AECO 701 Midterm Answers

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1 [50] **A Cake Eating Problem.** President von Clownstick is the leader of a small island. They have a limited number of resources and face the following infinite horizon problem:

$$\sum_{t=0}^{\infty} \beta^t ln(c_t) \tag{1}$$

Where $\beta \in (0, 1)$. The resource constraint is as follows:

$$c_t + k_{t+1} \leq k_t \tag{2}$$

where $c_t, k_{t+1} \geq 0$, and $k_0 > 0$ is given. Here, k_t is the amount of cake remaining.

a [30] Guess that the value function takes the form $V(k) = a_0 + a_1 ln(k)$. Solve for a_0 and a_1, find the optimal policy functions for k' and c, and then verify your guess.

Answer:

Guessing that the value function takes the form $V(k) = a_0 + a_1 ln(k)$ and plugging this into the recursive problem yields

$$RHS = \max_{k'} ln(k - k') + \beta[a_0 + a_1 ln(k')]$$

Taking the derivative of this with respect to k' yields

$$\begin{aligned}
FOC[k'] = -\frac{1}{k-k'} + \beta a_1 \frac{1}{k'} &= 0 \\
\frac{1}{k-k'} &= \beta a_1 \frac{1}{k'} \\
k' &= \beta a_1 (k - k') \\
k' + \beta a_1 k' &= \beta a_1 k \\
(1 + \beta a_1) k' &= \beta a_1 k \\
k' &= \frac{\beta a_1 k}{1 + \beta a_1}
\end{aligned}$$

The decision rule for c is straightforward:

$$\begin{aligned}
c &= k - \frac{\beta a_1 k}{1 + \beta a_1} \\
&= (1 - \frac{\beta a_1}{1 + \beta a_1})k \\
&= \frac{1}{1 + \beta a_1} k
\end{aligned}$$

Now, we can plug in these decision rules to solve for a_0 and a_1:

$$\begin{aligned} RHS^* &= ln(\frac{1}{1+\beta a_1}k) + \beta[a_0 + a_1 ln(\frac{\beta a_1 k}{1+\beta a_1})] \\ &= ln(k) - ln(1+\beta a_1) + \beta[a_0 + a_1\{ln(k) + ln(\beta a_1) - ln(1+\beta a_1)\}] \\ &= -(1+\beta a_1)ln(1+\beta a_1) + \beta[a_0 + a_1 ln(\beta a_1)] + (1+\beta a_1)ln(k) \end{aligned}$$

We know that $a_1 = (1+\beta a_1)$ and solving this gives us a_1:

$$\begin{aligned} a_1 &= (1+\beta a_1) \\ (1-\beta)a_1 &= 1 \\ a_1 &= \frac{1}{1-\beta} \end{aligned}$$

Now plugging this in for a_0 yields

$$\begin{aligned} a_0 &= -(1+\beta a_1)ln(1+\beta a_1) + \beta[a_0 + a_1 ln(\beta a_1)] \\ (1-\beta)a_0 &= -(\frac{1}{1-\beta})ln(1+\frac{\beta}{1-\beta}) + \frac{\beta}{1-\beta}ln(\frac{\beta}{1-\beta}) \\ a_0 &= \frac{-ln(1+\frac{\beta}{1-\beta}) + \beta ln(\frac{\beta}{1-\beta})}{(1-\beta)^2} \end{aligned}$$

Because this can be separated into an expression that follows $V(k) = a_0 + a_1 ln(k)$, our guess is verified.

b [20] Now suppose that they learn that with δ probability, they will be rescued and receive $V^R(k_R)$ where $k_R > k_0$ forever. Describe how their consumption path changes and how this depends on the value of δ.

Answer:

Now, there is a probability δ that they will be able to leave the island. We can write out their resulting recursive problem in the following way:

$$V(k) = \max_{c,k'} ln(c) + \beta[\delta V_R(k') + (1-\delta)V(k')] \tag{3}$$

$$\text{s.t. } c + k' = k \tag{4}$$

$$V_R(k) = \max_{c,k'} ln(c) + \beta[V_R(k')] \tag{5}$$

$$\text{s.t. } c + k' = k + k_R \tag{6}$$

Then the Euler Equation includes a gamble over two possible consumption states:

$$\frac{1}{c} = \beta[\delta\frac{1}{c'(k',\hat{y})} + (1-\delta)\frac{1}{c'(k',0)}] \tag{7}$$

You can easily appeal to Jensen's Inequality and note that more resources implies higher average c, which implies lower marginal utility, and hence more consumption today. The easier way to note that this is a closed economy with no positive investment, which means we can write the present value of lifetime resources as

$$\hat{K} = k_0 + \sum_{t=0}^{\infty} \delta k_R > k_0 \tag{8}$$

Because utility is strictly increasing in c,

$$\hat{K} = \hat{C} = k_0 + \sum_{t=0}^{\infty} \delta k_R > k_0 = \hat{C}_{No\ Escape} \tag{9}$$

where $\hat{C}$ is lifetime consumption and is clearly larger than the absence of leaving the island.

2 [50] **Artificial Expectations.** Consider a standard stochastic neoclassical growth model, where a representative agent

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$
$$s.t.\ y_{t+1} = f(k_{t+1}, \gamma_{t+1}),\ \gamma_{t+1} \overset{i.i.d.}{\sim} \Phi\ \forall t$$
$$y_t \geq c_t + k_{t+1} \forall\ t$$
$$y_0 = \bar{y}_0 \text{ given}$$

Timing:

- y_t summarizes state of world at the start of each period.
- c_t is chosen by the agent each period after observing the state.

a [10] Write the problem recursively and solve for the Euler Equation.

Answer:

We can write the problem recursively as

$$V(y) = \max_{c,k'} u(c) + \beta E[V(y')] \tag{10}$$
$$\text{s.t. } c + k' = y \tag{11}$$
$$y' = f(k', \gamma'), \gamma' \overset{i.i.d.}{\sim} \Phi \tag{12}$$

The only sublety is that you can get away with a single state variable because y summarizes the state of the world in the next time period and you know the evolution of the aggregate state, y'.Next, the Euler Equation is standard:

$$u'(c) = \beta E[f'(k_{t+1}, \gamma_{t+1})u'(c')] \tag{13}$$

b [10] Define the rational expectations equilibrium. Be careful to note i) the relevant equilibrium objects and functions, ii) the aggregation conditions that make this a rational expectations equilibrium.

Answer:

The recursive competitive equilibrium in this economy is functions $V(k)$, policy functions c, k', a set of prices, r such that

- c, k' solve the representative worker's problem given prices.
- Prices are determined competitively by the aggregate firm.
- Individual decisions are consistent with aggregate decisions and markets clear.

This final condition is of consequence here. A rational expectations equilibrium requires that aggregate capital solve $y' = E[f(k', \gamma)]$ where the expectation is over γ as well as the decisions of over agents in the economy. That is, taken any agent in the economy and given them the aggregate capital in the economy and they will on average correctly predict y' because they correctly understand the fundamentals of their world.

c [15] Now, suppose that the problem faced by the agent is given by the following:

$$v(a,y) = \max_{c,a'} u(c) + \beta \int_{\underline{y}}^{\bar{y}} v(a',y')\hat{f}(y')dy' \tag{14}$$

$$\text{s.t. } c + a' \leq (1+r)a + y \tag{15}$$

(note that this somewhat similar to the previous formulation, but has a simplified production/income environment). Agents are unaware that $\hat{f}$ is an approximation for the distribution of y', but you as the macroeconomist are. Please write out the Euler Equation under two different sets of beliefs:

- Exact beliefs, ie $\hat{f}(y') = f(y')$, and
- Beliefs measured with error, ie $\hat{f}(y') = y' + \epsilon'$, $y' \sim f(y')$, and $\epsilon \sim N(0,1)$, but may be correlated with y'.

Compare these two Euler equations under the condition that y' and ϵ' are uncorrelated. Provide intuition for your result.

Answer:

The key distinction in this set-up is that the income process is not correctly measured. The idea is similar to mis-measurement due to endogeneity. There is basically one idiosyncratic component, ϵ, and a common component y', and agents may not be able to distinguish between the two. Starting from our two Euler Equation's

$$u'(c) = \beta(1+r) \int_{\underline{y}}^{\bar{y}} u'(c')\hat{f}(y')dy' \tag{16}$$

In the case where beliefs are measured with error and they are correlated, we can think of agents as mistakenly thinking that the income process is actually an AR1 process. In this case, agents believe the variability of the shock is larger. Depending on the symmetry of the shocks, the means may be the same, but because agents in the second world are less-certain, they will consume less and save more.

d [15] Now suppose that y' and ϵ' are positively correlated and suppose that the interest rate were determined by the borrowing and savings decisions of agents in this economy. Try your best to describe what might happen (hint, the law of motion may not coincide with the rational expectations one!)

Answer:

This question was much trickier than intended. This was meant to ask you to think about a world in which high ability agents, $high-y$, also get lucky shocks and this is known by lenders in the economy. Under this circumstance, high income individuals are subject to less restrictive borrowing. In general equilibrium, the result is that high y retain a larger share of profits and with discrete numbers of individuals would collapse to a single person holding wealth. Under a representative agent, the information structure would prevent this.