Entrenched Beliefs, Slow Learning and Labor Force Participation*

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Abstract

We develop a partial equilibrium search model to show how initial labor market conditions can not only have a persistent effect on beliefs but can cause economy-wide average beliefs to deviate from their fundamental. When returns to the labor market are group-specific but unknown, individuals base their search decision on both their private information and the actions of others, the latter of which is encapsulated in a noisy public signal. The informativeness of the public signal depends on the aggregate action. When individuals are overly-optimistic or pessimistic, the degree of participation reduces the informational content of the signal, causing individuals to learn slowly and for beliefs to become entrenched.

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1 Introduction

Individuals' expectations about their labor market opportunities determine their realized labor market outcomes. Optimistic beliefs can encourage greater job-seeking and higher participation, while pessimistic beliefs can lead to more discouraged workers and raise non-employment. The prevalence of optimistic or pessimistic beliefs may in turn depend on initial labor market conditions, as young individuals who lack labor market experience may rely more on observed aggregate outcomes to learn about their labor market opportunities. Given both the important role of beliefs in driving current outcomes and the role that early labor market outcomes may play in shaping current beliefs, we examine how and when initial labor market conditions can give rise to slow learning and entrenched beliefs.

Our question is motivated by two stylized facts. The first stylized fact – which has been established by others in the literature such as Mueller et al. (2021) and Spinnewijn (2015) for example – is that average beliefs regarding job-finding tend to be *optimistic*, where optimism is defined as the case where the economy-wide average belief of one's ability to find a job is above the average realized job-finding rate.[1] The second stylized fact – which is new and a contribution of this paper – is that current individual beliefs about job-finding are affected by the initial labor market conditions faced by one's peers at the time of entry. Using data from the Survey of Consumer Expectations (SCE), we show that current beliefs on job-finding are positively and significantly affected by the initial labor force participation rate of *similarly-aged* individuals at the time of entry, but not by the initial labor force participation rate of the entire working population. Similarly, we find that current beliefs on job-finding are significantly negatively affected by the initial non-employment rate of similarly-aged individuals at the time of entry, and not by the initial non-employment rate of all individuals. In other words, only the outcomes of similarly-aged individuals matter, and *not* the outcomes of all individuals at the time of an individual's entry into the labor market. This suggests that information gleaned from the outcomes of peers are more useful in deciphering an individual's labor market opportunities.

Given our empirical findings, we then develop a partial equilibrium search model to rationalize these facts and show how initial conditions can lead to entrenched beliefs that significantly deviate from actual realizations. In our model, agents are born into the economy with uncertainty over their group's labor market prospects. By group, we refer to a set of individuals who draw the same fundamental value which influences their job-finding ability. Individuals are also initially endowed with a noisy private signal of their fundamental value. Searching for a job is costly, implying that experimenting by oneself to learn one's group fundamental value is also costly. Individuals, however, can observe the actions of others within their group to infer the value of their fundamental. In particular, they observe a noisy public signal of their group's participation

[1] We define pessimism in a similar way. That is, beliefs are pessimistic if the economy-wide average belief regarding job-finding is below the realized job-finding rate.

in the labor market. The public signal is endogenous, with the degree of participation affecting the amount of private information aggregated into the signal. Because individuals learn from the same noisy public signal, our model provides a rationale for why average beliefs can deviate from the fundamental value. Importantly, a model where agents have uncertainty about their own individual type as opposed to a group fundamental and learn from either private exogenous or private endogenous signals would predict average beliefs equal to the fundamental.[2] Intuitively, the individual noise observed in private signals or the randomness in private outcomes cancels out by law of large numbers, as such one can always show that the average belief in the economy must equal the average realization when learning is derived from private signals or outcomes about one's individual type. We return to this point in greater detail when we discuss alternative model set-ups. While one can get average beliefs to persistently deviate from the average realization in a model where individuals learn about a group fundamental from a public exogenous signal, such a model cannot generate the persistent effect initial conditions has on current beliefs. If the common public signal is exogenous, it is by construction independent of initial conditions. In contrast, our model by focusing on learning from endogenous public signals, illustrates how initial conditions can affect participation and consequently, the informational content of signals, thus allowing it to generate slow learning and entrenched beliefs.

Crucially, the informational content of these signals is endogenous. Either over-optimism or over-pessimism can reduce the amount of information aggregated in the signal. To see this, consider individuals who start their working lives in a boom. When the economy is expanding, many people participate regardless of their private information about their group's fundamental. Similarly, when the economy is contracting, relatively few individuals participate. When participation varies very little with respect to the underlying fundamental value, the public signal incorporates less information about the group's fundamental. When signals themselves are less informative, individuals put more weight on their prior, giving rise to slow learning and an entrenchment of beliefs. This also implies that individuals who start their working lives optimistic, can retain optimistic beliefs for a persistent amount of time when public signals do not reveal much information and individuals put more weight on their priors.

Given our framework, we calibrate the steady state of our model to labor market moments observed during the period 2013q2-2022q2, an interval of time corresponding to the SCE survey period. Specifically, we use information on quarterly realized and perceived job-finding rates, as well as the participation rate to discipline key parameters in our model. In terms of implementation, we assume that a group is equivalent to a cohort and we simulate cohorts for each period, drawing their group fundamental value from an unconditional distribution. Individuals within a cohort do not know their group fundamental value with certainty and must learn about it. Having calibrated the steady state of model, we then introduce aggregate shocks to examine

[2] We refer the reader to Appendix B for a proof of this statement.

how starting in either a strong expansion or deep recession can affect the evolution of beliefs. To simulate the shocks for each period, we back out the implied series of aggregate shocks by filtering empirical unemployment-to-employment transition rates and we feed this into our model. Our goal is to examine if our model with the same set of aggregate shocks as in the data, can explain the joint observations of optimistic beliefs and their dependence on initial conditions.

Overall, our calibrated model predicts that average perceived job-finding rates for the period spanning the SCE sample are about 2 percentage points above the realized job-finding rates, accounting for about one-third of the empirical gap between perceived and realized job-finding rates. Running the same regression in the model as in the data, we find that initial participation by one's cohort positively affects the average perceived job-finding rate, but does so at a decreasing rate, reflecting that how the information content of the signal can vary non-monotonically with participation rates. By causing participation rates to vary less with the underlying fundamental value, initial over-optimism and over-pessimism can delay learning. Focusing on cohorts that started in the Great Recession and during a strong expansion respectively, we find that these cohorts observe the least amount of learning in the early periods of their working life, and thus exhibit beliefs about job-finding which persistently deviate from their fundamental values, with the former being overly pessimistic and the latter being overly optimistic. In contrast, cohorts that enter the labor market when average productivity is close to its mean value, tend to observe less delayed learning, with initial conditions weighing less heavily on current beliefs.

We also extend our model to allow for learning from private outcomes in addition to endogenous public signals. While learning from private outcomes gives individuals an additional source of information, learning is still incomplete as a sub-set of individuals do not participate in the labor market. Because the average belief is affected by the history of public signals as well as one's own individual outcomes, learning can still be slow when participation does not vary much in the fundamental value and public signals are less informative. Consequently, beliefs about job-finding can be persistently optimistic (or pessimistic) and fail to converge to their true values even when we allow for learning from private outcomes.

Overall, our model offers a reason for why learning from endogenous public signals may be a factor behind persistent and optimistic beliefs. Unlike models of learning from private outcomes where average beliefs are equal to their fundamental values, learning from a common signal whose quality depends on the amount of information aggregated from the actions of others can give rise to slow learning and entrenchment of beliefs.

Related Literature This paper relates to the literature on labor market hysteresis. A large body of work argues that individuals who enter the labor market during a recession tend to observe persistently worse labor outcomes than those who enter during a boom (See for example Oreopoulos et al. (2012), Kahn (2010), Schwandt and Von Wachter (2019) and Wachter (2020) for a comprehensive overview). Most of the empirical literature on labor market scarring and hysteresis focuses on how poor initial labor market conditions can have a persistent impact on individuals' earnings and employment outcomes. In separate work, Malmendier and Shen (2018) show how experiencing a recession can persistently affect consumption behavior by affecting individuals' beliefs about future economic outcomes. We add to this literature by focusing on how initial outcomes can entrench beliefs about one's labor market prospects which in turn can affect future labor force participation decisions, long after the initial shock has dissipated.

Our work also relates to the literature on learning about labor market fundamentals. Much of the labor search literature has focused on learning either about one's productivity or fit for a job – as in Gonzalez and Shi (2010), Wee (2016), Doppelt (2016) and Bradley and Mann (2023) – or learning about aggregate fundamentals – as in Potter (2021) –primarily through the observation of one's own outcomes. Similar to Potter (2021) and Bradley and Mann (2023), we assume that agents do not know the true transition rates that they face. However, the uncertainty in our model concerns learning about a group fundamental, or in other words, the labor market prospects for a group of individuals who are otherwise similar to each other. Put more succinctly, agents in our model are learning about a group fundamental which governs the rate at which individuals transition between labor market states. Thus unlike both Potter (2021) and Bradley and Mann (2023), individuals in our model learn by observing the actions of *others* and engage in social learning, as the aggregate action conveys information about a group's fundamentals. As such, our paper focuses on how initial conditions can affect beliefs through the channel of social learning, and analyzes how the history of others' actions can weigh on one's belief about his shared group fundamental and thus the individual's own actions. This social learning channel is absent in the aforementioned papers but is relevant for rationalizing how average beliefs can be persistently optimistic and for understanding why initial conditions can matter for specific cohorts' beliefs about their job-finding ability even after many years in the labor market.

In related work, Fogli and Veldkamp (2011) also looks at the role of social learning in affecting labor force participation rates. In particular, Fogli and Veldkamp (2011) analyze how past information about the wage outcomes of preceding generations informs women of their returns to participating in the labor force. While their model focuses on learning from the outcomes of past generations, we focus on how initial labor market conditions through its effect on individuals' participation decisions can affect the informativeness of public signals, slowing the rate of learning and causing cohorts who start in a strong expansion or deep recession with more uncertainty about their fundamentals long after the initial shock has dissipated. Our paper builds on the literature on social learning and is closely related to work by Schaal and Taschereau-Dumouchel (2021). who show how herd-driven boom-bust cycles can arise when investors learn from the actions of others. In their model, initial optimism leads to high investment rates. Observing high aggregate investment in turn bolsters and reinforces initially optimistic beliefs and drives further investment. Our model's mechanism is similar, initial labor market conditions affect beliefs about the returns to participating in the labor market and thus affect actual participation. Our question however, differs from Schaal and Taschereau-Dumouchel (2021) and focuses on rationalizing the two aforementioned empirical findings in the SCE. Further, in our model, what drives persistently optimistic beliefs stems from a lack of learning, and not from mis-attribution of the fundamental value. Initial conditions which give rise to over-optimism (over-pessimism) about labor market opportunities cause individuals to over-participate (under-participate) no matter what their fundamental value. Because the signal component varies little with the fundamental, there is slower resolution of uncertainty regarding one's labor market opportunities, and thus, beliefs do not converge quickly to their true values.

Our work also relates to recent work by Menzio (2022) who considers a search equilibrium featuring workers with stubborn beliefs. Menzio (2022) departs from rational expectations and considers workers who believe that fundamentals are constant and equal to their unconditional mean even when they are stochastic. Consequently, expected job-finding rates are consistent with actual job-finding rates on average, but these expectations do not respond to the true aggregate state. Unlike Menzio (2022), we maintain rational expectations. Rather, it is because individuals in our model put some weight on the history of aggregate actions that past optimism and pessimism can cause perceived expected returns from participating in the labor market to diverge from actual returns.

The paper proceeds as follows. Section 2 documents a difference in beliefs and realizations in the labor market, and explores the cause. Section 3 presents a dynamic rational expectations model in which individuals update their beliefs after observing the labor market outcomes of similar individuals. Section 4 provides a simple example to show signal quality and participation are linked. Section 5 shows our calibration while Section 6 discusses the results from our baseline model. Section 7 discusses alternative models while Section 8 concludes.

2 Empirical Findings

In this section, we document our findings on perceived job-finding rates as well as how they are affected by initial group conditions. We start by documenting a divergence between beliefs about the likelihood of labor market outcomes and the realizations of those outcomes. Then we explore the causes of this difference.

2.1 Average perceived beliefs on job-finding

We use data from the Federal Reserve Bank of New York's Survey of Consumer Expectations (SCE). The SCE is a nationally representative survey conducted on a monthly basis on a rotating panel of approximately 1,300 respondents. Respondents are followed for up to 12 months. We use the data from 2013m6 to 2022m4, and focus on the responses of individuals aged 25-54. We focus on prime-aged individuals as this is typically a group of individuals with more stable employment paths and who would have "aged" out of learning about their own individual fit to particular jobs or careers. Thus, we would expect that these individuals have a better sense of their labor market opportunities. Table 1 shows how the prime-age sample in the SCE compares against that of prime-age workers sampled in CPS for the same time period in terms of their demographics. Overall, moments in the SCE is largely mimic that of the CPS.

Table 1: Comparison between SCE and CPS between 2013 and 2022

	SCE (Pooled)	SCE (Unemp.)	CPS (Pooled)	CPS (Unemp.)
Demographics				
High School or Less	0.602	0.747	0.622	0.762
College or More	0.398	0.253	0.378	0.238
Age	40.14	39.69	39.63	38.48
Male	0.475	0.366	0.485	0.519
White	0.811	0.739	0.795	0.714
Married	0.675	0.485	0.653	0.653
Observations	75,034	2,600	6,016,708	135,558

Note: This table presents descriptive statistics for prime-age workers from the SCE and CPS for the period 2013m6 to 2022m4.

A key question that the SCE asks individuals regards their perceived probability of finding a job within the next 3 months.[3] Both unemployed and employed individuals are asked this question, with the latter asked about her perceived job-finding probability over the next 3 months if she were to lose her job today. In addition to eliciting beliefs about job-finding rates, the SCE also contains information on realized search outcomes. Thus, we can observe how closely perceived job-finding rates tracks actual job-finding rates for a sub-set of individuals. Table 2 presents our results. The first row highlights the average perceived probability of finding a job within 3 months of across both unemployed and employed prime-age workers. The second row focuses on the unemployed and presents results on the average perceived job-finding rate as well as the average realized job-finding rate. To compute realized probability of finding a job within 3 months in the SCE, we use the fact that the monthly job-finding rate in the SCE is 0.20 and compute the realized probability of job-finding within 3 months as $1-(1-0.20)^3$. The perceived job-finding rate of the unemployed is about 6 percentage points higher than the average realization, slightly below the 8 percentage point gap recorded by Mueller et al. (2021). We also compute the job-finding rate of those with 0-6 months unemployment duration, a group we term as the "short-term unemployed".[4] Unlike Mueller et al. (2021) whose sample includes 20-65 year olds and covers the period 2013-2019, we find that for our sample which focuses on prime-age workers only and which includes periods after 2019, beliefs continue to be optimistic

[3] The SCE also asks the perceived probability of finding a job within the next 12 months. We focus on the 3 month probability as individuals may have more certainty about events over a shorter horizon.

[4] Since the long-term unemployed are traditionally viewed as those with unemployment duration exceeding 26 weeks, we view the complement of this group as the short-term unemployed.

even when we focus on individuals with 0-6 months unemployment duration.

Notably, it is not just the unemployed who exhibit optimistic beliefs. The SCE also asks employed individuals their beliefs about their job-finding probability out of unemployment. While we cannot observe a counterfactual realized job-finding rate out of unemployment for individuals who are employed, we can compare how their elicited beliefs stack up against the average job-finding rate in the economy by using information on realized job-finding rates in the CPS. The lower panel of Table 2 shows our results. Whether we compare to the average job-finding rate of all unemployed in the CPS or to those who are short-term unemployed, the average perceived job-finding rate of employed individuals exceeds the average realized job-finding rate. Overall, our results echo the findings of Mueller et al. (2021): average beliefs over job-finding are optimistic.

Table 2: Perceived vs Realized Job-finding rates

	Perceived	Realized
All	0.573	-
Unemployed	0.522	0.461
0-6mths	0.628	0.523
Employed	0.578	-
CPS (all)	-	0.489
CPS (0-6mths)	-	0.535

Note: This table presents the average perceived job-finding rate and realized job-finding rate for prime-age workers from the SCE and CPS for the period 2013m6 to 2022m4. The realized probability of find a job is imputed as $1-(1-UE)^3$ where UE is the monthly unemployment-to-employment transition rate observed in the data. Unless otherwise stated, all variables reported are computed from the SCE. For the employed, there is no counterfactual job-finding rate out of unemployment to observe within the SCE, as such we compare their outcomes to the job-finding rates observed from the CPS.

We next focus on how these beliefs are influenced by initial labor market conditions. Because we cannot observe the actual date an individual entered the labor market in the SCE or their actual number of years of experience in the labor market, we following Kahn (2010) and use potential experience to proxy for an individual's actual number of years in the labor market. To do this, we assume that an individual with less than a college degree entered the labor market when he or she was age 18, implying 12 years of schooling, and an individual with a college degree entered when he/she was 21, implying 16 years of schooling. We use a linear probability model and assess the impact of local initial labor market conditions on an individual's beliefs about her job-finding rate, where "local" refers to the labor market conditions in one's state. This results in the following specification:

$$y_{it} = \alpha + \pi_1 \text{LFPR}_{i,s,t} + \pi_2 \text{Init. LFPR}_{i,s} + \pi_3 \text{Init. LFPR (18-24)}_{i,s} + \pi_4 u_t + \pi_5 \text{LFPR}_t + \beta' X + \epsilon_{it} \quad (1)$$

where $\text{LFPR}_{i,s,t}$ is the current labor force participation rate (LFPR) observed in individual i who is in state s at time t, and Init. $\text{LFPR}_{i,s}$ is the labor force participation rate in their state when they first entered the labor market. Importantly, both $\text{LFPR}_{i,s,t}$ and Init. $\text{LFPR}_{i,s}$ refer to the state-wide labor force participation rate of individuals of all ages. In contrast, the variable Init. LFPR $(18\text{-}24)_{i,s}$ captures information on the initial labor force participation rate of those similar in age to the individual at the time of her entry into the labor market. Thus, the coefficient of interest is π_3; if individuals are learning about the relevant opportunities from the experiences of their peers, we would expect π_3 to be statistically significant. In contrast, we would expect state-wide variables, which describe the conditions for individuals of all ages, to be less informative of an individual's specific market opportunities. In other words, the variable Init. LFPR $(18\text{-}24)_{i,s}$ represents the *refinement* of information relevant to an individual's group. Our findings in Table 3 describe which variables matter and thus, which pieces of information affect an individual's belief about her current job-finding rate (columns 1 and 2). We also show that our finding is robust to including local unemployment measures (columns 3 and 4).

Our findings in Table Table 3 are remarkably consistent: in all specifications, the current aggregate unemployment rate negatively affects current beliefs about one's job-finding rate, suggesting that individuals do take into account aggregate conditions when forming expectations. Focusing on Column 1 of Table 3, we find that initial conditions also matter for one's belief. Specifically, the initial local labor force participation rate has a positive effect on an individual's expected job-finding rate. This coefficient, π_2, however becomes statistically insignificant once information on the initial local labor force participation rate for those aged 18-24 has been incorporated. Column 2 of Table 3 shows that π_3 is positive and statistically significant, and that a 1 percentage point increase in the local initial labor force participation rate of those aged 18-24 raises an individual's expected job-finding rate by 0.29. Even when we include additional information on local and initial unemployment rates (Column 3), as well as state fixed effects (Column 4), our results survive: that is, initial state-wide participation rates do not matter, instead what matters is the initial participation rate of peers an individual is likely to see as similar at the time of entry. These results suggest that once the aggregate state is taken into account – where the aggregate state here is represented by the current aggregate unemployment rate – the information that is relevant for forming beliefs about labor market opportunities and one's job-finding rate are the inferences one can glean from peers similar to the individual, as opposed to information on all agents in the economy. Further, our results underscore how information on initial conditions of similar peers continue to have an impact on current beliefs.

One might be interested in understanding why the inclusion of initial unemployment rates do not inform current beliefs. These empirical findings directly motivate the construction of our hypothesized model, in which we allow individuals to learn from the actions of others as such actions aggregate and partially reveal the private information that each individual possesses. The initial local participation rate of those aged 18-24 aggregates the private information of agents similar to the individual at the time of entry. In contrast, the unemployment rate captures

Table 3: Expectations about labor market and LFPR

	Expected Job Finding			
	(1)	(2)	(3)	(4)
Local current LFPR	-0.28	-0.34	-0.13	0.02
	(0.21)	(0.23)	(0.28)	(0.49)
Local initial LFPR	0.46^{**}	0.24	-0.14	0.12
	(0.21)	(0.24)	(0.30)	(0.43)
Local initial LFPR (18-24)	-	0.29^{***}	0.32^{***}	0.22^{*}
	-	(0.09)	(0.09)	(0.13)
Local current u	-	-	-0.80^{*}	-0.41
	-	-	(0.40)	(0.37)
Local initial u	-	-	0.06	0.15
	-	-	(0.24)	(0.28)
Local initial u (18-24)	-	-	-0.03	-0.06
	-	-	(0.12)	(0.12)
Aggregate u (25-54yrs)	-2.20^{***}	-1.99^{***}	-1.11^{**}	-1.46^{***}
	(0.30)	(0.31)	(0.48)	(0.49)
Aggregate LFPR (25-54yrs)	1.19	1.47^{*}	0.69	0.80
	(0.82)	(0.87)	(0.87)	(0.87)
Controls	Yes	Yes	Yes	Yes
State Fixed Effects	No	No	No	Yes
R^2	0.03	0.03	0.03	0.03
Observations	55,438	51,397	43,894	43,894

Note: This table presents regression results from individuals aged 25 to 54 years in the labor force on their expected probability of finding a job in the next 3 months using data from the SCE from 2013m6 to 2022m4. Regression controls include potential experience, potential experience squared, education dummies, dummy for whether the individual is a male, a dummy for whether the individual is Caucasian, a dummy for whether the individual is married or cohabits, and a dummy for whether the individual is currently employed. Potential experience is calculated as age less 12 years of schooling if the individual has less than a college degree, and age less 16 years of schooling if the individual has a college degree. "Aggregate u_t" represents the national unemployment rate while variables affixed with "Local" denote state-level employment statistics. Initial conditions are determined at age 18 if the person has less than a college degree, or the at age 22 if the person has a college degree. All data on unemployment rates are taken from the BLS. Column 1 represents the most parsimonious regression with only aggregate and local labor market conditions as well as the local labor force participation rate at the time of an individual's entry into the labor market. Column 2 adds initial local labor market information for the relevant age group (18-24). Columns 3 repeats this specification but includes information on local and initial unemployment rates. Finally, Column 4 repeats specification 3 but with state fixed effects. Standard errors are presented in parentheses. Statistical significance is denoted by the following: $***$ p<0.01, $**$ p<0.05, $*$ p<0.1

the outcomes of those who participated, but does not necessarily convey the aggregated private information of individuals which the participation rate captures. The reason becomes evident when we replace local and initial labor force participation rates with non-employment rates as shown in Table A1 in Appendix A. Because non-participation is an endogenous choice of the worker, this information contains a closely related signal to labor force participation. In fact, we arrive at a similar result: the initial local non-employment rate of those aged 18-24 at the time of entry matters for current beliefs, but not that of the state-wide initial local non-employment rate, nor that of the initial local unemployment rate. Non-employment rates, unlike unemployment rates, capture information on outcomes as well as the aggregation of private information.

Taken together, our findings underscore how 1) average beliefs on job-finding are optimistic and above realized job-finding rates and 2) how local initial conditions of one's relevant group matter for current individual expectations of labor market opportunities. We now turn to outlining how model rationalizes these facts.

3 Model

In this section, we adapt a model of social learning as outlined in Chamley (2004) and Schaal and Taschereau-Dumouchel (2021) to fit a labor market context.

3.1 Environment

Time is discrete. The economy is populated with a unit mass of cohorts. The age of a cohort is denoted by τ and each cohort lives a total of T periods. Thus, at any point in time, there exist $1/T$ measure of individuals aged τ. Workers are risk neutral and discount the future with factor β. Workers can either be employed or non-employed. Non-employed workers consume home production b. Employed workers earn a wage w. Non-employed individuals in every period can choose whether to search and be unemployed, or to stay out-of-the-labor-force. Participating in the labor market is costly. A non-employed worker who chooses to participate in the labor force and search for a job incurs flow cost c.

All agents know aggregate productivity, a_t at the start of date t. Aggregate productivity evolves according to an AR(1) process:

$$a_t = \rho a_{t-1} + \varepsilon_t$$

where the innovation ε_t is drawn from a normal distribution with mean μ_ε and variance σ_ϵ^2. In addition to aggregate productivity, all cohorts at the time of entry draw a group fundamental, z. The group fundamental represents the productivity or skill of the cohort and is drawn at the time a cohort enters the economy. Specifically, for each cohort, the group fundamental, z, is an iid draw from the distribution $\Pi(z)$, and remains constant throughout the cohort's lifetime. Unlike a_t, individuals do not know z.

Transitions into employment are affected by both aggregate productivity and a group fundamental. In particular, the exogenous job-finding rate is a function of both z, and a_t. Job-finding rates, $f(a_t, z)$ are increasing in a_t and z, $f_a(a_t, z) > 0$ and $f_z(a_t, z) > 0$. Individuals are also subject to an exogenous separation rate δ which is invariant to both a_t and z. Because individuals do not know z, they have uncertainty over the job-finding rates they would face, and thus, the value from participating in the labor market.

Information At the time of a cohort's entry, individuals receive an exogenous private signal about z once and for all, where for each individual i the private signal is given by:

$$s_i = z + \epsilon_i$$

where $\epsilon_i \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$. At the end of every period, individuals also observe a noisy endogenous public signal of the labor force participation rate of their cohort. Denote $\ell_t(\tau)$ as the true labor force participation rate at date t of a cohort aged τ, and let $\widehat{\ell}_t(\tau)$ be the associated noisy public signal of the participation rate, where

$$\widehat{\ell}_t(\tau) = \ell_t(\tau) + \xi_t(\tau) \tag{2}$$

$\xi_t(\tau)$ is the noise in signal $\widehat{\ell}_t(\tau)$ where $\xi_t(\tau) \sim \mathcal{N}\left(0, \sigma_\xi^2\right)$ is treated as measurement error in the reporting of the participation rate of individuals aged τ. Denote $\mathcal{I}_{t-1}(\tau)$ as the public information set relevant to agents of age τ at the start of period t. The public information set consists of the history of observed public signals observed up to the start of date t, i.e., $\mathcal{I}_{t-1}(\tau) = \left\{\widehat{\ell}_{t-1}(\tau-1), \ldots, \widehat{\ell}_1(1)\right\}$.

At the start of period t, and based on public information only, i.e., $\mathcal{I}_{t-1}(\tau)$, the distribution of public beliefs – that is, the beliefs an outsider would form if he only has access to public information – is denoted by $H_{t-1}(z, \tau)$ with $h_{t-1}(z, \tau)$ as the associated prior density. As in Chamley (2004) and Schaal and Taschereau-Dumouchel (2021), since individuals only learn from public signals that are evolving over time, and the private signal, s_i, is not time-varying, we only need to track the evolution of public beliefs, $h_{t-1}(z, \tau)$. Private beliefs at any date t can be recovered from public beliefs as we will show later below.

Based on the public information set, $\mathcal{I}_{t-1}(\tau)$, and their private signal s_i, individuals form their own private beliefs $h_{it}(z, \tau)$, and choose each period whether or not to participate in the labor market. As aforementioned, individuals incur cost c from participating and searching the labor market for a job. If they fail to find a job at the end of date t, they remain non-employed. We will assume that prior to observing their own private signal, s_i, all agents have the prior $h_0(z, \tau = 1) = \pi(z)$, that is, all agents start with the unconditional distribution that z is drawn from as their prior.

Timing The timing of the model is as follows: at the start of the period, aggregate productivity, a_t, is observed and separation shocks occur. We assume that those are separated must wait one period before they can search the labor market. Individuals, given their private beliefs on z, choose whether or not to participate in the labor force and search for a job. If they choose to search, they incur the cost c and proceed to the search and matching stage. Once search and matching is over, individuals observe noisy public signals on participation. Finally, production occurs and beliefs are updated. In what follows we describe the value functions if z was known and the choice problem of an individual in our economy.

3.2 Values

Consider a cohort with group fundamental z. Suppose z is known. When z is known, the problem of the non-employed individual is straightforward. In each period, she chooses to enter the labor force if the net benefit of doing so is greater than the value of staying out of the labor force. That is, she solves the following problem each period:

$$V^N(a_t, z, \tau) = \max\left\{V^U(a_t, z, \tau) - c, V^O(a_t, z, \tau)\right\}$$

where $V^U(a_t, z, \tau)$ is the value of unemployment for an individual of age τ, with group fundamental z and aggregate productivity a_t. $V^O(a_t, z, \tau)$ is the value of staying out of the labor force for that same individual. The true value of staying out of the labor force, $V^O\left(a_t, z, \tau\right)$, is given by:

$$V^O\left(a_t, z, \tau\right) = b + \beta \mathbb{E}_{a_{t+1}|a_t} V^N\left(a_{t+1}, z, \tau + 1\right)$$

In words, the individual who stays out of the labor force receives home production, b, in the current period and has continuation value from the expected value of non-employment next period. Similarly, the value of unemployment is given as:

$$V^U(a_t, z, \tau) = f(a_t, z)\widetilde{V}^W(a_t, z, \tau) + \left[1 - f(a_t, z)\right] V^O(a_t, z, \tau)$$

where

$$\widetilde{V}^W(a_t, z, \tau) = w + \beta \mathbb{E}_{a_{t+1}|a_t} V^W(a_{t+1}, z, \tau + 1)$$

With probability $f(a_t, z)$, the unemployed individual finds a job and receives a wage w and continuation value of expected employment next period. With probability $1 - f(a_t, z)$, the individual fails to find a job and in that case has the same value as an individual who stays out of the labor force.

Finally the value of employment is given by:

$$V^W(a_t, z, \tau) = \delta V^O(a_t, z, \tau) + \left[1 - \delta\right] \widetilde{V}^W(a_t, z, \tau)$$

With probability δ, the employed individual enters into non-employment and enjoys value $V^O(a_t, z, \tau)$. With probability $1 - \delta$, the worker remains employed.

Thus far, we have outlined the values if z was known. However, the individual in our model does not know her group fundamental z and makes her participation choice based on her beliefs. The next section outlines the individual's choice problem.

3.3 Choices and learning under uncertainty

While the above section characterized the true values of labor market outcomes for an individual in group z, individuals do not know their group's fundamental z initially though they may learn it over time. Instead, when making their decision of whether to become unemployed and thus whether to participate in the labor force, individuals instead solve the following problem given public information $\mathcal{I}_{t-1}(\tau)$ and private signal s_i:

$$\mathbb{E}V^N(a_t, z, \tau) = \max\left\{\mathbb{E}V^O\left(a_t, z, \tau\right), \mathbb{E}V^U\left(a_t, z, \tau\right) - c\right\}$$

Let $h_{it}(z, \tau)$ be the private belief that the state is z given her private signal of s_i. Given prior public belief, $h_{t-1}(z, \tau)$, and her signal s_i, the individual's private belief at time t is given as:

$$h_{it}(z, \tau) = \frac{h_{t-1}\left(z, \tau\right)\varphi\left(s_i|z\right)}{\int h_{t-1}\left(\widetilde{z}, \tau\right)\varphi\left(s_i \mid \widetilde{z}\right)d\widetilde{z}} \tag{3}$$

where $\varphi(s_i|z)$ is the density of observing signal s_i given state z, and is equivalent to $\varphi(s_i|z) = \phi\left([s_i - z]/\sigma_\epsilon\right)$. Since ϕ is a standard normal, the distribution of private signals satisfy the monotone likelihood ratio property (MLRP), that is for any $z_2 > z_1$ and $s_2 > s_1$, we have:

$$\frac{\varphi(s_2|z_2)}{\varphi(s_2|z_1)} \geq \frac{\varphi(s_1|z_2)}{\varphi(s_1|z_1)}$$

In words, this means that a higher signal of s is more likely to occur when z is higher.

Then given s_i, an individual participates if the expected net value of unemployment is at least as high as the expected value of staying out of the labor force:

$$\int\left[V^U\left(a_t, z, \tau\right) - c - V^O\left(a_t, z, \tau\right)\right]h_{it}\left(z, \tau\right)dz \geq 0 \tag{4}$$

Substituting (3) into (4), and focusing on the individual who is just indifferent between participating and staying out of the labor force, we can solve for $s_t^*(\tau)$:

$$\int\left[V^U\left(a_t, z, \tau\right) - c - V^O\left(a_t, z, \tau\right)\right]\frac{h_{t-1}\left(z, \tau\right)\varphi\left(s_t^*(\tau)|z\right)}{\int h_{t-1}\left(\widetilde{z}, \tau\right)\varphi\left(s_t^*(\tau) \mid \widetilde{z}\right)d\widetilde{z}}dz = 0 \tag{5}$$

where $s_t^*(\tau)$ is the signal the individual needs to observe to be just indifferent. Because $\varphi(s_i|z)$ satisfies MLRP, this implies that all non-employed individuals aged τ with $s_i \geq s_t^*(\tau)$ enter the labor force as unemployed individuals. Accordingly, the share of non-employed individuals aged τ who participate in the labor market, $p_t(\tau)$, is given by:

$$p_t(\tau) = 1 - \Phi\left(\frac{s_t^*(\tau) - z}{\sigma_\epsilon}\right)$$

And the non-employment rate of those aged τ at the end of the period is given by:

$$n_t(\tau) = [1 - f(a_t, z)p_t(\tau)]\, n_{t-1}(\tau - 1) + \delta\, [1 - n_{t-1}(\tau - 1)]$$

where $n_{t-1}(\tau-1)$ is the non-employment rate of individuals aged $\tau - 1$ at the end of $t-1$. By definition, the total non-employed and employed of age τ sum to 1, i.e., $n_t(\tau) + e_t(\tau) = 1$. Note that out of the non-employed of aged $\tau - 1$ last period, a share $f(a_t, z)p_t(\tau)$ participated and found a job. Thus, $1 - f(a_t, z)p_t(\tau)$ of this group remain non-employed. The second terms on the RHS of the above equation represents inflows into non-employment from those aged $\tau - 1$ who were employed at the end of last period.

Finally, we can characterize the labor force participation rate of those aged τ at the end of the period t as one less those participated in the labor force:

$$\ell_t(\tau) = 1 - n_{t-1}(\tau - 1)\, [1 - p_t(\tau)] = 1 - m_t(\tau)n_{t-1}(\tau - 1)$$

where $m_t(\tau) = 1 - p_t(\tau)$ and is the share of non-employed at the end of $t - 1$ who didn't participate in t. Implicitly, this counts those who are newly separated from their jobs at the start of t, $\delta(a_t, z)[1 - n_{t-1}(\tau - 1)]$ as being part of the labor force in period t.

Signals and updating While $p_t(\tau), n_t(\tau)$ and $\ell_t(\tau)$ represent the actual outcomes at date t for individuals aged τ, individuals do not observe these actual outcomes. Instead, they observe the noisy signal $\widehat{\ell}_t(\tau)$ as described in Equation (2).

Since agents know the structure of the model and know the public belief $h_{t-1}(z, \tau)$, all agents of age τ can compute the threshold signal, $s_t^*(\tau)$, above which individuals choose to participate. Thus, individuals can compute a counterfactual $p_t(\tau; z), n_t(\tau; z)$ and $\ell_t(\tau; z)$ for any z possible. This mean that for any z, individuals can compute the implied noise level:

$$\xi_t(\tau; z) = \widehat{\ell}_t(\tau) - \ell_t(\tau; z)$$

where $\ell_t(\tau; z) = 1 - n_{t-1}(\tau - 1; z)[1 - p_t(\tau; z)]$. Given this implied noise and applying Bayes' rule, the public posterior belief at the end of t and start of $t + 1$ can be characterized as:

$$h_t(z, \tau + 1) = \frac{h_{t-1}(z, \tau)\phi\left(\frac{\xi_t(\tau; z)}{\sigma_\xi}\right)}{\int h_{t-1}(\widetilde{z}, \tau)\phi\left(\frac{\xi_t(\tau; \widetilde{z})}{\sigma_\xi}\right) d\widetilde{z}} \tag{6}$$

Since the evolution of public beliefs depend on $\widehat{\ell}_t(\tau)$, it is useful to note that both the signal and $\ell_t(\tau)$ are linear functions of $p_t(\tau)$. The degree of participation, as captured by $p_t(\tau)$, is in turn affected by the level of optimistic or pessimistic beliefs, which itself manifests through the level of $s_t^*(\tau)$. For example, when agents are pessimistic and $s_t^*(\tau)$ is very large, $p_t(\tau)$ tends towards zero, implying that the observed noisy signal $\widehat{\ell}_t(\tau)$ – as a linear function of $p_t(\tau)$ – does not vary very much in z. The degree of participation then matters crucially for the informativeness of the signal. We outline this in the following simple example.

4 A simple example: signal quality and participation

To see how participation affects the information content of a signal, we construct the following one-shot example. For ease of exposition, we will silence the dependence on τ.

Suppose that individuals only live one period. At date 0, this implies $n_0 = 1$. In this static model, this also implies that realized labor force participation at the end of date 1 is given by:

$$\ell_1 = (1 - p_1)n_0 = p_1$$

Assume that the group fundamental z is drawn from a normal distribution with mean μ_z and variance σ_z^2 at time 0, i.e., $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Let the mean and variance of the public belief at date 0 then be given by $\mu_0 = \mu_z$ and $\sigma_0^2 = \sigma_z^2$. As per Equation 4, individuals only participate if they expect the net value of search to be greater than or equals to the value of staying out of the labor force. Consequently, all individuals with $s_i > s^*$ participate, and the participation rate is given by:

$$\ell_1 = p_1 = 1 - \Phi\left(\frac{s^* - z}{\sigma_\epsilon}\right)$$

For exposition purposes, it is useful to focus on the complement of the participation rate, i.e., the measure of non-participation, which is given by:

$$m_1 = \Phi\left(\frac{s^* - z}{\sigma_\epsilon}\right)$$

Suppose we consider a first-order linear approximation of m_1 and further denote $\widetilde{m}_1$ as a noisy signal of m_1:

$$\widetilde{m_1} = \Phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) + \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)(z - \mu_z) + \xi$$

where as before, ξ is the noise in the public signal and $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$. Since all agents can compute the threshold s^*, and since agents know the distributions from which z and ξ are drawn from, i.e., they know μ_z and σ_ϵ^2, the following signal $\widehat{m}_1$ is informationally equivalent to $\widetilde{m}_1$:

$$\widehat{m}_1 = \widetilde{m}_1 - \Phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) + \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)\mu_z = \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)z + \xi$$

Having characterized the form of the noisy public signal, $\widehat{m}_1$, we can now describe how the informational content varies with optimism and pessimism through s^*:

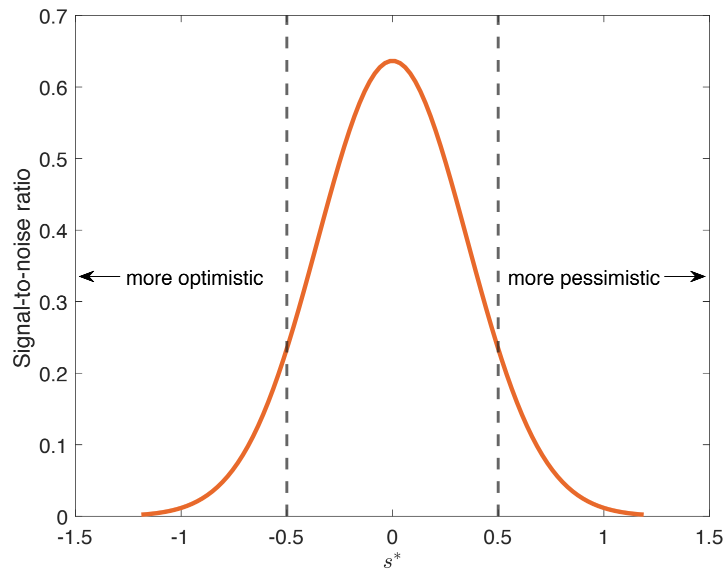


Figure 1: Signal-to-noise ratio non-monotonically changing in s^*

Proposition 1. *The informational content of the signal varies non-monotonically with s^* and thus, with the degree of optimism.*

Proof. Since z and ξ are drawn from normal distributions, we can characterize the informational content of the signal by focusing on the signal-to-noise ratio. Applying Bayes Rule, the signal-to-noise ratio is given by:

$$\text{signal-to-noise ratio} = \left[\phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)\right]^2 \frac{\sigma_z^2}{\sigma_\xi^2} \tag{7}$$

Since the term in square brackets is just the probability density of observing s^*, the signal-to-noise ratio varies non-monotonically with s^*. $\square$

Figure 1 plots the signal-to-noise ratio as defined in (7) and shows how this ratio is hump-shaped in s^*. Intuitively, for very low values of s^* which corresponds to a situation where agents are very optimistic, almost all individuals participate. This causes the signal of participation, or in this case, non-participation, $\widehat{m}_1$, to not display much variation in z. Accordingly, the signal-to-noise ratio is low, and there is little informational content to be gained from the signal. Conversely, for very high values of s^*, a case where individuals are very pessimistic, almost no individual participates. This again implies that the signal $\widehat{m}_1$ varies little in z, and thus the informational content for very high values of s^* is also low.

Importantly, when the informational content of a signal is low, individuals put little weight on the signal and more weight on their prior. In fact, the posterior mean of the public belief can be characterized as:

$$\mu_1 = \frac{\left[\phi\left(\frac{s^*-\mu_z}{\sigma_\epsilon}\right)\sigma_z\right]^2}{\left[\phi\left(\frac{s^*-\mu_z}{\sigma_\epsilon}\right)\sigma_z\right]^2+\sigma_\xi^2}\widehat{m}_1 + \left(1-\frac{\left[\phi\left(\frac{s^*-\mu_z}{\sigma_\epsilon}\right)\sigma_z\right]^2}{\left[\phi\left(\frac{s^*-\mu_z}{\sigma_\epsilon}\right)\sigma_z\right]^2+\sigma_\xi^2}\right)\mu_z$$

When the signal-to-noise ratio is small, i.e., when $\phi\left(\frac{s^*-\mu_z}{\sigma_\epsilon}\right)$ approaches zero, the individual puts more weight on her prior, $\mu_0 = \mu_z$, and little weight on the signal observed, $\widehat{m}_1$. Consequently, learning slows down and the individual's belief is little changed from her initial prior. While the simple example illustrates how participation can affect the informational content of signals, we now turn to calibrating our model to quantify how much initial conditions can cause beliefs to persistently deviate from actual realizations.

5 Calibration

A period in our model is a quarter. We calibrate our model economy to a deterministic steady state, covering the period 2013m6-2022m4. This interval of time represents the coverage of the SCE data used in our analysis. We set the discount factor, β, to be 0.99 and the wage w to equal 1. We set the value of home production to be 0.4 which is similar to the replacement ratio used in Shimer (2005). Finally, in the CPS, we observe a monthly separation rate of 0.026, which implies a quarterly separation rate of 0.076. We assume that individuals' working lives begin at age 20 and end at age 65. Consequently, we set the total amount of periods an individual lives to $T = 45 \times 4 = 180$ quarters. The rest of the parameters are internally calibrated.

Internally calibrated parameters All parameters are jointly calibrated. We assume that the group fundamental z is drawn from a Beta distribution with support $z \in [0, 1]$ and governed by parameters (A_z, B_z). In the CPS, we compute the probability of finding a job within 3 months as $1-(1-UE)^3$ where UE refers to the monthly unemployment-to-employment transition rate. Thus, to pin down the parameters (A_z, B_z), we target a mean quarterly job-finding rate of 0.488 with standard deviation 0.053.[5]. The average labor force participation rate for prime-age workers reported in the CPS for the period 2013m6-2022m6 is about 0.82. We use this moment to target the cost of participating in the labor market, c.

Finally, we use information on the dispersion in perceived job-finding rates among prime-aged individuals and among those aged 18-24 to calibrate the standard deviation in the noise terms, σ_ϵ and σ_ξ. Because we calibrate to a deterministic steady state where there are no fluctuations in aggregate productivity a, we use the information from *predicted* perceived job-finding rates. That is, we run a regression on observables, control for the aggregate state and compute predicted

[5] We use monthly data on UE rates and compute the probability of find a job for each month as $1-(1-UE)^3$. We take the average and dispersion of this series for the period 2013m6 to 2022m4 to be the mean and standard deviation of our quarterly job-finding rate

Table 4: Internally Calibrated parameters

Parameter	Description	Value	Target	Model	Data
A_z	Beta dist parameter	6.07	Mean job-finding rate, f	0.499	0.489
B_z	Beta dist parameter	14.37	Std dev. job-finding rate, f	0.051	0.053
c	Participation cost	3.27	Prime-age participation	0.818	0.820
σ_ϵ	Dispersion in ϵ	0.36	Std dev. perceived f, 18-24	0.056	0.060
σ_ξ	Dispersion in ξ	0.14	Std dev. perceived f, 25-54	0.043	0.046

Notes: Dispersion in perceived job-finding rates for the relevant age group is computed as the standard deviation in predicted perceived job-finding rates after controlling for aggregate fluctuations

values purged of the fluctuations in aggregate activity.

To pin down σ_ϵ, we target the dispersion in beliefs in the first period of agents' working life in the model to be equal to the predicted dispersion in job-finding beliefs among those aged 18-24 in the data.[6] We do this as the only information individuals have at the start of the first period of their working lives in our model is their private signal. Subsequently, in all other periods, the information set at the start of period t, i.e., $\mathcal{I}_{t-1}(\tau)$, includes the history of public signals observed. As such, the dispersion in job-finding beliefs in the first period are mainly influenced by the dispersion in s_i. We thus target an initial dispersion in beliefs of 0.06, the average dispersion in beliefs observed for those aged 18-24.

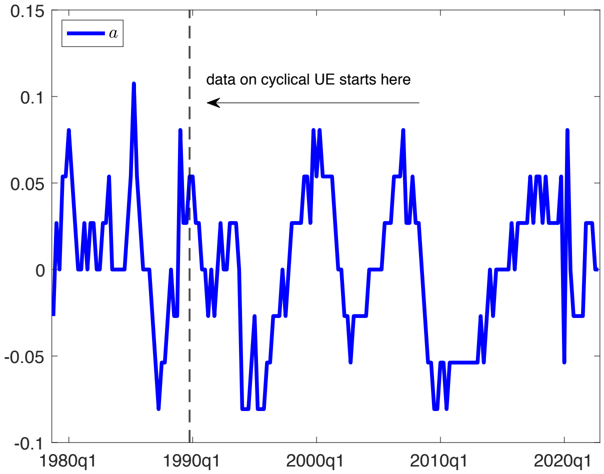
To pin down σ_ξ, we use information on perceived job-finding rates for prime-age individuals. Specifically, we pin down σ_ξ by targeting the dispersion in the predicted perceived job-finding rate of those aged 25-54. As aforementioned, an individual's belief in the first period is mainly influenced by s_i, while in all latter periods, it is affected by not only the private signal but increasingly by, the history of public information. Thus, the degree of noise in the public signal affects how beliefs evolve and we target the dispersion in perceived job-finding rates purged of the aggregate fluctuations of those aged 25-54 help to inform σ_ξ. Table 4 shows our results. Overall, our model moments match the targeted data moments fairly well. In terms of non-targeted moments, our calibrated model predicts a non-employment rate of 0.29 for prime-age individuals, close to the empirical non-employment rate of 0.22 for the period 2013m6-2022m4.

6 Quantitative Results

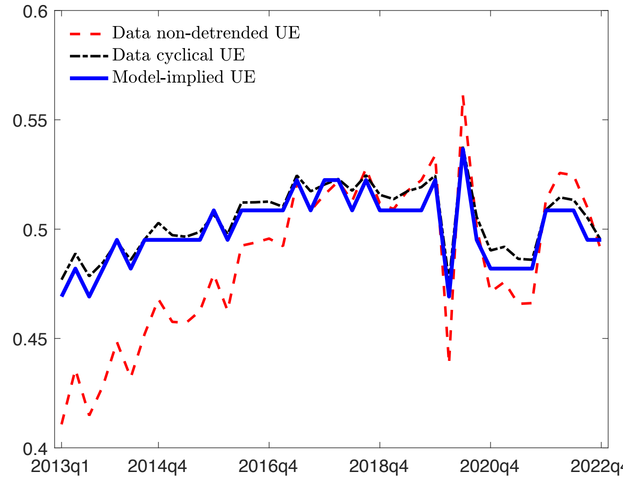
Our main thought experiment is to examine how beliefs in our model economy would evolve if we fed it the same series of aggregate shocks as observed in the data. To this end, we apply a Hodrick-Prescott (HP) filter to seasonally adjusted quarterly log unemployment-to-employment (UE) transition rates of prime-aged workers for the period 1989q1 to 2023q2.[7] In filtering the

[6] Because of small sample sizes and because we also do not observe the actual year that the individual actually entered the labor market, we use all the information for those aged 18-24.

[7] The panel nature of the CPS allows us to construct UE transitions from 1989 onwards.



(a) Cyclical component of UE rates



(b) Data vs model-implied job-finding rates

Figure 2: Model-implied a_t and job-finding rate

data, we follow Shimer (2005) and apply a smoothing parameter of 10^5. We derive the cyclical component as the difference between log UE and its trend, and treat this cyclical component as our series of aggregate shocks for the period 1989q1 to 2022q2. Because the oldest cohort in the SCE sample is aged 54 in 2013, this would imply that they were aged 20 in the year 1979. Since this is out of the sample of aggregate shocks we can derive from the HP-filtered empirical job-finding rate, we simulate the shocks for the earlier periods that are out-of-sample by assuming that the shock process for a_t is AR(1) and has the same standard deviation of 0.02 as our cyclical UE rate.[8] We further run an AR(1) regression on our derived cyclical UE series and set the persistence to the coefficient on its first lag, i.e., we set the persistence of a_t to 0.83.

Figure 2a shows the aggregate shock series we use while Figure 2b shows how our model-implied job-finding rate given mean z performs against the raw non-detrended UE observed in data as well as the cyclical UE rate derived from filtering the data.[9] The divergence between the red and black lines in Figure 2b indicate that our filtered series of shocks attributes part of the low job-finding rates observed in the slow recovery from the Great Recession to trend. As a result, our model-implied job-finding observes less of a decline than the raw data for the SCE survey period of 2013m6 to 2022m4.

For each cohort c, we draw a z from the distribution Beta(A_z, B_z). The first cohort corresponds to the group that is age 54 in 2013, and the last cohort we simulate for is age 25 in 2022. For each period t in 2013q2 to 2022q2, we compute the realized average job-finding as the mean actual job-finding rate faced by all cohorts that are prime-age in that period t. In our model,

[8] Denote UE_c as the cyclical component, then the implied cyclical UE rate is equal to $0.49\exp(UE_c)$.

[9] Under a Beta distribution, the mean z is given by $A_z/(A_z + B_z)$. To construct our model-implied job-finding rate, we assume that the mean is given by $f(a_t, \bar{z})$ where $\bar{z}$ refers to mean z.

prime-age agents have labor market experience between 21 to 140 quarters during a period t. Similarly, we compute the average perceived job-finding rate as the mean perceived belief across all cohorts who are between 21 to 140 quarters of experience (model age) in a period t.

Figure 3 shows how the average perceived job-finding rate performs relative to the realized job-finding rate over this period of time. Similar to the data, we find that average beliefs on job-finding are *optimistic*, that is the average perceived job-finding rate among prime-aged individuals is above the average realized job-finding rate. Our calibrated model predicts that perceived job-finding rates are on average 2 percentage points higher than realized job-finding rates, accounting for about one-third of the empirical gap between perceived and actual job-finding rates.[10]

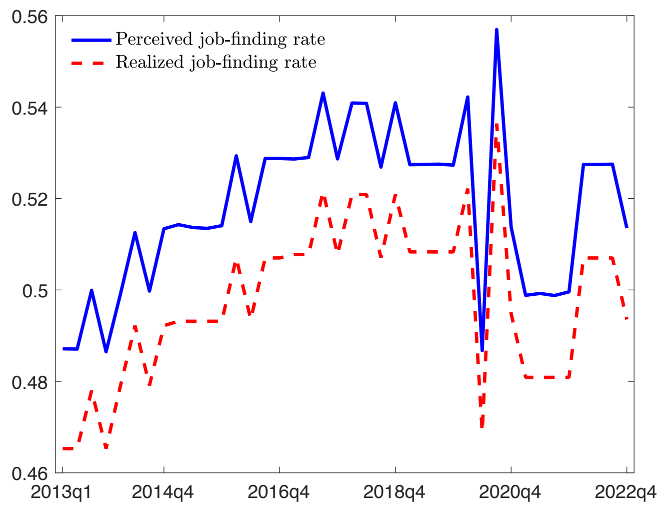


Figure 3: Average beliefs on job-finding are optimistic

It is not the case that all cohorts in our calibrated model have optimistic beliefs. Rather, initial conditions are important in the context of our model for determining how beliefs about the job-finding rate evolve. To see this, consider two cohorts, a cohort that entered the labor market during the onset of the Great Recession in 2009 and a cohort that entered during a boom in 2000 where the aggregate shock is about 3 standard deviations above its mean. Figure 4 shows how their beliefs evolve across time.

Focusing on top panel of Figure 4, the cohort that entered during the onset of the Great Recession starts off highly pessimistic, with the threshold signal required to participate in the first period, s_1^*, being in the 99th percentile. As agents perceive labor markets to be very weak, very few individuals participate. In fact, the share of non-employed who participate in that first quarter, p_1, is about 1 percent. As highlighted in Section 4, a lack of participation in turn

[10]The gap when comparing beliefs of all participants and the actual job-finding rate in the CPS is about 0.57-0.51=0.06, while the gap when using information only on the SCE unemployed is about 0.52-0.46= 0.06

implies that very little of the private information of individuals is aggregated into the public signal. Consequently, the public signal bears little informational content on the true value of z, leading individuals in this cohort to put more weight on their prior and to learn the true value of z very slowly. Because the Great Recession was both deep and protracted, participation remains low for a sustained period of time, implying that public signals during this period are largely uninformative. Thus, the cohort that entered during the Great Recession learns their fundamental value of z very slowly and their priors are little changed. This gives rise to persistently pessimistic beliefs regarding their job-finding rate, as depicted in the top panel of Figure 4. Note that agents in our model always know the true value of the aggregate productivity shock, a_t. As such, pessimism here only reflects what they believe about their fundamental z.

While the gap between the perceived and realized job-finding rates does eventually narrow for this cohort, it does so slowly. The left panel of Figure 5 shows the distribution of public beliefs in period 0, 1 and 64 for the cohort that entered during the Great Recession. Because participation was extremely low in the first period, the distribution of beliefs in the first period after entry h_1 - as depicted by the blue dashed line - overlaps with h_0, their initial distribution of beliefs in period 0, and as depicted by the black solid line. Over time, as the economy improves, agents participate more, incorporating a larger amount of private information into the public signal. This causes beliefs to converge closer to the true value. The red dashed-dot line in the left panel of Figure 5 shows that 64 quarters (16 years) later, the mean belief of this cohort is closer to its true fundamental z value although there still remains considerable uncertainty as can be seen from the dispersion in beliefs in h_{64}. This is in spite of the fact that the share of non-employed who participate 64 quarters later, p_{64}, is about 86 percent. Thus, even though agents are refining their beliefs as participation rises with the recovery in the economy, the delay in learning causes the level of uncertainty regarding their true z to remain elevated.

Conversely, the bottom panel of 4 shows a cohort that entered during a boom where the aggregate shock was 3 standard deviations above the mean and how this cohort's beliefs remain persistently optimistic. In this case, because of the large positive aggregate shock, agents in this cohort perceive labor markets to be very strong, and the threshold signal required to participate in the first period, s_1^* is below the 1st percentile. As highlighted in Section 4, this represents the other extreme case where there is little informational content in the public signal. When almost all agents choose to participate – as is in this case where the share of non-employed who participate in the first period, p_1 is close to 100 percent – the signal component varies very little across z. Consequently, learning is also slow for agents in this cohort as they again put more weight on their prior. The middle panel of Figure 5 shows that for this cohort that started in a boom, the distribution of beliefs in period 1 and period 0, h_1 and h_0, completely overlap, as depicted by the dashed blue and solid black lines, respectively. Learning does take place over time as 64 quarters later, the mean belief is closer to that cohort's true z value, and the

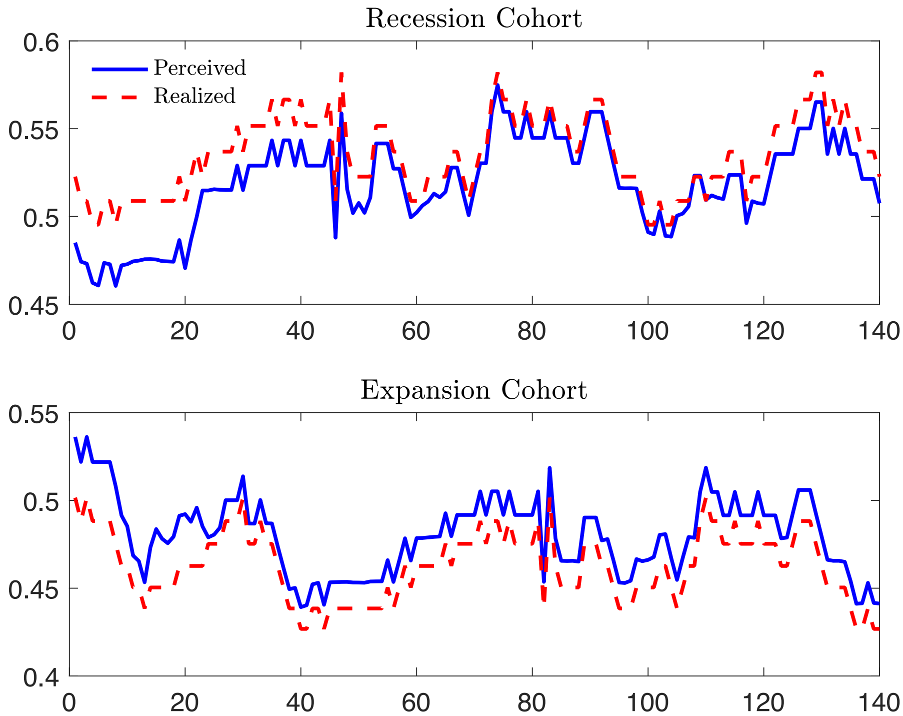


Figure 4: Persistence in beliefs

dispersion in beliefs is smaller as exemplified by the red dot-dash line. Nonetheless, the delay in learning stemming from the lack of informative signals early in this cohort's working life causes average beliefs to be persistently optimistic.

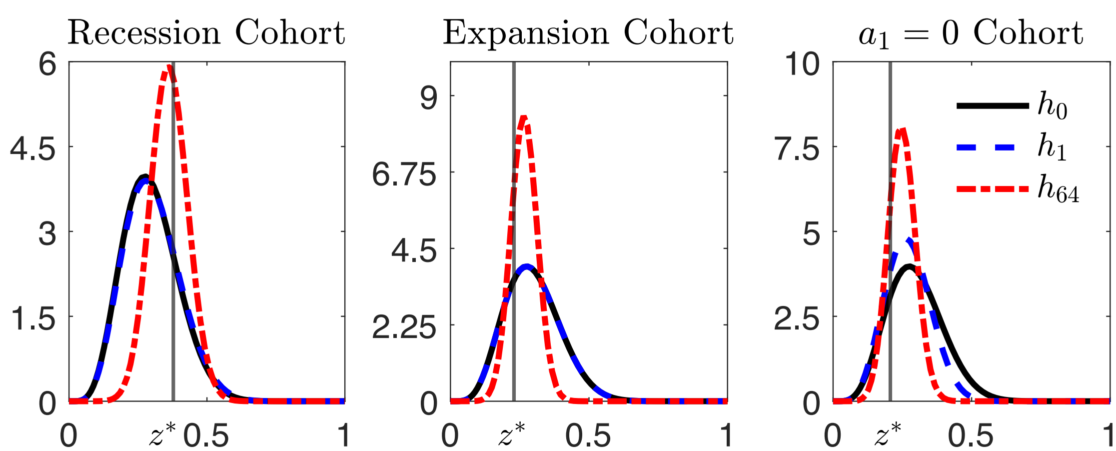


Figure 5: Evolution of beliefs across different initial conditions

The previous two examples highlighted cases where both very adverse and very positive initial conditions can cause the public signal to be uninformative, leading to very little updating of priors and an entrenchment of pessimistic and optimistic beliefs. This does not imply that public signals are always uninformative. In our model, entering the labor market during a less expansionary period or during a milder recession generates moderate initial levels of participation, giving rise to more informative public signals that incorporate the private information of individuals. In the case when the cohort starts out in an economy that is neither expanding nor contracting, agents participate a moderate amount and the public signal while noisy holds informational content that agents use to update their beliefs. The rightmost panel of Figure 5 shows that for this cohort, the distribution of public beliefs at the end of period 1, h_1, is not only shifted to the right of the distribution of beliefs at date 0, h_0, but is also more precise as shown by the less dispersed distribution depicted by the blue dashed line relative to the solid black line.

To understand how much initial conditions can affect current beliefs, we simulate 1000 individuals in each cohort and run the same regression as we do in data. That is, we regress perceived job-finding rates of each individual against the initial labor force participation rate of a cohort in the first four quarters of their working life and the aggregate state. Since both the modeler and agents in the model know exactly the aggregate state a_t, including variables such as the prime-age unemployment rate and the prime-age labor force participation rate do not provide more information as they are functions of a_t. Thus, to avoid multi-collinearity, we leave out these two variables.

Table 5 shows our results. Our calibrated model predicts that a one-percentage point increase in the cohort's initial labor force participation rate raises current beliefs about job-finding by about 7 percentage points. This value is smaller than observed in the data, as documented in Table 3. Since our model highlights that the degree of participation has non-monotonic effects on the informativeness of the public signal, Column 2 shows that even after we account for non-linearity by including the square of a cohort's initial labor force participation rate in the regression, a one percentage point increase in labor force participation rate has a positive effect on one's current perceived job-finding rate, although this effect is smaller the higher the initial labor force participation rate. Overall, our results are qualitatively consistent with our earlier empirical result that initial conditions matter for current beliefs even after accounting for the aggregate state.

In summary, our calibrated model predicts that average beliefs about job-finding are overall optimistic. Initial conditions play a large role in determining how informative public signals are and the speed with which agents learn their true fundamental values. Very adverse or very positive initial conditions can cause agents to be overly pessimistic and optimistic, leading to extreme participation levels that are not informative of the true value. Consequently, agents learn slowly, giving rise to a persistently optimistic or pessimistic beliefs.

	Expected Job Finding	
	(1)	(2)
Initial LFPR_c	0.066***	0.237***
	(0.000)	(0.001)
Initial LFPR_c^2		−0.176***
		(0.001)
a_t	0.689***	0.705***
	(0.003)	(0.003)
Observations	1,200,000	1,200,000
R^2	0.119	0.140

Notes: This table presents the results from a regression of individual perceived job-finding rates on initial labor force participation of their cohort and the aggregate state a_t. Initial LFPR_c is the average labor force participation rate of a cohort c in the first four quarters of their working life while a_t is the series of aggregate shocks we feed into the model. Standard errors are in parenthesis.

Table 5: Regression on perceived job-finding rates from model-simulated data

7 Extensions

7.1 Ruling out private learning about individual fixed effects

A natural question that arises is whether models focusing on learning about individual fixed effects as opposed to a group fundamental could have sufficed. In what follows, we show analytically why such models fail at generating average beliefs which are optimistic, and instead must predict that average beliefs are equal to their average realization. In all our examples, we assume that an individual's type or fixed effect is an iid draw from some distribution. Because other peoples' fixed effects or types are not informative of the agent's own individual fixed effect, only private signals or private outcomes are relevant for learning. As such, in our examples, we focus on learning from only private signals or private outcomes. We will further assume in all our examples that agents start with the unconditional prior, although our results will also survive if each agent starts with unbiased beliefs that are centered around their true fundamental value.

7.1.1 Learning from private exogenous signals

Consider an economy populated with a measure 1 of individuals. Suppose each agent draws her fixed effect or true type z from normal distribution $N(\mu_z, sigma_z^2)$. Each agent's type is a permanent iid draw from the unconditional distribution. Assume further that each period, an individual i observe a signal of the form $s_{it} = z + \epsilon_{it}$, where ϵ_{it} is drawn from a normal distribution $N(0, \sigma_\epsilon^2)$. We will assume that agents observe these signals regardless of whether they participate in the labor market or not.[11] Then the measure of individuals who draw z as their fixed effect is $\phi(\frac{z-\mu_z}{\sigma_z})$ and conditional on drawing z, the probability density of drawing signal s is given by $\phi(\frac{s-z}{\sigma_\epsilon})$.

Because all subsequent periods are a repeat of the process described in the first period, it is sufficient for us to focus on learning in the first period. As such, we will drop the t subscript. We will also write things in terms of precision, ρ, which is the inverse of the variance: i.e. $\rho = 1/\sigma^2$, as this will aid in exposition. Then given linear Gaussian signals, we can characterize an individual's posterior distribution of beliefs by its mean and precision. For an individual i who drew z and signal s_i, her posterior precision is given by:

$$\rho_i' = \rho_\epsilon + \rho_z = \rho'$$

The posterior precision is independent of the values of s_i and z, and so we can substitute ρ_i' with ρ'. The posterior mean for individual i who drew signal s_i and fixed effect z is given by:

$$\mu_i'(s \mid z) = \frac{\rho_\epsilon}{\rho'} s_i + \left(1 - \frac{\rho_\epsilon}{\rho'}\right)\mu_z$$

Summing over all possible s and z, we can compute the average belief.

Proposition 2. *In a model where agents are trying to learn their individual fixed effects from private exogenous signals, the economy-wide average belief is equal to the average realization.*

Proof. See Appendix B.1 □

Since all individuals draw their type from the unconditional distribution and since by law of large numbers, the noise term cancels out, the average belief in the economy is equal to the average realization, μ_z.

7.1.2 Learning from private endogenous signals

In this example, we consider a model where individuals learn about their fixed effect or type from private outcomes only. Consider again an economy with a measure 1 of individuals. For ease of exposition, we will assume that a type maps into an individual's true job-finding rate and abstract from aggregate shocks. Suppose that for each individual, the individual's true job finding rate p^* is drawn from an unconditional distribution, $G(p)$. We require that

$$\int p^* g(p^*)\, dp^* = \overline{p}$$

[11] If we had instead assumed that individuals observed signals conditional on participating, our results would not change since agents start with the unconditional prior and so if one agent finds it worthwhile to participate, all agents find it worthwhile to participate.

where $\overline{p}$ is the mean of the unconditional distribution $G(p)$. This implies that $g(p^*)$ is the measure of individuals who draw $p = p^*$.

Individuals do not know their job-finding rate p^*, but start out with the unconditional prior $g(p)$. Denote $h_0(p; p^*) = g(p)$ be the prior of an individual who has true job-finding rate p^*. Further, let q be equal to 1 if the individual finds a job and 0 otherwise. Then the posterior belief of an individual with true job-finding rate p^* can be characterized by $h_1(p \mid q = 1; p^*)$ for those who found a job

$$h_1(p \mid q = 1; p^*) = \frac{p h_0(p; p^*)}{p^*} = \frac{p g(p)}{p^*}$$

and by $h_1(p \mid q = 0; p^*)$ for those who did not find a job:

$$h_1(p \mid q = 0; p^*) = \frac{(1-p) h_0(p; p^*)}{1-p^*} = \frac{(1-p) g(p)}{1-p^*}$$

Proposition 3. *In a model where agents are trying to learn their individual true job-finding rate from private endogenous signals, the economy-wide average belief is equal to the average realization.*

Proof. See Appendix B.2 □

Intuitively, since agents draw their individual job-finding rates from the unconditional distribution, and since the law of large numbers means that the randomness induced by search frictions cancels out, this implies the average belief must equal the average realization which is the unconditional mean.

In summary, while individual beliefs can be different from their true values, a model where agents learn about their individual fixed effect or individual job-finding rate from private signals cannot predict a persistent divergence in the average belief from the average realization.

7.2 Sensitivity analysis: degree of noise in public signal

In this section, we conduct sensitivity analysis by varying the noise in the public signal. We consider two cases: 1) a noisier public signal by setting $\sigma_\xi = \sigma_\epsilon = 0.36$, and 2) a more precise public signal by halving the dispersion in ξ, i.e., setting $\sigma_\xi = 0.07$. In all our experiments, we re-draw the fundamental values z for each cohort and the noise terms for each period. When comparing average beliefs on job-finding vs average realizations, we find that both the economy with a larger σ_ξ and the economy observing a smaller σ_ξ generate perceived job-finding rates that are on average about 1 percentage point above their average realized values, accounting for about 16 percent of the empirical gap.

While both model economies generate similar results, they do so for different reasons. Focusing first on the economy where σ_ξ is small, intuitively, when public signals are less noisy,

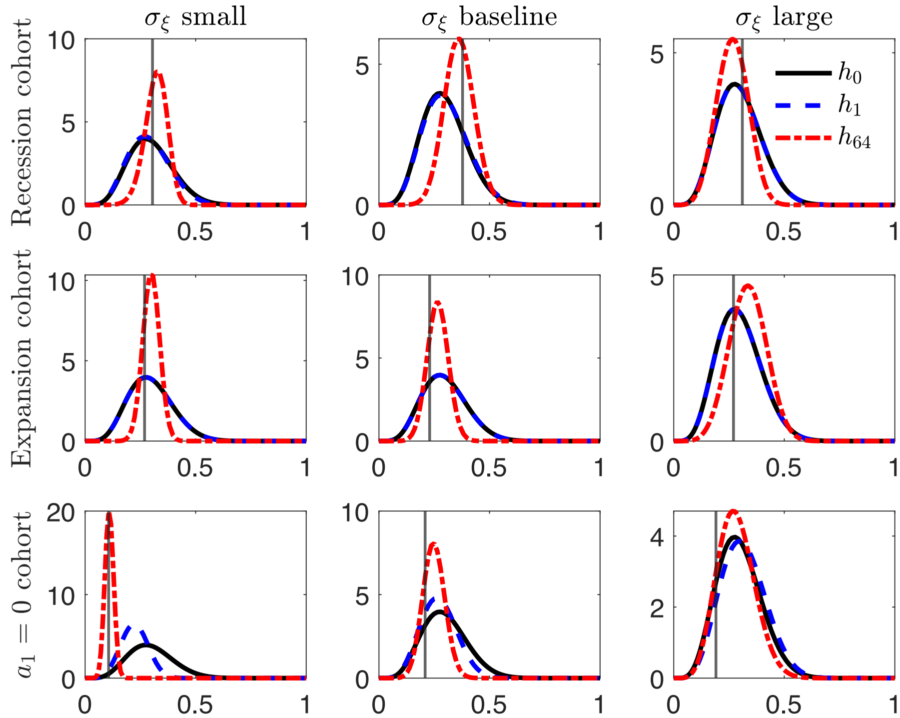


Figure 6: Evolution of beliefs for different σ_ξ

individuals extract more information from these signals, allowing them to learn at a faster rate, and for the beliefs to converge more rapidly to the truth.

Delving deeper, the first column (denoted σ_ξ small) in Figure 6 showcases how the distribution of beliefs evolve for cohorts that enter in a recession, expansion or when $a_1 = 0$. As in our baseline model, learning is fastest when the cohort enters during periods in which the economy is neither undergoing a large recession nor a large positive shock. The reason is that cases in which almost none or all individuals participate deplete the value of the signals received about an individual's group. This can be seen in the bottom panel of Column 1 where beliefs are tightly centered around the true fundamental value for the cohort which entered when $a_1 = 0$. Initial conditions still affect the speed at which individuals learn by affecting the degree of participation and thus the amount of private information aggregated into the public signal. Relative to the results from the baseline model (second column, σ_ξ baseline), individuals in the economy featuring less noisy public signals observe tighter beliefs that are more closely centered around their true fundamental values. Since signals are less noisy and more informative, the gap between perceived and realized job-finding rates are smaller as individuals are more likely to learn their true cohort value. Consequently, average beliefs in the economy with a smaller σ_ξ are only 1.2 percentage points above average realized values.

In contrast, the economy with a larger σ_ξ features very slow learning as signals are noisy and very uninformative. Notably, Column 3 of Figure 6 shows that for this economy, the distribution of beliefs does not change very much from the initial distribution h_0. This is true even for the cohort that starts out in an economy where aggregate productivity is at its mean value, as shown in the bottom panel of Column 3. Moreover, the degree of uncertainty remains elevated in their posterior beliefs. When signals are very uninformative, individuals rely more on their priors. In our model, all cohorts start out with the same prior which is the unconditional distribution from which z is drawn from, $\Pi(z)$, implying that in the economy with a large σ_ξ, current beliefs are little changed from this initial prior. When each cohort draws their fundamental value z from the same distribution which forms their initial prior and when current beliefs are little changed from the initial prior, the gap between average perceived and average realized job-finding rates is small. Thus, the economy with noisier public signals (large σ_ξ) predicts a smaller gap than our baseline model.

Taking stock, the gap between perceived and realized job-finding rates is non-monotonic in the degree of noise in the public signal. Less noisy signals lead to faster learning and beliefs converge closer to their true values. Highly noisy signals are treated as uninformative, causing individuals to mostly maintain their prior and gaps between the average perceived and realized job-finding rates are also smaller, but only because individuals start out with unbiased priors, i.e., the unconditional distribution from which z is drawn. Overall, we argue that our analysis highlights that to account for the presence of optimistic beliefs in the data, one needs to assume a moderate amount of noise in the public signal, as in our baseline model.

7.3 Noisy signal of non-employment

In this section, we examine how our model would perform if we instead allowed individuals to observe a noisy signal of the non-employment rate instead of the labor force participation rate. That is, we allow the public signal that individuals observe to be $\widehat{n}_t(\tau) = n_t(\tau) + \eta_t(\tau)$, where $\eta_t(\tau)$ is the noise term. Effects of participation on the quality of the signal can be asymmetric in this case. Consider the limit case where all individuals of age τ choose to participate $p_t(\tau) = 1$, the noisy signal then becomes:

$$\widehat{n}_t(\tau) = [1 - f(a_t, z)]n_{t-1}(\tau - 1) + \delta[1 - n_{t-1}(\tau - 1)] + \eta_t(\tau)$$

In this case, when all non-employed participate, the only convolution in the signal stems from the noise term $\eta_t(\tau)$. Thus, in this special case where $p_t(\tau) = 1$, the signal becomes very uninformative only if the noise term is very large. When the noise is moderate, individuals can still learn from the public signal even if agents are highly optimistic and all individuals choose to participate. Thus, deep recessions and strong expansions can generate differential rates of learning. We emphasize though that this does not imply that individuals learn perfectly in a boom, rather our results suggests that this particular signal is more informative in a strong expansion than in a deep recession.

To see this, we take the model with the noisy signal of non-employment and re-calibrate it to the same moments. Table A2 in Appendix C shows our model fit. Because the parameters governing the unconditional distribution of group fundamentals are different, we re-draw cohorts from this new distribution.[12] We also re-draw the noise shocks for each cohort in each time period as the calibrated variance in the noise terms differs from that computed under our baseline model. We, however, feed in the same estimated aggregate shock series.

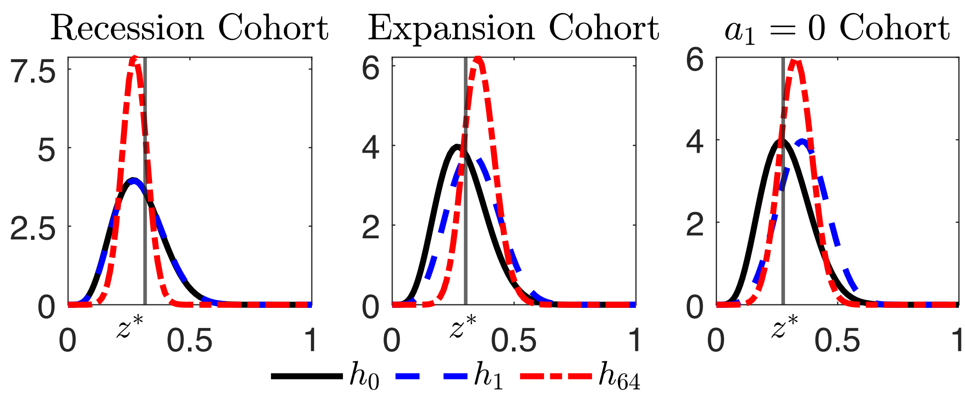
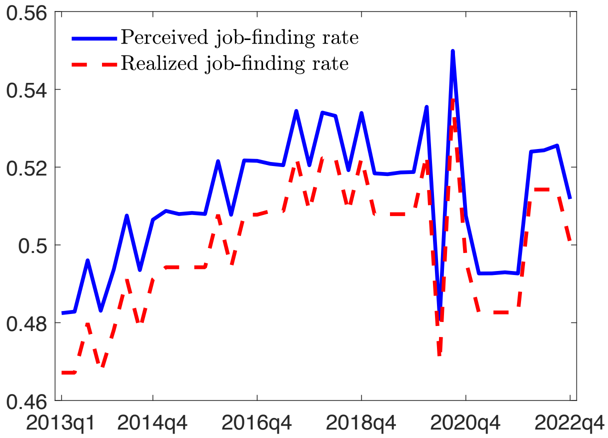


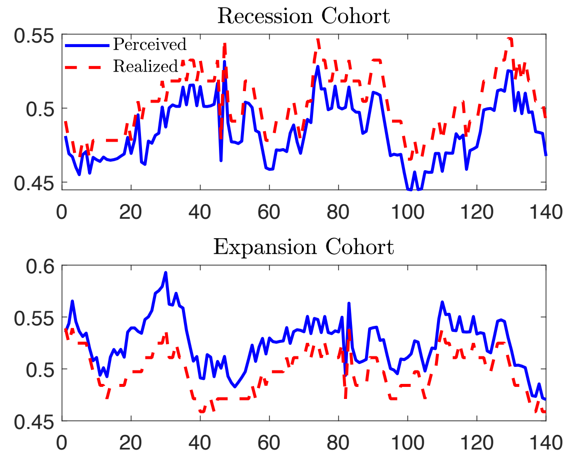
Figure 7: Evolution of beliefs with non-employment rate as noisy signal

Figure 7 shows how the distribution of beliefs evolve for different cohorts. As before, we focus on three types of cohorts to explain the mechanisms in our model. Specifically, Figure 7 shows the distribution of beliefs across different time periods for a cohort that entered i) during the onset of the Great Recession (left panel), ii) during a highly expansionary period where aggregate productivity is 3 standard deviations above mean (middle panel) and iii) when aggregate productivity was at its mean. Similar to our baseline model, the cohort that entered during the expansion – where aggregate productivity was 3 standard deviations above its mean – perceive labor markets to be very strong and the share of non-employed who participated is close to 1. Unlike our baseline model, however, this cohort derives information from the noisy signal of non-employment. Since $n_0 = 0$, the noisy public signal in period 1 is equivalent to $\widehat{n}_1(1) = [1 - f(a_t, z)] + \eta_1(1)$. So long as the noise in the signal is moderate, agents learn from the signal. The middle panel of Figure 7 shows that the distribution of beliefs in date 1, h_1 (blue dashed line), does not overlap with the distribution of beliefs at date 0, h_0 (black solid line). In contrast, very few individuals in the cohort that entered during the onset of the Great Recession participated in the first period, and hence the public signal of the non-employment rate is uninformative. These differences contribute to differential learning across large recessions and booms, with signals being more informative during strong expansions. Nonetheless, it should be

[12] That is for the model with noisy non-employment rates, beta distribution parameters A_z and B_z are different than that computed under our baseline model.



(a) Averages across time



(b) Deep recession vs strong expansion

Figure 8: Perceived vs realized rates under noisy non-employment rates

noted that while there is differential learning across recessions and booms, learning is still not complete in this model with noisy signals of the non-employment rate. Figure 8a shows that a gap, albeit smaller, still exists between perceived and realized job-finding rates of prime age workers. Moreover, focusing on the Great Recession and strong expansion cohorts, Figure 8b shows that initial pessimism and optimism for these cohorts remain highly persistent. Perceived job-finding rates remain below (above) the cohort's realized job-finding rate for those started in the Great Recession (strong expansion). This persistence in beliefs and lack of incomplete learning leads our model with noisy non-employment rates to predict about a 1.3 percentage point difference between the two rates, about 22 percent of the gap observed in data.

7.4 Learning from private outcomes and public signals

Thus far, we have abstracted from learning from private outcomes. Computationally, this is a non-trivial problem to solve as once we include learning from private outcomes, we need to keep track of the *distribution* of distribution of beliefs. In particular, when observing the public signal, individuals need to infer the beliefs of others to compute the counterfactual participation rate for each z. Thus, unlike our baseline model, where we only needed to keep track of the public belief and could use Bayes rule to back out individual beliefs at any point in time, we now need to keep track of each individual's belief and thus the distribution of distribution of beliefs. This greatly increases the dimensionality of the problem.

To make the problem tractable, we instead assume that the fundamental z takes on only two values, $z \in \{z_H, z_L\}$ where z_H is assumed to be 1.5 times the mean value of z in our baseline model and z_L is half the value of mean z. A binary z allows us to characterize each individual's distribution of beliefs by one variable, the probability that the fundamental is z_H. This greatly reduces the complexity of the problem. Appendix D details the model framework when z is binary and agents can learn from private outcomes. We do not re-calibrate the model and all other parameters are assumed to be the same as in the baseline model.

Importantly, because z only takes on two values, it should be noted that a cohort that draws z_H can never have optimistic beliefs defined as average beliefs being above their fundamental value. This is because individuals form expectations of their z drawn, which has to be a convex combination of z_L and z_H. Thus, for a cohort that draws z_H, their belief at any point in time must be less than or equals to z_H. Similarly, a cohort that draws z_L can never have pessimistic beliefs (average beliefs below their fundamental value). Given these technicalities, we emphasize that the goal of this exercise with a binary z is to highlight how beliefs can be persistent and do not converge to their true value even when we allow for learning from private outcomes.

Figure 9 shows how perceived job-finding rates can persistently diverge from realized job-finding rates. To construct this figure, we focused on the cohort that started when initial aggregate productivity was at its mean value, i.e., $a_1 = 0$.[13] We consider two scenarios, a cohort that draws group fundamental equal to z_L (left panel) and a cohort that draws group fundamental equal to z_H (right panel). Both scenarios show that the average perceived job-finding rate diverges significantly from the average realized rate. Under the scenario where the cohort drew z_L, the gap between perceived and realized rates is much larger although it narrows over time. Notably, this cohort (which drew z_L) had an initial participation rate of 5 percent in the first period. Since most individuals did not participate in the first period, they have no new private outcomes to observe, making them reliant on the information from the public signal. Lack of participation makes the public signal even noisier in the first period. As such, learning slows down, and the average perceived rate persistently lies above its realized counterpart.

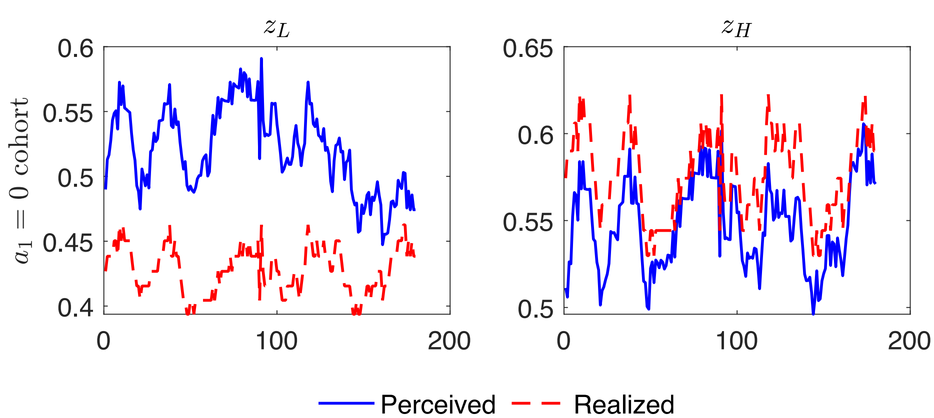


Figure 9: Beliefs on job-finding rates can persistently diverge from realized job-finding rates

[13] This is the cohort that enters the market in 1998 and we use the estimated cyclical shocks from the series as shown in Figure 2a.

Conversely, under the scenario where the cohort drew z_H, the gap between perceived and realized rates is smaller. While this gap does shrink over time, there still remains a persistent gap between the average perceived and realized rates. Participation in the first period is moderate, implying that some individuals are able to gain some information from private outcomes in addition to the public signal. Nonetheless, since only a portion of individuals have access to information from private outcomes, while all individuals have access to the same common public signal, learning is still incomplete. Thus, perceived rates do not converge to their actual values.

8 Conclusion

We developed a partial equilibrium model to show how average beliefs can persistently diverge from their fundamental values. Initial conditions affect participation which in turn affects the noise in endogenous public signals through the aggregation of private information. Severe recessions and strong expansions have the effect of muting the degree to which the signal component varies with the fundamental value, leading to less informative signals and slower learning. While we have limited our analysis to one public signal, one could ask whether beliefs continue to diverge from realized values when individuals are simultaneously exposed to many public signals. In this case, one could hypothesize that individuals might have limited capacity to understand all the information before them, even if they have access to all types of signals. We leave this question for future research

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Appendix

A Data Appendix

Table A1 replicates the regression exercise in Equation 1 but with non-employment rates. Similar to Table 3, the coefficient on the local initial non-employment rate aged 18-24 is negative and statistically significant, suggesting that initial conditions of one's peers matter for current beliefs.

Table A1: Expectations about labor market and Non-employment

	Expected Job Finding			
	(1)	(2)	(3)	(4)
Local current NE/POP	0.08	0.01	0.14	-0.05
	(0.18)	(0.25)	(0.30)	(0.52)
Local initial NE/POP	-0.29^{*}	0.13	0.15	-0.15
	(0.17)	(0.25)	(0.31)	(0.45)
Local initial NE/POP (18-24)	-	-0.28^{***}	-0.36^{***}	-0.25^{*}
	-	(0.07)	(0.10)	(0.15)
Local current u	-	-	-0.88^{**}	-0.38
	-	-	(0.40)	(0.41)
Local initial u	-	-	-0.05	0.25
	-	-	(0.31)	(0.37)
Local initial u (18-24)	-	-	0.23	0.12
	-	-	(0.17)	(0.18)
Aggregate u (25-54yrs)	-2.21^{***}	-1.81^{***}	-1.12^{**}	-1.45^{***}
	(0.34)	(0.36)	(0.48)	(0.49)
Aggregate LFPR (25-54yrs)	1.26	1.16	0.70	0.81
	(0.81)	(0.87)	(0.87)	(0.87)
Controls	Yes	Yes	Yes	Yes
State Fixed Effects	No	No	No	Yes
R^2	0.03	0.02	0.03	0.03
Observations	55,438	43,894	43,894	43,894

Note: This table presents results from prime-aged individuals on their expected probability of finding a job in the next 3 months using SCE data for the period 2013m6-2022m4. Regression controls include potential experience, potential experience squared, education dummies, a dummy for whether the individual is a male, a dummy for whether the individual is Caucasian, a dummy for marriage or cohabitation, and a dummy for whether the individual is currently employed. Potential experience is calculated as age less 12 years of schooling if the individual has less than a college degree, and age less 16 years of schooling if the individual has a college degree. "Aggregate u_t" represents the national unemployment rate while variables affixed with "Local" denote state-level employment statistics. Initial conditions are determined at age 18 if the person has less than a college degree, or the at age 22 if the person has a college degree. All data on unemployment rates are taken from the BLS. Column 1 represents the most parsimonious regression with only aggregate and local labor market conditions as well as the local non-employment rate at the time of an individual's entry into the labor market. Column 2 adds initial local labor market information for the relevant age group (18-24). Columns 3 includes information on local and initial unemployment rates. Finally, Column 4 repeats specification 3 but with state fixed effects. Standard errors are presented in parentheses. Statistical significance is denoted by the following: $***$ p<0.01, $**$ p<0.05, $*$ p<0.1

B Ruling out private learning

B.1 Proof of Proposition 2

To arrive at the average belief, we integrate across all possible s and z:

$$\begin{aligned}
\int\int \{\alpha s + (1-\alpha)\,\mu_z\}\,\phi\left(\frac{s-z}{\sigma_\epsilon}\right) ds\phi\left(\frac{z-\mu_z}{\sigma_z}\right) dz &= \alpha\int z\phi\left(\frac{z-\mu_z}{\sigma_z}\right) dz + (1-\alpha)\mu_z \\
&= \alpha\mu_z + (1-\alpha)\mu_z \\
&= \mu_z
\end{aligned}$$

Focusing on the inner integral with respect to signal s, by law of large numbers, the noise term cancels out, leaving the group of individuals who drew fixed effect z to have average expectation z. Since all individuals draw their fixed effect from the unconditional distribution, this must also imply that by integrating across z, the average belief is equal to the average realization.

B.2 Proof of Proposition 3

Aggregating across all individuals, we can compute the average belief as:

$$\begin{aligned}
\text{Average belief} &= \int\left\{\int p\left[h_1\left(p \mid q=1;p^*\right)p^* + h_1\left(p \mid q=0;p^*\right)\left(1-p^*\right)\right] dp\right\} g(p^*)dp^* \\
&= \int\left\{\int p\left[pg(p) + (1-p)g(p)\right] dp\right\} g(p^*)dp^* \\
&= \int\left\{\int pg\left(p\right) dp\right\} g(p^*)dp^* \\
&= \int \overline{p}g(p^*)dp^* = \overline{p}
\end{aligned}$$

Thus, the average belief is equal to the average realization.

C Calibration of model with noisy non-employment rates

We calibrate to the same moments as our baseline model. In this version, individuals no longer observe a noisy signal of their cohort's participation rate, $\widehat{\ell}_t(\tau)$, but instead observe a noisy signal of their cohort's non-employment rate, $\widehat{n}_t(\tau)$. Table A2 shows our calibrated results. Overall, our model moments largely match their data-counterparts.

D Learning from private outcomes and public signals

We outline a version of the model of learning from private outcomes and endogenous public signals when the fundamental z is binary. In doing so, we highlight the assumptions made for our solution method. For ease of exposition, we will silence the dependence on age τ to keep the

Table A2: Internally calibrated parameters: non-employment model

Parameter	Description	Value	Target	Model	Data
A_z	Beta dist parameter	5.85	Mean job-finding rate, f	0.493	0.489
B_z	Beta dist parameter	14.26	Std dev. job-finding rate, f	0.048	0.053
c	Participation cost	3.19	Prime-age participation	0.812	0.820
σ_ϵ	Dispersion in ϵ	0.38	Std dev. perceived f, 18-24	0.054	0.060
σ_η	Dispersion in η	0.16	Std dev. perceived f, 25-54	0.038	0.046

Notes: Dispersion in perceived job-finding rates for the relevant age group is computed as the standard deviation in predicted perceived job-finding rates after controlling for aggregate fluctuations

notation simple.

D.1 When z is binary

Let z take on two values, i.e., $z \in \{z_H, z_L\}$. Further denote $h_{it} = Pr\left(z_H\right)$, where h_{it} captures the belief of individual i at the end of period t that the fundamental is z_H. Let the unconditional probability that Nature draws $z = z_H$ at date 0 be given by h_0.

As per the baseline model, at the start of date 0, nature draws the fundamental z. Individuals at the start of date 0 also see private exogenous signal $s_i = z + \epsilon_i$ where $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The individual's belief at the start of date t can be denoted as $\widetilde{h}_{it}$. Thus, $\widetilde{h}_{i1}$ is the individual's belief at the start of date 1 after observing private signal s_i, and for this specific case, $\widetilde{h}_{i1} = \frac{h_0\varphi(s_i|z_H)}{h_0\varphi(s_i|z_H)+(1-h_0)\varphi(s_i|z_L)}$. For all other periods i.e., $t > 1$, $\widetilde{h_{it}} = h_{it-1}$. We will outline how individuals' beliefs evolve at any date t.

Suppose an individual i starts the period with belief $\widetilde{h}_{it}$. As per the baseline model, individuals participate so long as their expected net value of search is at least as large as the expected value of staying out of the labor force. The marginal individual who is just indifferent between participating and staying out of the labor market can be characterized as having belief $\widetilde{h}_t^*$ where $\widetilde{h}_t^*$ is implicitly defined by the following indifference condition:

$$\widetilde{h}_t^* \Xi\left(a_t, z_H\right) + \left[1 - \widetilde{h}_t^*\right] \Xi\left(a_t, z_L\right) = 0$$

and $\Xi(a_t, z) = V^U(a_t, z) - c - V^O(a_t, z)$. This implies that any individual with private belief $\widetilde{h}_{it} \geq \widetilde{h}_t^*$ will participate. Define Γ_t as the time t distribution *of distribution of beliefs* in the economy at the start of period t and γ_t as the associated density. Γ_t is known to the modeler, but not to agents in the economy. Because the fundamental is binary, this means we can describe each individual's belief by their perceived probability that the fundamental is z_H, and thus Γ_t is the distribution over $\widetilde{h}_t$. We defer discussing how Γ_t evolves to Appendix D.2. The share of non-employed who participate in period t is then characterized by:

$$p_t = 1 - \Gamma_t(\widetilde{h}_t^*)$$

And the actual participation rate is given by $\ell_t = 1 - (1 - p_t)n_{t-1}$. Individuals instead observe the noisy signal $\widehat{\ell}_{t-1}$. Denote with *hats*, the intermediate belief individuals have after observing the public signal, i.e., $\widehat{h}$ is the belief that the fundamental is z_H after observing the public signal $\widehat{\ell}$ and is characterized as:

$$\widehat{h}_{it} = \frac{\widetilde{h}_{it}\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_H]}{\sigma_\xi}\right)}{\widetilde{h}_{it}\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_H]}{\sigma_\xi}\right) + \left(1 - \widetilde{h}_{it}\right)\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_L]}{\sigma_\xi}\right)} \tag{A1}$$

To understand how individuals derive $\ell_t(z_H)$ and $\ell_t(z_L)$, we will need to characterize how individuals compute the counterfactual labor force participation rate, $\ell_t(z)$ for $z \in \{z_H, z_L\}$. To do this, we assume that individuals are told the previous period's distribution of agents' prior beliefs i.e., they observe the distribution $\Gamma_{t-1}(\widetilde{h})$, where $\gamma_{t-1}(\widetilde{h})$ is the associated density. Since the public signal from last period is observed by all individuals, they can compute what the distribution of beliefs in the economy is after the public signal is observed. Denote $G_{t-1}(\widehat{h})$ as the updated distribution of beliefs after public signal $\widehat{\ell}_{t-1}$ is observed. Given $G_{t-1}(\widehat{h})$, for any belief $\widehat{h}$, individuals can compute the density of individuals with belief $h_{it-1}(\widehat{h})$ at the end of the period for any z. That is, they can compute the economy-wide distribution of beliefs at the end of period $t-1$, and thus, the distribution of beliefs at the start of period t. Denote this counterfactual distribution of beliefs at the start of t as $\Psi_t(\widetilde{h}, z)$, for any given z. Using $\Psi_t(\widetilde{h}, z)$, they can then compute the counterfactual $p_t(z)$ and hence, $\ell_t(z)$. In particular, given $\Psi_t(\widetilde{h}, z)$, they can compute the counterfactual share of non-employed who participate for a given z as:

$$p_t(z) = 1 - \Psi_t(\widetilde{h}_t^*, z)$$

and the counterfactual participation rate as:

$$\ell_t(z) = 1 - [1 - p_t(z)]n_{t-1}(z)$$

We outline in Appendix D.2 how the distributions Γ_t, G_t and Ψ_t are derived. In terms of realized vs perceived job-finding rates, the actual realized job-finding rate is given by $f(a_t, z)$ where $z \in \{z_H, z_L\}$, while the average perceived job-finding rate is given by:

$$\text{average perceived job-finding rate} = \int \left[\widetilde{h}f(a_t, z_H) + (1 - \widetilde{h})f(a_t, z_L)\right]\gamma_t(\widetilde{h})d\widetilde{h}$$

Note that the average perceived job-finding rate is the rate calculated using the beliefs prior to seeing the public signal, as this is the belief individuals use when deciding whether to participate in the labor market.

At the end of period t, given counterfactual $\ell_t(z_H)$ and $\ell_t(z_L)$, and observing her own outcome, the individual i updates her posterior belief, h_{it}. Equation A2 describes how beliefs evolve:

$$h_{it} = \begin{cases} \frac{\widehat{h}_{it} f(a_t,z_H)}{\widehat{h}_{it} f(a_t,z_H)+\left(1-\widehat{h}_{i1}\right)f(a_t,z_L)} & \text{if participate and find job,} \\ \frac{\widehat{h}_{it}[1-f(a_t,z_H)]}{\widehat{h}_{it}[1-f(a_t,z_H)]+\left(1-\widehat{h}_{it}\right)[1-f(a_t,z_L)]} & \text{if participate and didn't find job,} \\ \widehat{h}_{it} & \text{did not participate} \end{cases} \tag{A2}$$

The first line of Equation A2 captures the case where the individual participates, $\widetilde{h}_{it} > \widetilde{h}^*_t$, and finds a job where $f(a_t, z)$ is the probability she finds a job if the state is z. The second line captures the case where the individual participates, $\widetilde{h}_{it} > \widetilde{h}^*_t$, but does not find a job, where the latter occurs with probability $1 - f(a_t, z)$ if the state is z. Finally, the last line captures the case where the individual does not participate. In that case, she has no new private information and her belief is equivalent to the belief she had after observing the public signal.

End-of-period beliefs, h_{it} form the basis of beliefs at the start of the next period, i.e., $\widetilde{h}_{it+1} = h_{it}$. The whole process then repeats itself.

D.2 Evolution of distribution of beliefs

Thus far, we have not specified how the distribution of *distribution* of beliefs evolves. We focus first on describing how $\Gamma_t(\widetilde{h})$ evolves. To do this, we first describe what the distribution of beliefs looks like at the start of date 1, before generalizing the characterization to any t period.

At date 0, each individual i observes a signal $s_i = z + \epsilon_i$ where ϵ_i is drawn from a normal distribution with mean zero and variance σ^2_ϵ. This implies that the density of individuals at date 1 with private signal equal to s is given by $\phi\left(\frac{s-z}{\sigma_\epsilon}\right)$. We also know that given a signal of value s_i, an individual has belief equivalent to:

$$\widetilde{h}_{i1} = \frac{h_0 \phi\left(s_i \mid z_H\right)}{h_0 \phi\left(s_i \mid z_H\right) + \left(1 - h_0\right) \phi\left(s_i \mid z_L\right)} = \nu(s_i)$$

Since $\widetilde{h}_{i1}$ is monotonic in s_i, we can define the inverse function as $s_i = \nu^{-1}\left(\widetilde{h}_{i1}\right) = \Omega\left(\widetilde{h}_{i1}\right)$. Then applying a change of variable, define the distribution of beliefs at the start of period 1 as:

$$\Gamma_1\left(\widetilde{h}\right) = \int_{-\infty}^{\Omega(\widetilde{h})} \phi\left(\frac{s-z}{\sigma_\epsilon}\right) ds$$

Accordingly, the density of beliefs equal to $\widetilde{h}$ in period 1 is given by:

$$\gamma_1\left(\widetilde{h}\right) = \phi\left(\frac{\Omega\left(\widetilde{h}\right) - z}{\sigma_\epsilon}\right)\Omega'\left(\widetilde{h}\right)$$

After observing the public signal $\widehat{\ell}_1$, the individual updates her belief according to Equation A1. Let the updated belief be summarized as $\widehat{h} = \chi(\widetilde{h}) = \frac{\widetilde{h}_{it}\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_H]}{\sigma_\xi}\right)}{\widetilde{h}_{it}\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_H]}{\sigma_\xi}\right) + \left(1-\widetilde{h}_{it}\right)\phi\left(\frac{\widehat{\ell}_t - \ell_t[z_L]}{\sigma_\xi}\right)}$. Similarly, let $\Theta(\widehat{h}) = \widetilde{h}$ be the inverse of equation A1. Given this relationship, the distribution of beliefs in period 1 after the public signal has been observed can be written as:

$$G_1\left(\widehat{h}\right) = \int_{-\infty}^{\Theta\left(\widehat{h}\right)} \gamma_1\left(\widetilde{h}\right) d\widetilde{h} = \int_{-\infty}^{\Theta\left(\widehat{h}\right)} \phi\left(\frac{\Omega\left(\widetilde{h}\right) - z}{\sigma_\epsilon}\right)\Omega'\left(\widetilde{h}\right) d\widetilde{h}$$

And the density of individuals with belief $\widehat{h}$ in period 1 is given by:

$$g_1\left(\widehat{h}\right) = \gamma_1\left(\Theta\left[\widehat{h}\right]\right)\Theta'\left(\widehat{h}\right) = \phi\left(\frac{\Omega\left(\Theta\left[\widehat{h}\right]\right) - z}{\sigma_\epsilon}\right)\Omega'\left(\Theta\left[\widehat{h}\right]\right)\Theta'\left(\widehat{h}\right)$$

In period 1, all individuals with $\widetilde{h}_{i1} \geq \widetilde{h}_1^*$ or equivalently $\Omega(\widehat{h}_{i1}) \geq \widetilde{h}_1^*$ participate, and a fraction $f(a_1, z^*)$ of the share of non-employed who participate find a job, where z^* is equal to the true value of z drawn. These individuals revise their belief up to $\widetilde{h}_{i2}$. A fraction $1 - f(a_1, z^*)$ of the participants fail to find a job and revise down their belief. Finally all non-participants maintain the same belief as they had after observing the public signal. This implies for a given belief $\widehat{h}_{i1}$, the density of individuals with $\widetilde{h}_{i2}$ evolves according to:

$$\gamma_2(\widetilde{h}) = \begin{cases} g_1\left(\widehat{h}\right) f\left(a_1, z^*\right) & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_1^* \text{ and } \widetilde{h} = \frac{\widehat{h} f(a_1, z_H)}{\widehat{h} f(a_1, z_H) + \left(1-\widehat{h}\right) f(a_1, z_L)}, \\ g_1\left(\widehat{h}\right)\left[1 - f\left(a_1, z^*\right)\right] & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_1^* \text{ and } \widetilde{h} = \frac{\widehat{h}[1-f(a_1, z_H)]}{\widehat{h}[1-f(a_1, z_H]) + \left(1-\widehat{h}\right)[1-f(a_1, z_L])}, \\ g_1\left(\widehat{h}\right) & \text{for } \Theta(\widehat{h}) < \widetilde{h}_1^* \text{ and } \widehat{h} = \widetilde{h} \end{cases}$$

Once we have $\gamma_2(\widetilde{h})$ and thus $\Gamma_2(\widetilde{h})$, we have the distribution of beliefs at the start of period 2. Given γ_2 and Γ_2, we can derive the distribution G_2 and density g_2. The process then repeats itself. Consequently, the two main laws of motion can be summarized by:

$$g_t\left(\widehat{h}\right) = \gamma_t\left(\Theta\left[\widehat{h}\right]\right)\Theta\left(\widehat{h}\right) \tag{A3}$$

and

$$\gamma_{t+1}(\widetilde{h}) = \begin{cases} g_t\left(\widehat{h}\right) f\left(a_t, z^*\right) & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_t^* \text{ and } \widetilde{h} = \frac{\widehat{h} f(a_t, z_H)}{\widehat{h} f(a_t, z_H) + \left(1-\widehat{h}\right) f(a_t, z_L)}, \\ g_t\left(\widehat{h}\right)\left[1 - f\left(a_t, z^*\right)\right] & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_t^* \text{ and } \widetilde{h} = \frac{\widehat{h}[1-f(a_t, z_H)]}{\widehat{h}[1-f(a_t, z_H)] + \left(1-\widehat{h}\right)[1-f(a_t, z_L)]}, \\ g_t\left(\widehat{h}\right) & \text{for } \Theta(\widehat{h}) < \widetilde{h}_t^* \text{ and } \widehat{h} = \widetilde{h} \end{cases} \quad \text{(A4)}$$

D.2.1 Deriving Ψ_t

As aforementioned, we assume that at the start of t, individuals know the distribution $\Gamma_{t-1}(\widetilde{h})$ but not $\Gamma_t(\widetilde{h})$. Since all individuals observe the public signal in the previous period, this also implies they also know $G_{t-1}(\widehat{h})$. We now can describe how individuals form the counterfactual distribution of beliefs at the start of t, $\Psi_t(\widetilde{h}, z)$, for any z. Specifically, after observing last period's public signal, for any belief $\widehat{h}_{it-1}$, the counterfactual density of individuals with belief $\widetilde{h}_{it}$ at the start of t is characterized by:

$$\psi_t(\widetilde{h}, z) = \begin{cases} g_t\left(\widehat{h}\right) f\left(a_t, z\right) & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_t^* \text{ and } \widetilde{h} = \frac{\widehat{h} f(a_t, z_H)}{\widehat{h} f(a_t, z_H) + \left(1-\widehat{h}\right) f(a_t, z_L)}, \\ g_t\left(\widehat{h}\right)\left[1 - f\left(a_t, z\right)\right] & \text{for } \Theta(\widehat{h}) \geq \widetilde{h}_t^* \text{ and } \widetilde{h} = \frac{\widehat{h}[1-f(a_t, z_H)]}{\widehat{h}[1-f(a_t, z_H)] + \left(1-\widehat{h}\right)[1-f(a_t, z_L)]}, \\ g_t\left(\widehat{h}\right) & \text{for } \Theta(\widehat{h}) < \widetilde{h}_t^* \text{ and } \widehat{h} = \widetilde{h} \end{cases} \quad \text{(A5)}$$

The main difference between Equation A4 and A5 is that the former describes how the distribution evolves given the true value of z and thus given the true job-finding rate, whereas the latter computes how the counterfactual distribution using the job-finding rates of any z. With the counterfactual distribution $\Psi_t(\widetilde{h}, z)$, individuals can compute $p_t(z), \ell_t(z), n_t(z)$. Note that $\Psi_t(\widetilde{h}, z)$ is derived from the previous period's distribution of beliefs, whereas $\Gamma_t(\widetilde{h})$ represents the true distribution of prior beliefs in the economy at the start of t. If we had instead provided information on $\Gamma_t(\widetilde{h})$, then individuals would be able to compute the true p_t each period, and there would be no learning from the public signal. Individuals also do not learn from $\Gamma_{t-1}(\widetilde{h})$ directly, i.e., they do not update individual beliefs based on information of $\Gamma_{t-1}(\widetilde{h})$. We make this assumption to maintain computational tractability.