Instructor: *Professor Griffy*
Due: *Feb., 15th 2023*
AECO 701

# Problem Set 2

**Problem 1: Markov Chains** We can apply Markov Chains in a variety of circumstances that involve dynamics. We will think of a simple "lake model" of employment dynamics. There are three states: employed, unemployed, and non-participant (out of the labor force). Denote these as $E$ for employed, $U$ for unemployed, and $N$ for non-participant. The transition probabilities are as follows: $E \to E$: 0.9; $E \to U$: 0.1; $E \to N$: 0.0; $U \to E$: 0.5; $U \to U$: 0.5; $U \to N$: 0.0; $N \to E$: 0.0; $N \to U$: 0.0; $N \to N$: 1.0

**a)** Write down the transition equation in the following way: $x'_{t+1} = x'_t A$. Define each state in $x$ clearly and denote the transition matrix, $A$, correctly.

**b)** Is there a unique stationary distribution? Why or why not?

**c)** Use a computer programming language to find the ergodic distribution for the following initial condition: $x'_0 = [0.55\ 0.05\ 0.4]$. Given an initial condition, is this distribution stationary?

**d)** **Sectoral Decline**: Suppose now that we are modeling an individual sector in the economy. Over time, this sector's reliance on labor is declining. Unfortunately for workers in this sector, they have a great deal of sector-specific human capital and do not have enough general human capital to find jobs in other sectors. The transition probabilities for $E$ and $N$ remain unchanged, but now the transition probabilities for $U$ are given by $U \to E$: 0.5; $U \to U$: 0.45; $U \to N$: 0.05. Write down the transition equation. Does this have a unique stationary distribution? Use the same initial condition and find the resulting distribution for large $t$.

**e)** Suppose now that the government institutes a worker retraining program. Keeping the probabilities for transitions out of unemployment fixed at their values from part *d*, this new policy makes the transition probabilities from $N$: $N \to E$: 0.1; $N \to U$: 0.0; $N \to N$: 0.9. Use the initial distribution from part (c) and find the distribution for large $t$.

**f)** Draw two random series uniformly distributed. To do this, draw 3 numbers from a uniform distributed between 0 and 1. The first number is the (un-normalized) measure of workers starting employed; the second unemployed; the third, non-participant. Normalize by the sum of these three numbers. Simulate the Markov Chain for this series over 1000 periods and plot this series. Repeat this a second time and plot this series again. Discuss the differences between your results.