AECO 701 Midterm

Name:
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1 [40] **Neoclassical Growth with Vintage Capital.** Consider an economy with mass one of identical consumers whose preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t ln(c_t) \tag{1}$$

where β discounts future utility. Consumers supply one unit of labor inelastically. In this economy, there are two sectors, one that employs "vintage" capital and one that employs "modern" capital. Aggregate productivity grows at rate γ, but is subject to idiosyncratic shocks:

$$A_{t+1} = (1+\gamma)A_t + \epsilon_t, \quad \epsilon \sim N(0, \sigma_\epsilon^2) \tag{2}$$

where $\gamma > 0$. Following each period, "vintage" capital depreciates completely. In other words, capital only lasts for two periods, but does not depreciate at all during its lifespan. Note that modern capital is equal to investment from the previous period, while vintage capital is equal to investment from two periods before, so the economy choice of investment each period is equal to the stock of modern capital in the next period. Additionally, capital employs aggregate technology during its first period of use. In other words, production in the "vintage" sector during the current period is given by the following:

$$Y_t^v = A_{t-1} k_{t-1}^{\alpha} (L_t^v)^{1-\alpha} \tag{3}$$

Modern Sector:

$$Y_t^m = A_t k_t^{\alpha} (L_t^m)^{1-\alpha} \tag{4}$$

The superscript v denotes output from the vintage sector, while the superscript m denotes production from the modern sector. Labor can be freely allocated between either of the two sectors. Aggregate resources are equal to the sum of output from both sectors.

a [20] Write the planner's problem in recursive form, being careful to note the state variables and choice variables.

b [15] Solve for the first-order conditions of the problem.

c [5] Show the conditions under which the planner would assign more labor to the vintage sector than the modern sector.

2 [20] **Contraction Mapping Theorem.** Blackwell's Sufficiency Conditions for a contraction are given by the following:

T is monotone if for $f(x) \leq g(x)\ \forall x \in X$, then

$$Tf(x) \leq Tg(x) \quad \forall x \in X \tag{5}$$

T discounts if for some $\beta \in (0, 1)$ and any $a \in \mathcal{R}_+$

$$T(f + a)(x) \leq Tf(x) + \beta a \quad \forall x \in X \tag{6}$$

a [20] Prove that the Bellman Operator satisfies both of these conditions.

3 [40] **Guess and Verify.** Consider an economy with an infinitely-lived representative consumer. Lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t (\gamma ln(c_t) + (1-\gamma) ln(1-l_t)) \tag{7}$$

Where $\beta, \gamma \in (0,1)$. Each individual is endowed with 1 unit of time each period, which can be split between working and leisure. The resource constraint is as follows:

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} l_t^{1-\alpha} \tag{8}$$

Where $\theta > 0$ and $\alpha \in (0,1)$.

a [15] Write down the planner's problem.

b [20] Guess that the value function takes the form $V(k) = a_0 + a_1 ln(k)$. Assume that labor and leisure are constants and solve for a_0 and a_1, as well as policy functions for consumption, leisure, labor, and capital tomorrow (c(k), l(k), k'(k)).