Quantitative Macro-Labor: Inequality in Heterogeneous Agent Models

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Announcements

- Today: Continue heterogeneous agent models.
- Start writing down and solving your model.
- Due in 3 weeks!
- No office hours today (please email if you have questions!)

Model Project

- Write down and solve a model that can explain your empirical regularities.
- Solving a model is tricky. Try to get a solution for "calibrated" parameters.
- Try to pick a model somewhat similar to the code I've provided.
- Start with a model that we have either talked about or is widely used.
- I'll try to help as much as I can.
- We start presentations after Thanksgiving!

Thinking about Inequality

- We started this semester discussing the role of inequality between individuals in determining outcomes.
- What is the source of this inequality?

Huggett, Ventura, and Yaron (2011)

- Questions:
  - How much of life-cycle inequality is ex-ante? i.e., how much is beyond an individual's control?
  - Which of the initial conditions are important?
- Outline:
  - Life-cycle production economy with
  - Risk-aversion, borrowing constraints, and human capital accumulation,
  - competitively determined prices, social security (GE)
- ex-ante heterogeneity in wealth, human capital, learning ability.

Model Environment

- Life-cycle model: age discrete, indexed by j; retirement at age J_R die at age J
- Agents:
  - Age-j, born at time t households: (j, t, k, h, ℓ)
  - competitive firms who set interest rate and wages.
- Technology:
  - Endogenous human capital accumulation.
  - Borrowing constraints. $k \geq \underline{k}$
  - Uncertainty over rental rate of labor (standard inc. mkts.) & rate of return to human capital.
- Initial heterogeneity:
  - Initial wealth (a_0), human capital (h_0), and learning ability (ℓ).
- Prices determined by overlapping generations.

Household's Problem

- Sequential Formulation for agent born at time t:

$$\max_{\{c_j, l_j, s_j, h_j, k_j\}_{j=1}^{J}} E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)] \tag{1}$$

$$\text{s.t. } c_j + k_{j+1} = (1 + r_{t+j-1})k_j + e_j - T_{j,t+j-1}(e_j, k_j) \tag{2}$$

$$e_j = R_{t+j-1} h_j (1 - s_j) \forall\ j < J_R,\ 0\ else \tag{3}$$

$$h_{j+1} = e^{z_{j+1}} H(h_j, s_j, a) \tag{4}$$

- GE Objects:
  - Prices: interest rate & wages.
  - Social security income T
- Multiple generations alive at same time. Hard to solve!
- Luckily, no aggregate uncertainty, can solve in stationary equilibrium.

Firm's Problem

- Firms are competitive.
- i.e., they pay the marginal product.
- Overlapping generations balanced growth model.
- Firm's problem:

$$\max_{K,L} \pi F(K, LA) - RL - rK \tag{5}$$

$$A_{t+1} = (1+g)A_t \tag{6}$$

- Cross-cohort growth rates of human capital (effectively).

Balanced Growth Competitive Equilibrium

- A balanced-growth equilibrium is a collection of decisions for each cohort at each age, a set of factor prices, government taxes and expenditures, and an initial distribution ψ such that
  1. Agents optimize taking as given the factor prices.
  2. Prices are formed competitively from the firm's problem.
  3. Resource feasibility: expenditures = output in the aggregate.
  4. Balanced government budget.
  5. Balanced growth: c, k, T, G grow over time, while portfolio allocation and interest rate remains constant.
  6. Aggregates are consistent with individual policy rules: $K = \sum_j \int k d\psi$, $L = \sum_j \int \epsilon d\psi$, $C = \sum_j \int c d\psi$, $T = \sum_j \int T(.) d\psi$

Household's Problem

- Sequential Formulation for agent born at time t:

$$\max_{\{c_j, l_j, s_j, h_j, k_j\}_{j=1}^{J}} E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)] \tag{7}$$

$$\text{s.t. } c_j + k_{j+1} = (1 + r_{t+j-1})k_j + e_j - T_{j,t+j-1}(e_j, k_j) \tag{8}$$

$$e_j = R_{t+j-1} h_j (1 - s_j) \forall\, j < J_R,\ 0\ else \tag{9}$$

$$h_{j+1} = e^{z_{j+1}} H(h_j, s_j, a) \tag{10}$$

- GE Objects:
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- Multiple generations alive at same time. Hard to solve!
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Empirical Preliminaries

- Model parameters: $\sigma = 2, \beta = \frac{1}{1+r_F}$
- Power utility + unemp leisure: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$
- HC Evolution: $h' = e^{\epsilon}(h + H(h, \ell, s)) = e^{\epsilon}(h + \ell \times (hs)^{\alpha_H})$
- Initial conditions:
  - $(k_0, h_0, \ell) \sim LN(\psi, \Sigma)$
  - Correlations $\rho_{KH}, \rho_{KL}, \rho_{HL}$
- Most results: $\sigma_K = 0$

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Identification

- They preset parameters to non-controversial values ($\sigma = 2$, etc.)
- Controversial problem: disciplining human capital accumulation.
- Specifically, how to separate depreciation from time allocation.
- Their approach:
  - Argue that households stop accumulating human capital at some point. Call this age t.
  - Then, any change in human capital from ages t to $t + n$ is due to depreciation.
  - Use this & second-moments to identify human capital.

Identification II

- Initial conditions. How to discipline them?
- Here, assume that model is correct and pick initial conditions that best fit the data.
- Initial human capital: shifts the intercept of the earnings profile.
- Learning ability: rotates (change the slope) of the earnings profile.
- Match first and second (and third) moments of earnings distributions/profiles for these initial conditions.
- Match age-23 wealth for initial wealth.

Empirical Preliminaries

TABLE 2—PARAMETER VALUES: BENCHMARK MODEL

Category	Symbol	Parameter value
Demographics	(J, J_R, n)	$(J, J_R, n) = (53, 39, 0.012)$
Preferences	$\beta, u(c) = c^{(1-\rho)}/(1-\rho)$	$(\beta, \rho) = (0.981, 2)$
Technology	(γ, δ, g)	$(\gamma, \delta, g) = (0.322, 0.067, 0.0019)$
Tax system	$T_j = T_j^{ss} + T_j^{inc}$	$T_j^{ss}(e_j) = 0.106 e_j$ for $j < J_R$ $T_j^{ss}(e_j) = -0.4\bar{e}$ otherwise T_j^{inc}—see text
Human capital shocks	$z \sim N(\mu, \sigma^2)$	$(\mu, \sigma) = (-0.029, 0.111)$
Human capital technology	$h' = \exp(z')H(h,s,a)$ $H(h,s,a) = h + a(hs)^{\alpha}$	$\alpha = 0.70$
Initial conditions	$\psi = LN(\boldsymbol{\mu}_{\mathbf{x}}, \Sigma)$	$\boldsymbol{\mu}_{\mathbf{x}} = (\mu_h, \mu_a) = (4.66, -1.12)$ $(\sigma_h^2, \sigma_a^2, \sigma_{ha}) = (0.213, 0.012, 0.041)$

Sources of Inequality

- Sequential Formulation for agent born at time t:

$$\max_{\{c_j,l_j,s_j,h_j,k_j\}_{j=1}^{J}} E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)] \tag{11}$$

$$\text{s.t. } c_j + k_{j+1} = (1 + r_{t+j-1})k_j + e_j - T_{j,t+j-1}(e_j, k_j) \tag{12}$$

$$e_j = R_{t+j-1} h_j (1 - s_j) \forall\, j < J_R,\ 0 \ else \tag{13}$$

$$h_{j+1} = e^{z_{j+1}} H(h_j, s_j, a) \tag{14}$$

- Initially, agents differ in terms of k_0, h_0, ℓ
- Agents face uncertainty over their future stream of consumption.
- Differences in human capital $\rightarrow e_j \downarrow \propto \Delta h$

Sources of Inequality

- Sequential Formulation for agent born at time t:

$$\max_{\{c_j, l_j, s_j, h_j, k_j\}_{j=1}^{J}} E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)] \tag{15}$$

$$\text{s.t. } c_j + k_{j+1} = (1 + r_{t+j-1})k_j + e_j - T_{j,t+j-1}(e_j, k_j) \tag{16}$$

$$e_j = R_{t+j-1} h_j (1 - s_j) \forall\, j < J_R,\ 0\ else \tag{17}$$

$$h_{j+1} = e^{z_{j+1}} H(h_j, s_j, a) \tag{18}$$

- Agents also make a portfolio allocation decision.
- Uncertainty important: human capital is a risky asset; capital is riskless.
- What does this mean in terms of initial inequality?

Thinking about Uncertainty in Macroeconomic Models

- Returning to our Euler Equation.
- Wealthy agents:

$$u'(c_{i,t}) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \quad \underbrace{u'(c_{i,t+1})}_{Closer\ to\ Linear} \quad] \tag{19}$$

- Poor Agents:

$$u'(c_{i,t}) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(c_{i,t+1})}_{Non-\ Linear}] \tag{20}$$

- What does this mean for portfolio allocation?

Some Exploration I

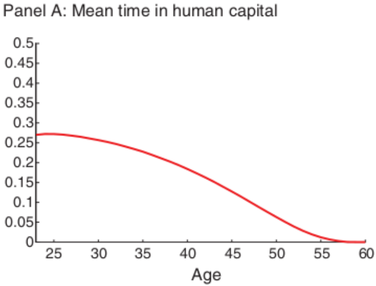

Figure 3. Properties of Human Capital by Age

Some Exploration II

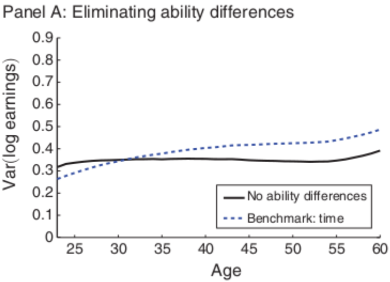

FIGURE 4. DISPERSION OF EARNINGS BY AGE

Initial Conditions

TABLE 3—PROPERTIES OF THE INITIAL DISTRIBUTION

Statistic	Benchmark model	Benchmark model with initial wealth differences
Mean learning ability (a)	0.329	0.328
Coefficient of variation (a)	0.112	0.124
Mean initial human capital (h_1)	116.9	117.5
Coefficient of variation (h_1)	0.487	0.476
Correlation (a, h_1)	0.746	0.655

Note: Entries show properties of the distribution of initial conditions for the parameters that best match the profiles of mean earnings, earnings dispersion, and skewness.

Initial Conditions vs. Shocks

TABLE 5—SOURCES OF LIFETIME INEQUALITY

Statistic	Benchmark model	Benchmark model with initial wealth differences
Fraction of variance in lifetime utility due to initial conditions	0.640	0.661
Fraction of variance in lifetime earnings due to initial conditions	0.615	0.613
Fraction of variance in lifetime wealth due to initial conditions	0.615	0.626

Notes: Entries show the fraction of the variance in the statistic accounted for by initial conditions (initial human capital, learning ability, and initial wealth). Wealth differences are measured directly from PSID data as explained in the text.

Initial Conditions and Inequality

TABLE 6—CHANGES IN INITIAL CONDITIONS

Variable	Change in variable	Equivalent variation (%)	Change in lifetime wealth (%)
Human capital	+1 st. deviation	39.3	47.5
	−1 st. deviation	−28.3	−31.7
Learning ability	+1 st. deviation	5.7	8.1
	−1 st. deviation	−2.6	−3.9
Initial wealth	+1 st. deviation	7.1	5.0
	−1 st. deviation	−1.6	−1.3

Notes: The table states equivalent variations and the percentage change in the expected lifetime wealth associated with changes in each initial condition. The baseline initial condition is set equal to the mean log values of initial human capital, learning ability, and wealth. Changes in initial conditions are also in log units.

What are your thoughts?

- ?

Conclusion

- Inequality in a heterogeneous agents environment.
- Next time: Estimation/linearization?
- Start your final projects! Don't wait to last minute.