{Moral\ Hazard}\underbrace{(1-w)}_{Profits}\}
\end{aligned}
$$

Current wages change the optimization in two ways: first, it decreases profits (the last part). Second, it decreases the probability that the worker leaves to a new job, which is known as the "hold-up problem," which roughly translates to being unable to recoup investments because the firm faces the moral hazard that the worker may find a better paying job.

b) (10) Suppose that labor markets are competitive, which in this context means that workers are paid their marginal product, $w = 1$, $w \sim F(1, 0)$ and they always find a job if they are looking, $\lambda_E = 1$. Rewrite the firm's problem and then show that optimal training provision, τ is zero. Answer:
We can start from the above and adjust the firm's value function

$$
\begin{aligned}
J_1(h, w) &= \max_\tau h - wh - \tau^2 + \beta\{(1-\lambda_E(1-\int_w^{\bar{w}} dF(w')))[\tau(h+1)(1-w) + (1-\tau)h(1-w)]\} \\
&= \max_\tau h - h - \tau^2 + \beta\{(1-1(1-\int_w^{\bar{w}} dF(w')))[\tau(h+1)(1-1) + (1-\tau)h(1-1)]\} \\
&= \max_\tau h - h - \tau^2 + \beta\{(1-1(1-1))[\tau(h+1)(1-1) + (1-\tau)h(1-1)]\} \\
&= \max_\tau h - h - \tau^2 + \beta\{(1-1(1-1))[\tau(h+1)(1-1) + (1-\tau)h(1-1)]\} \\
&= \max_\tau -\tau^2
\end{aligned}
$$

Clearly, the optimum is $\tau = 0$. Here, we see that the firm's inability to recoup costs (i.e., profits are zero), means that they will not invest in human capital.

c) (15) Now assume that wages are uniformly distributed $w \sim U(0, 1)$. Rewrite the worker and firms problems with this distribution. Answer:
The worker's problem doesn't actually change, except the inclusion of the uniform distribution allows you to write the value function as

$$
\begin{aligned}
V_1(h, w) &= wh + \beta\{\lambda_E \int_{\underline{w}}^{\bar{w}} \max\{w'g(w, h), wg(w, h)\}dF(w') + (1-\lambda_E)wg(w, h)\} \\
&= wh + \beta\{\lambda_E \int_w^{\bar{w}} w'g(w, h)dF(w') + (1-\lambda_E(1-F(w)))wg(w, h)\}
\end{aligned}
$$

we could go further than this once we know the functional form for $g(w, h)$, but the worker has no real role in this problem and will always accept any wage $w' \geq w$. Why is this the case? They only live for

one more period, so have no option value to rejecting their job.
The firm's optimization problem is given by

$$\begin{aligned}J_1(h,w) &= \max_{\tau} h - wh - \tau^2 + \beta\{(1-\lambda_E(1-\int_w^{\bar{w}} dF(w')))[\tau(h+1)(1-w) + (1-\tau)h(1-w)]\} \\ &= \max_{\tau} h - wh - \tau^2 + \beta\{(1-\lambda_E(1-F(w)))[\tau(h+1)(1-w) + (1-\tau)h(1-w)]\}\end{aligned}$$

The next step warrants a bit of discussion. Because $w \sim U(0,1)$, we can write the PDF as simply w:

$$\begin{aligned}J_1(h,w) &= \max_{\tau} h - wh - \tau^2 + \beta\{(1-\lambda_E(1-w))[\tau(h+1)(1-w) + (1-\tau)h(1-w)]\} \\ &= \max_{\tau}(1-w)h - \tau^2 + \beta\{(1-\lambda_E(1-w))[(\tau+h)(1-w)]\}\end{aligned}$$

Then their first-order condition is given by

$$\begin{aligned}FOC[\tau] &= -2\tau + \beta\{(1-\lambda_E(1-w))(1-w)\} = 0 \\ \rightarrow \tau &= \frac{\beta\{(1-w) - \lambda_E(1-w)^2\}}{2}\end{aligned}$$

d) (10) How much does output change moving from a frictional world in part 3 to a frictionless world in part 2. For simplicity, we will assume that all workers are ex-ante identical. In the perfectly competitive world, $w = 1$, and $h = 1$. In the frictional world $w = \frac{1}{2}$, and $h = 1$. Also assume that in the frictional world, $w \sim U(0,1)$ and $\lambda_E = 1$. Is the competitive equilibrium pareto optimal? Why or why not? Answer: We can start from the FOC above and plug in the values given to determine the level of training:

$$\begin{aligned}\tau &= \frac{\beta\{(1-w) - \lambda_E(1-w)^2\}}{2} \\ &= \frac{\beta\{\frac{1}{2} - \frac{1}{4}\}}{2} \\ &= \beta\frac{1}{8}\end{aligned}$$

We know that $\beta > 0$, therefore the optimal investment, $\tau^* > 0$. Output becomes

$$\begin{aligned}Y_F &= h_0 + \tau_0(h_0+1) + (1-\tau_0)h_0 - \tau_0^2 \\ &= 1 + 2\tau_0 + (1-\tau_0)1 - \tau_0^2 \\ &= 2 + \tau_0 - \tau_0^2 \\ &= 2 + \beta\frac{1}{8} - (\beta\frac{1}{8})^2\end{aligned}$$

In the frictionless or competitive world,

$$\begin{aligned}Y_C &= h_0 + \tau_0(h_0+1) + (1-\tau_0)h_0 - \tau_0^2 \\ &= 1 + 2\tau_0 + (1-\tau_0)1 - \tau_0^2 \\ &= 2 + \tau_0 - \tau_0^2 \\ &= 2\end{aligned}$$

because firms never choose to invest as they cannot recoup their costs and therefore $\tau_0 = 0$. This would sufficie for an argument that the competitive equilibrium is not pareto optimal, as it contains a substantial hold-up externality.

2 [50] **Temporary Unemployment Compensation**. At the beginning of each period an unemployed worker draws one offer to work forever at wage w (which she may accept or reject). Wages are i.i.d. draws from the c.d.f. F, where $F(\underline{w}) = 0$ and $F(\overline{w}) = 1$. The worker seeks to maximize $\mathbb{E}\sum_{t=0}^{\infty} \beta^t y_t$, where y_t is the worker's wage or unemployment compensation, if any. The worker is entitled to unemployment compensation in the amount $\gamma > 0$ only during the *first* period that she is unemployed. After one period on unemployment compensation, the worker receives none.

a) (15) Write the Bellman equations for this problem.

Answer:

We'll start by defining the superscript "1" to denote the first period of an unemployment spell, and the superscript "+" denote any future periods in such spell.

$$\begin{aligned}
V_E(w) &= w + \beta V_E(w) \\
V_U^1(w) &= \max\left\{V_E(w), \gamma + \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\} \\
V_U^+(w) &= \max\left\{V_E(w), \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\}
\end{aligned}$$

b) (10) Show and explain how the worker's reservation wage and her "hazard of leaving unemployment" (i.e. the probability of accepting a job offer) varies with the duration of unemployment.

Answer:

The reservation wage is given by the wage such that an individual is indifferent between working and remaining unemployed. Here, depending on how many periods the worker has been unemployed, the reservation wage will be different. Denote these w_R^1 and w_R^+. Further, we can rewrite $V_E(w) = \frac{w}{1-\beta}$.

$$\begin{aligned}
w_R^1 &= (1-\beta)\gamma + (\beta - \beta^2) \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w') \\
w_R^+ &= (\beta - \beta^2) \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')
\end{aligned}$$

We can see that $w_R^1 - w_R^+ = (1-\beta)\gamma > 0$. That is, the reservation wage is decreasing as an unemployment spell continues. In the first equation above, the reservation wage is proportional to unemployment benefits *plus* the option value of waiting for another offer (where one knows that there will be no benefits in the future).

Next, note that the hazard of leaving unemployment is given by the probability that a wage is drawn at least as large as the reservation wage.

$$\Pr(w \geq w_R^i) = 1 - F(w_R^i) \qquad i \in \{1, +\}$$

Noting the discussion above about the result that $w_R^1 > w_R^+$, we can easily determine that $\Pr(w \geq w_R^1) \leq \Pr(w \geq w_R^+)$, that is the hazard of leaving unemployment is increasing with the length of an unemployment spell.

For parts (c) and (d), assume that the worker is also entitled to unemployment compensation if she quits a job. As before, the worker receives unemployment compensation in the amount of γ during the first period of an unemployment spell, and zero during the remaining part of the spell. (In order to re-qualify for the benefits, the worker must find a job and work at least one period.)

The timing of events is as follows. At the very beginning of a period, a worker who was employed in the previous period must decide whether or not to quit. If she quits, she draws a new wage offer as described previously, and if she accepts the offer she immediately starts earning that wage without suffering any period of unemployment.

c) (15) Write the Bellman equations for this problem. [Hint: Define a value function $V_E(w)$ for a worker who was employed the previous period with wage w, before any decision to quit (and receive some new draw w') occurs.]
Answer:

$$V_E(w) = \max\left\{w + \beta V_E(w), \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')\right\}$$
$$V_U^1(w) = \max\left\{w + \beta V_E(w), \gamma + \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\}$$
$$V_U^+(w) = \max\left\{w + \beta V_E(w), \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')\right\}$$

d) (10) Characterize the reservation strategy of an employed worker and then prove that the reservation wage for someone unemployed longer than one period is equal to 0 (if you cannot prove it, give some intuition as to why it must be the case). Answer:
First denote the reservation wage of an employed person as w_R^E (this gives the cutoff for when a worker will decide to quit or stay at a job). It is defined by the following.

$$w_R^E + \beta V_E(w_R^E) = \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Because the problem is the same every period for an employed worker, her quit / stay decision will always be the same. We can utilize this fact to simplify the above.

$$w_R^E = (1-\beta)\int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Regarding the proof, suppose that $w_R^+ > 0$. Then we must have that

$$\underbrace{w_R^+ + \beta V_E^+(w_R^+) = \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')}_{\text{reservation statement for “+” unemployed}} < \underbrace{\int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w') = w_R^E + \beta V_E(w_R^E)}_{\text{reservation statement for employed workers}}.$$

Since $w + \beta V_E(w)$ is (weakly) increasing in w, the above statement necessarily implies that $w_R^+ < w_R^E$. Because we are talking about reservation strategies, if we were to plug in w_R^+ into the employed worker's problem, we know she will quit.

$$V_E(w_R^+) = \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')$$

Finally, we can utilize the above result to plug into the reservation statement for "+" workers (and do some moving around).

$$w_R^+ + \beta \overbrace{\int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w')}^{V_E(w_R^+)} = \beta \int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w')$$

$$w_R^+ = \beta \left[\int_{\underline{w}}^{\overline{w}} V_U^+(w')dF(w') - \int_{\underline{w}}^{\overline{w}} V_U^1(w')dF(w') \right] < 0 \qquad \text{(contradiction)}$$

The above is a contradiction, and so we cannot have $w_R^+ > 0$. Because wages are (weakly) positive, we must have $w_R^+ = 0$. Intuitively, since after the first period of unemployment a worker does not receive any benefits, accepting an offer and then quitting is at least as good as rejecting an offer and drawing again next period.