AECO 701 Midterm

Name:
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1 [50] **Guess and Verify.** Consider the decision problem of a man with his volleyball on a deserted island. He faces an infinite horizon and the only food he has is a single cake that he must optimally divide over time. His problem is given by

$$\sum_{t=0}^{\infty} \beta^t ln(c_t) \tag{1}$$

Where $\beta \in (0,1)$. The resource constraint is as follows:

$$c_t + k_{t+1} \leq k_t \tag{2}$$

where $c_t, k_{t+1} \geq 0$, and $k_0 > 0$ is given. Here, k_t is the amount of cake remaining.

a [10] Write down the recursive form of this problem.

b [25] Guess that the value function takes the form $V(k) = a_0 + a_1 ln(k)$. Solve for a_0 and a_1, and then find the optimal policy functions for k' and c.

c [15] Verify this guess.

2 [50] **Contraction Mapping Theorem and House-Keeping.** Blackwell's Sufficient Conditions for a contraction are given by the following:

T is monotone if for $f(x) \leq g(x)\ \forall x \in X$, then

$$Tf(x) \leq Tg(x) \quad \forall x \in X \tag{3}$$

T discounts if for some $\beta \in (0,1)$ and any $a \in \mathcal{R}_+$

$$T(f+a)(x) \leq Tf(x) + \beta a \quad \forall x \in X \tag{4}$$

Now consider a neoclassical growth model in which a representative agent solves the following:

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\gamma}(1-l_t)^{1-\gamma})^{\sigma} - 1}{1-\sigma} \tag{5}$$

Where $\beta, \gamma \in (0,1)$ and $\sigma > 0, \sigma{=}1$. Each individual is endowed with 1 unit of time each period, which can be split between working and leisure. The resource constraint is as follows:

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} l_t^{1-\alpha} \tag{6}$$

Where $\theta > 0$ and $\alpha \in (0,1)$.

a [10] Write the problem recursively and define the Recursive Competitive Equilibrium in this economy.

b [10] Write down the planner's problem. Is it possible for the planner's allocation to be equivalent to the competitive allocation? Yes or no and why.

c [15] Take a derivative of the representative agents value function in the recursive problem with respect to current period capital, k. Do not assume the Envelope Theorem applies and include any implicit differentiation on other endogenous variables that would result. Now use the first order conditions of the problem to show that only the direct effects of k need be considered (i.e., you can ignore its effect on endogenous variables).

d [15] Use Blackwell's Sufficient Conditions to show that the recursive formulation of the competitive equilibrium is a contraction.