Quantitative Macro-Labor: Beliefs and Participation

Professor Griffy

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Announcements

- Briefly review beliefs and rational expectations.
- Show a model of inequality derived from beliefs.
- Presentation schedule?

Portfolio Problem

- Consider a portfolio allocation problem in which an agent chooses between one of two assets:
  1. Asset a_{safe} offers a return of r_F, which is known with certainty.
  2. Asset a_{risky} offers a return of r_R, $r_R > r_F$ with probability λ and 0 with probability $(1 - \lambda)$,
- Agents maximize a static portfolio problem:

$$V(m) = \max_{a_{safe}, a_{risky}} \lambda u(c'_H) + (1 - \lambda) u(c'_L) \quad (1)$$

$$c'_H = (1 + r_F) a_{safe} + (1 + r_R) a_{risky} \quad (2)$$

$$c'_L = (1 + r_F) a_{safe} + a_{risky} \quad (3)$$

$$m = a_{safe} + a_{risky} \quad (4)$$

Parameter Uncertainty

- What if λ is type-specific?
- Asset a_{risky} offers a return of r_R
  1. with probability λ_H for a high-type and λ_L for a low-type
  2. and 0, with probability $(1 - \lambda_H)$ for a high-type and $(1 - \lambda_L)$ for a low-type.
  3. Agents endowed with prior belief that they are high type, $\theta^i \in [0, 1]$
- Prior beliefs drawn from uniform distribution $g(\theta^i) \sim U(0, 1)$.
- Belief distribution is agent-specific: $h(\theta^i)$ may differ based on history. Initially $h(\theta^i) = g(\theta^i) = \theta$
- For simplicity ignore bandit problem.

Signal Extraction

- Bayes theorem:

$$h(\theta'|c') = \frac{f(c'|\lambda)b(\lambda|\theta)g(\theta)}{f(y)} \tag{5}$$

- Binomial likelihood:

$$f(c_H|\lambda) = \lambda^{1_{c'=c_H}}(1-\lambda)^{1-1_{c'=c_H}} \tag{6}$$

- Binomial likelihood:

$$b(\lambda|\theta) = \lambda_H \text{with prob.} \theta = \lambda_L \text{with prob.} (1-\theta) \tag{7}$$

- Prior Distribution ($U(0,1)$):

$$g(\theta) = \theta, \theta \in [0,1],\ 0\ else \tag{8}$$

- Updating ($f(y) = 1$):

$$h(\theta'|c' = c'_H) = \frac{\lambda_H \theta}{\lambda_H \theta + \lambda_L(1-\theta)} \tag{9}$$

$$h(\theta'|c' = c'_L) = \frac{(1-\lambda_H)\theta}{(1-\lambda_H)\theta + (1-\lambda_L)(1-\theta)} \tag{10}$$

Discussion

- Updating ($f(y) = 1$):

$$h(\theta'|c' = c'_H) = \frac{\lambda_H \theta}{\lambda_H \theta + \lambda_L(1-\theta)} \quad (11)$$

$$h(\theta'|c' = c'_L) = \frac{(1-\lambda_H)\theta}{(1-\lambda_H)\theta + (1-\lambda_L)(1-\theta)} \quad (12)$$

- Thoughts about this updating:
  - Can beliefs persist? What would drive this?
  - What if parameters are group-specific?

Entrenched Beliefs, Slow Learning and Labor Force Participation

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Question

Can poor initial labor market outcomes slow learning and entrench beliefs ?

Motivation

Data from the SCE: expectations on labor market prospects

- Stylized Fact 1: Average beliefs about job-finding rate are optimistic (Mueller et al 2021) (Spinnewijn 2015) (Conlon et al 2018)

- Stylized Fact 2: Initial aggregate outcomes *(of group)* matter for current expectations of job-finding

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- Stylized Fact 2: Initial aggregate outcomes *(of group)* matter for current expectations of job-finding

Want a model that can explain why average beliefs are not equal to fundamentals and where initial aggregate outcomes impact current beliefs

Mechanism

- Uncertainty over group fundamentals: labor market prospects for a group
  - By group: individuals who are similar to each other, e.g., same gender, cohort, etc.
- Participation is costly
- Social learning: learn by observing the actions of others similar to you
- Degree of participation affects informativeness of signal

Mechanism

- Uncertainty over group fundamentals: labor market prospects for a group
- Participation is costly
    - $\implies$ experimentation is costly
- Social learning: learn by observing the actions of others similar to you
- Degree of participation affects informativeness of signal

Mechanism

- Uncertainty over group fundamentals: labor market prospects for a group
- Participation is costly
- Social learning: learn by observing the actions of others similar to you
  - Learn by looking at noisy public signals (such as share who participates)
- Degree of participation affects informativeness of signal

Mechanism

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Mechanism

- Uncertainty over group fundamentals: labor market prospects for a group
- Participation is costly
- Social learning: learn by observing the actions of others similar to you
- Degree of participation affects informativeness of signal
- Noisier signals $\implies$ slows learning and entrenchment of beliefs.

Rationalizing the stylized facts

- Learning about group fundamental from endogenous public signals: $\implies$ same common signal, average beliefs can deviate from average realization

  - Learning about individual types from exogenous or endogenous private signals $\implies$ economy-wide average belief= average realization ▸ indtype_endo ▸ indtype_exo

- Initial endogenous labor market outcomes affect info. quality of signals, and rate of learning, thus impacting current beliefs

- This paper: jointly rationalizes optimistic beliefs and initial conditions mattering for current beliefs

▸ privateendo

Rationalizing the stylized facts

- Learning about group fundamental from endogenous public signals: $\implies$ same common signal, average beliefs can deviate from average realization

- Initial endogenous labor market outcomes affect info. quality of signals, and rate of learning, thus impacting current beliefs

  - Subjective beliefs in form of constant gain learning (Branch and Evans (2006)): initial conditions matter less and less for current beliefs subjective

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privateendo

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Roadmap

- Empirical findings (Brief)
  - Fact 1 well-documented, 8 pp gap (Mueller et al 2021), 6 pp gap (this paper) fact1
  - Focus more on fact 2
- PE model of learning from endogenous public signals
- Quantitative analysis
  - Model explains 1/4 to 1/3 of empirical gap + persistence of beliefs
- Extensions (in paper): 1) learning from other public signals, 2) learning with private outcomes and endogenous public signals

Data

Key question in SCE

"... what do you think is the percent chance that within the coming 3 months, you will find a job that you will accept? "

Stylized Fact 2: Initial outcomes matter for expectations

- Regress expected job-finding on local initial participation (LFPR). Local=state. Proxy for initial entry with age 18 if < college, age 22 if >=college. nepop

	Expected Job Finding			
	(1)	(2)	(3)	(4)
Local current LFPR	-0.24	-0.33	-0.08	0.01
	(0.24)	(0.26)	(0.31)	(0.56)
Local initial LFPR	0.46*	0.27	-0.16	0.16
	(0.23)	(0.27)	(0.33)	(0.47)
Local initial LFPR (18-24)	-	0.27***	0.32***	0.28**
	-	(0.10)	(0.10)	(0.14)
Local current u	-	-	-0.55	-0.22
	-	-	(0.41)	(0.41)
Aggregate u (25-54yrs)	-2.36***	-2.18***	-1.54**	-1.82***
	(0.30)	(0.31)	(0.51)	(0.54)
Controls	Yes	Yes	Yes	Yes
State Fixed Effects	No	No	No	Yes
Observations	50,581	40,023	40,023	40,023

Environment

- Economy populated with unit mass of cohorts
- Each cohort has measure 1 of individuals who live T periods
- Denote the age of a cohort by τ
- Time is discrete, agents are risk-neutral and discount future with factor β

Environment

- Individual can be employed or non-employed
- Employed earn a wage w
- Non-employed consume home production b
- Non-employed can choose to search and be *unemployed* or stay out-of-the-labor-force (OLF)
- Incur flow cost c if participate and search for a job

Environment

- All agents know aggregate productivity a_t which evolves according to:

$$a_t = \rho a_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- Group fundamental z is iid draw from distribution $\Pi(z)$ at time cohort enters

- Both a_t and z affect job-finding rate rates: $f(a_t, z)$: $f_a(\cdot), f_z(\cdot) > 0$

- Individuals do **not** know their group fundamental z and have to learn about it

Information

- At entry ($\tau = 0$), agents endowed with exogenous private signal once and for all:

$$s_i = z + \epsilon_i \quad \text{where } \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

- Agents observe an endogenous public signal of z at end of each period:

  - Noisy signal of participation rate of individuals aged τ at date t: $\widehat{\ell}_t(\tau)$

$$\widehat{\ell}_t(\tau) = \ell_t(\tau) + \xi_t(\tau) \quad \text{where } \forall \tau, \xi_t(\tau) \sim \mathcal{N}(0, \sigma_\xi^2)$$

Public beliefs

- Public info at start of t relevant to age τ: $\mathcal{I}_{t-1}(\tau) = \{\widehat{\ell}_{t-1}(\tau - 1), \ldots, \widehat{\ell}_1(1)\}$

- Public belief at start of t (end of $t - 1$) for age τ: $h_{t-1}(z, \tau)$.

  - Public prior = belief that outsider forms if only have access to $\mathcal{I}_{t-1}(\tau)$.

- Individuals know $\mathcal{I}_{t-1}(\tau)$ and their own $s_i \implies$ private beliefs $h_{it}(z, \tau)$

- s_i drawn once, $\mathcal{I}_t(\tau)$ evolving, $h_{it}(z, \tau)$ recovered from $h_{t-1}(z, \tau)$

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Timing

- Aggregate productivity a_t realized and separation shocks occur with probability δ
- Non-employed given private beliefs $h_{it}(z)$ choose whether to participate
- Search and matching occurs
- Public signal, $\widehat{\ell}_t$, observed
- Update beliefs

Values

If z is known:

- Value of non-employment of age τ, fundamental z, and aggregate productivity a_t:

$$V^N(a_t, z, \tau) = \max\left\{V^O(a_t, z, \tau), V^U(a_t, z, \tau) - c\right\}$$

- Value of OLF:

$$V^O(a_t, z, \tau) = b + \beta \mathbb{E}_a V^N(a_{t+1}, z, \tau + 1)$$

- Value of unemployment

$$V^U(a_t, z, \tau) = f(a_t, z)\widetilde{V}^W(a_t, z, \tau) + [1 - f(a_t, z)] V^O(a_t, z, \tau)$$

where

$$\widetilde{V}^W(a_t, z, \tau) = w + \beta \mathbb{E}_a V^W(a_{t+1}, z, \tau + 1)$$

- Value of employment

$$V^W(a_t, z, \tau) = [1 - \delta]\widetilde{V}^W(a_t, z, \tau) + \delta V^O(a_t, z, \tau)$$

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Individual's choice problem

But z is **not** known:

- Each individual i makes choice of whether to participate based on her **beliefs**:

$$\max\left\{\mathbb{E}_{it}V^{O}\left(a_t,z,\tau\right),\mathbb{E}_{it}V^{U}\left(a_t,z,\tau\right)-c\right\}$$

- Using Bayes rule and given s_i, back out private beliefs from public beliefs as:

$$h_{it}\left(z,\tau\right)=\frac{h_{t-1}\left(z,\tau\right)\varphi\left(s_i\mid z\right)}{\int h_{t-1}\left(\widetilde{z},\tau\right)\varphi\left(s_i\mid\widetilde{z}\right)d\widetilde{z}}\qquad\text{where }\varphi\left(s_i|z\right)=\phi\left(\frac{s_i-z}{\sigma_\epsilon}\right.$$

- Exists $s_t^*(\tau)$ such that:

$$\int\left[V^{U}\left(a_t,z,\tau\right)-V^{O}\left(a_t,z,\tau\right)-c\right]\frac{h_{t-1}\left(z,\tau\right)\varphi\left(s_t^*\left[\tau\right]\mid z\right)}{\int h_{t-1}\left(\widetilde{z},\tau\right)\varphi\left(s_t^*\left[\tau\right]\mid\widetilde{z}\right)d\widetilde{z}}dz$$

Individual's choice problem

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- Using Bayes rule and given s_i, back out private beliefs from public beliefs as:

$$h_{it}(z,\tau) = \frac{h_{t-1}(z,\tau)\,\varphi(s_i \mid z)}{\int h_{t-1}(\widetilde{z},\tau)\,\varphi(s_i \mid \widetilde{z})\,d\widetilde{z}} \qquad \text{where } \varphi(s_i|z) = \phi\left(\frac{s_i - z}{\sigma_\epsilon}\right.$$

- Individual participates so long as:

$$\int \left[\underbrace{V^U(a_t,z,\tau) - c}_{\text{net value of search}} - V^O(a_t,z,\tau)\right] h_{it}(z,\tau)\,dz \geq 0$$

- Exists $s_t^*(\tau)$ such that:

$$\int \left[V^U(a_t,z,\tau) - V^O(a_t,z,\tau) - c\right] \frac{h_{t-1}(z,\tau)\,\varphi(s_t^*[\tau] \mid z)}{\int h_{t-1}(\widetilde{z},\tau)\,\varphi(s_t^*[\tau] \mid \widetilde{z})\,d\widetilde{z}}\,dz$$

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Actual outcomes vs. signals

- All individuals with $s_i \geq s_t^*(\tau)$ participate, share of non-employed who participate:

$$p_t(\tau) = 1 - \Phi\left(\frac{s_t^*(\tau) - z}{\sigma_\epsilon}\right)$$

- and true non-employment rate is:

$$n_t(\tau) = [1 - f(a_t, z)\, p_t(\tau)]\, n_{t-1}(\tau - 1) + \delta\, [1 - n_{t-1}(\tau - 1)]$$

- and actual labor force participation rate at end of period:

$$\ell_t(\tau) = 1 - n_{t-1}(\tau - 1)\,[1 - p_t(\tau)] = 1 - m_t(\tau) n_{t-1}(\tau - 1)$$

where $m_t(\tau)$ is share of non-employed at end of $t - 1$ who didn't participate in t.

Actual outcomes vs. signals

- But individuals only observe noisy signal of $\ell_t(\tau)$

$$\widehat{\ell}_t(\tau) = \ell_t(\tau) + \xi_t(\tau)$$

Updating

- Agents know structure of model and public belief $\Longrightarrow$ can compute $s_t^*(\tau)$.

- For any z, individuals can compute counterfactual $p_t(\tau; z)$, $n_t(\tau; z) \Longrightarrow \ell_t(\tau; z)$:

$$\ell_t(\tau; z) = 1 - n_{t-1}(\tau - 1; z)\left[1 - p_t(\tau; z)\right]$$

Updating

- If fundamental $= z$, observing $\widehat{\ell}_t(\tau) \implies$ noise of magnitude $\widehat{\ell}_t(\tau) - \ell_t(\tau; z)$

- Given noisy signals, posterior public belief becomes:

$$h_t(z, \tau + 1) = \frac{h_{t-1}(z, \tau) \overbrace{\phi\left(\frac{\widehat{\ell}_t(\tau) - \ell_t(\tau; z)}{\sigma_\xi}\right)}^{\textbf{probability density of observing noise of this magnitu}}}{\int h_{t-1}(\widetilde{z}, \tau)\, \phi\left(\frac{\widehat{\ell}_t(\tau) - \ell_t(\tau; \widetilde{z})}{\sigma_\xi}\right) d\widetilde{z}}$$

Informativeness of signal varies in aggregate action

Initial conditions affects s^* and thus, participation,
Participation matters for informativeness of endogenous public signal

A simple example

- 1-shot model. $n_0 = 1, \ell_1 = 1 - n_0 (1 - p_1) = p_1$

- Further assume $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$

- Participate if $s_i \geq s^* \implies$:

$$p_1 = 1 - \Phi\left(\frac{s^* - z}{\sigma_\epsilon}\right)$$

- Denote $m_1 = 1 - p_1$, i.e., measure of non-participation

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A simple example

- Take linear approximation and suppose $\widetilde{m}_1$ is a noisy signal of m_1

$$\widetilde{m}_1 = \Phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) + \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)(z - \mu_z) + \xi \qquad \text{where } \xi \sim \mathcal{N}(0, \sigma$$

- Since s^*,μ_z and σ_ϵ^2 known, $\widetilde{m}_1$ informationally equivalent to $\widehat{m}_1$

$$\widehat{m}_1 = \widetilde{m}_1 - \Phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) + \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right)\mu_z = \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) z + \xi$$

- Noisy signal $\widehat{m}_1$:

$$\widehat{m}_1 = \phi\left(\frac{s^* - \mu_z}{\sigma_\epsilon}\right) z + \xi$$

- Signal-to-noise ratio given by:

$$\left[\phi\left(\frac{s_1^* - \mu_z}{\sigma_\epsilon}\right)\right]^2 \frac{\sigma_z^2}{\sigma_\xi^2}$$

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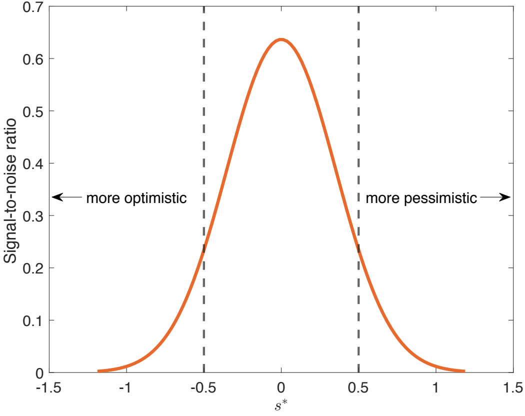
Signal-to-noise ratio non-monotonically changing with s^*



Calibration

- Calibrate to deterministic steady state
- Period is a quarter, cohort lives $T = 180$ quarters, perpetual youth
- Set parameters: $\beta = 0.99$, $w = 1$, $b = 0.4$, $\delta = 0.076$
- z drawn from Beta distribution $z \sim Beta(A_z, B_z)$
- Noise terms normally distributed:$\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$, $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$
- Job finding $f(a, z) = \exp(-(1 - a - z))$

Calibration

- Parameters to calibrate: $\{A_z, B_z, \sigma_\epsilon, \sigma_\xi, c\}$

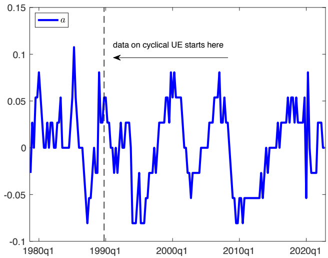
Parameter	Description	Value	Target	Model	Data
A_z	Beta dist parameter	6.07	Mean job-finding rate, f	0.499	0.490
B_z	Beta dist parameter	14.37	Std dev. job-finding rate, f	0.051	0.052
c	Participation cost	3.27	Prime-age participation	0.818	0.820
σ_ϵ	Dispersion in ϵ	0.36	Std dev. perceived f, 18-24	0.056	0.060
σ_ξ	Dispersion in ξ	0.14	Std dev. perceived f, 25-54	0.043	0.046

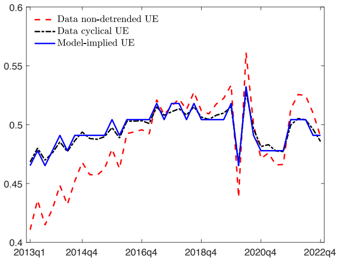
Notes: Dispersion in perceived job-finding rates for the relevant age group is computed as the standard deviation in predicted perceived job-finding rates after controlling for aggregate fluctuations

Thought Experiment

- Simulate model with same number of cohorts as SCE data (40)

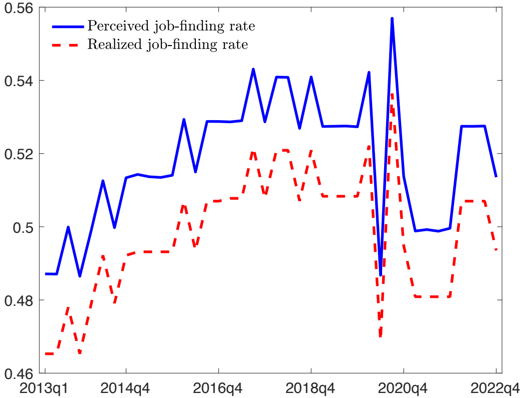
- Aggregate shocks = cyclical component of empirical job-finding rates





Model predictions

- Average perceived job-finding rate about 2 percentage points higher than realized, ≈ 1/3 (6pp) to 1/4 of gap (8pp)



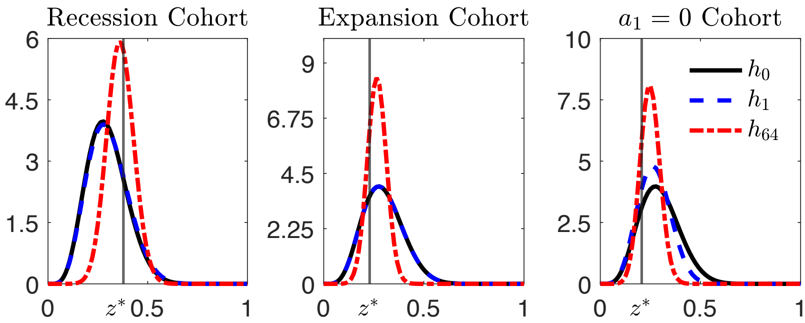
Model Predictions

- Run same regression in model as in data
- Key independent variable: initial labor force participation rate of cohort (1st year)

	Expected Job Finding	
	(1)	(2)
Initial LFPR$_c$	0.066***	0.237***
	(0.000)	(0.001)
Initial LFPR$_c^2$		$-$0.176***
		(0.001)
a_t	0.689***	0.705***
	(0.003)	(0.003)
Observations	1,200,000	1,200,000
R^2	0.119	0.140

Initial conditions matter

- Recession cohort (2009): p_1 extremely low, h_1 little unchanged from h_0
- Expansion cohort (2000): p_1 extremely high, h_1 also little unchanged
- $a_1 = 0$ cohort (1998): p_1 moderate, public signal more informative, faster learning



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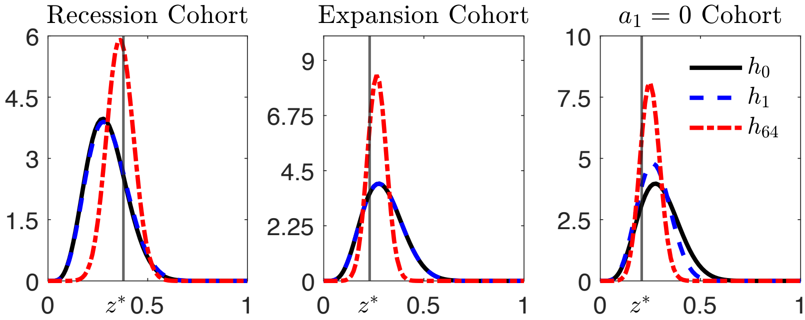

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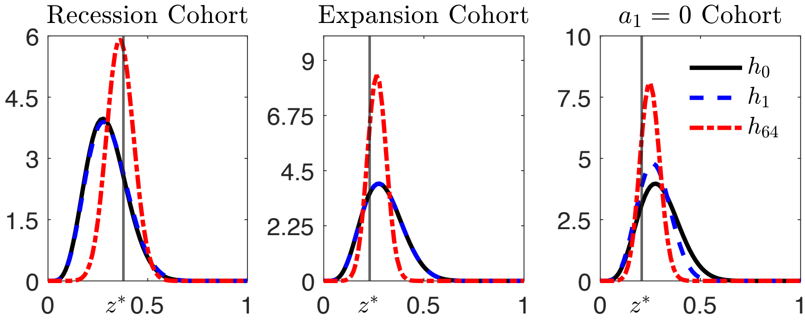
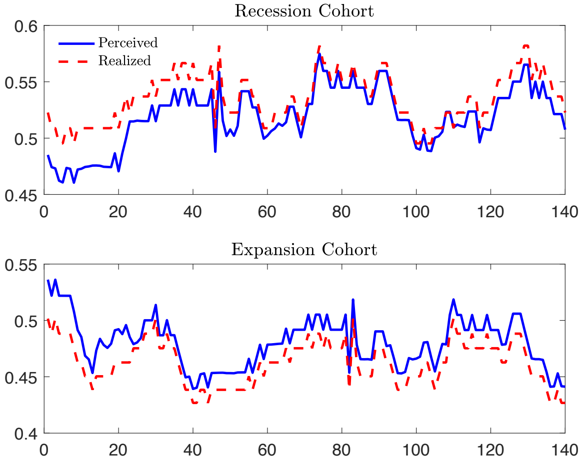


Entrenched beliefs

◦ Persistently pessimistic (optimistic) beliefs for deep recession (expansion) cohorts



Overview

- PE model to rationalize how initial group labor market outcomes can impact learning and entrench beliefs
- Optimism and pessimism can affect informativeness of signal
- Model predicts optimistic 1) job-finding beliefs and 2) initial outcomes weigh on current beliefs, making them persistently optimistic, pessimistic

Conclusion

- Two ways to approach quantitative macro:
  - Seek permission: look for empirical regularities and write down model to try and explain them.
  - Ask forgiveness: write down model and then look for empirical regularities consistent with equilibrium.
- Both are valid ways to approach quantitative macro, and both can involve sunk costs.
- Final due date for full project? Sometime around Dec 12th.

Learning from private endogenous signals

- Suppose true job-finding rate p^* drawn
- All individuals start off with prior beliefs $f_0(p; p^*)$ such that:

$$\int p f_0(p; p^*) dp = p^*$$

- Suppose individuals search. Let $s = 1$ if individual finds job, $s = 0$ otherwise.
- Average beliefs equal fundamental

back

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$$f_1(p \mid s=1;p^*) = \frac{p f_0(p;p^*)}{p^*}$$

and

$$f_1(p \mid s=0;p^*) = \frac{(1-p) f_0(p;p^*)}{1-p^*}$$

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$$\begin{aligned}\int p f_1(p)\,dp &= \int p\left[f_1(p \mid s=1)\,p^* + f_1(p \mid s=0)\,(1-p^*)\right]dp \\ &= \int p\left[p f_0(p;p^*) + (1-p)\,f_0(p;p^*)\right]dp \\ &= p^*\end{aligned}$$

back

Learning from private outcomes

- 2nd period, suppose employed don't search anymore, and fraction m non-employed also don't search

- Posterior density , non-participants:

- Average beliefs equal fundamental back

Learning from private outcomes

- 2nd period, suppose employed don't search anymore, and fraction m non-employed also don't search

- Posterior density , non-participants:

$$f_2(p \mid s_1 = j; p^*) = f_1(p \mid s_1 = j; p^*) \quad \text{for } j \in \{0, 1\}$$

and unemployed who searched and found job in period 2

$$f_2\left(p \mid s_1 = 0, s_2 = 1; p^*\right) = \frac{p f_1(p \mid s_1 = 0; p^*)}{p^*} = \frac{p\,(1-p)\,f_0\,(p; p^*)}{p^*\,(1-p^*)}$$

and unemployed who searched but didn't find job in period 2

$$f_2\left(p \mid s_1 = 0, s_2 = 0; p^*\right) = \frac{(1-p) f_1(p \mid s_1 = 0; p^*)}{(1-p^*)} = \frac{(1-p)^2\, f\,(p}{(1-p^*)^2}$$

- Average beliefs equal fundamental back

Learning from private outcomes

- 2nd period, suppose employed don't search anymore, and fraction m non-employed also don't search
- Posterior density , non-participants:
- Average beliefs equal fundamental Suppressing dependence on p^* in f:

$$\begin{aligned}\int pf_2(p)\,dp &= \int p\left\{f_2(p \mid s_1 = 1)\,p^* + mf_2(p \mid s_1 = 0)(1-p^*)\right.\\ &+ (1-m)\int pf_2(p \mid s_1 = 0, s_2 = 1)\,p^*(1-p^*)dp\\ &+ (1-m)\int pf_2(p \mid s_1 = 0, s_2 = 0)(1-p^*)^2 dp\\ &= \int p\left\{p + (1-m)(1-p) + m[1-p]\right\} f(p)\,dp\\ &= p^*\end{aligned}$$

back

Learning about individual type, exogenous signals

- Measure 1 population. Each individual's type z is iid draw from distribution $N(\mu_z, 1/\rho_z)$ where $\rho_z = 1/\sigma_z^2$.
- Assume job-finding, p is function of z. E.g. $p(z) = \frac{\exp(z)}{1+\exp(z)}$
- Every period, noisy signal of z

$$s_{it} = z_i + \epsilon_{it} \quad \text{where} \quad \epsilon_{it} \sim N(0, 1/\rho_\epsilon)$$

- End of period 1, individual's posterior precision given by:

$$\rho' = \rho_\epsilon + \rho_z$$

posterior mean:

$$\mu_i'(s \mid z) = \frac{\rho_\epsilon}{\rho'} s + \left(1 - \frac{\rho_\epsilon}{\rho'}\right) \mu_z$$

Learning about individual type, exogenous signals

- Denote $\alpha = \frac{\rho_\epsilon}{\rho'}$. Integrate across s and z to get average belief in economy:

$$\int \int \{\alpha s + (1-\alpha)\,\mu_z\} \overbrace{\phi\left(\frac{s-z}{\sigma_\epsilon}\right)}^{\text{conditional on } z\text{, pdf of drawing } s} ds \underbrace{\phi\left(\frac{z-\mu_z}{\sigma_z}\right)}_{\text{measure drawing}}$$

- μ_z is also average realization in economy.

- Thus, average belief = average realization back

Learning about individual type, endogenous signals

- Measure 1 population. Suppose each individual draw job-finding rate from $G(p)$
- $g(p^*)$ measure draw $p = p^*$
- Require

$$\int p^* g(p^*) dp = \overline{p}$$

where $\overline{p}$ = mean of $G(p)$.

back

Learning about individual type, endogenous signals

- Suppose each individual has initial unbiased beliefs $f_0(p) = g(p)$.
- Let $s = 1$ be event find job, $s = 0$ event do not find job
- Given true individual job-finding rate p^*, posterior:

$$f_1(p \mid s = 1; p^*) = \frac{p f_0(p)}{p^*}$$

$$f_1(p \mid s = 0; p^*) = \frac{(1 - p) f_0(p)}{1 - p^*}$$

Learning about individual type, endogenous signals

- Average economy-wide belief:

$$\int \left\{ \int p f_1(p; p^*) dp \right\} g(p^*) dp^*$$

- Plugging in posterior beliefs, inner integral is:

$$\int p \overbrace{\left[f_1(p \mid s = 1; p^*) p^* + f_1(p \mid s = 0; p^*)(1 - p^*) \right]}^{pg(p)+(1-p)g(p)} dp = \overline{p}$$

- So average economy wide beliefs:

$$\int \left\{ \int p f_1(p; p^*) dp \right\} g(p^*) dp^* = \int \overline{p} g(p^*) dp^* = \overline{p}$$

Average belief = fundamental

back

Constant gain learning

- Literature has looked at subjective beliefs, particularly constant-gain learning because best fit survey expectations (Branch and Evans (2006))
- Individuals have true z^*, start with some prior z_0 at date 0.
- Every period, individuals observe a noisy signal of z^*, $\nu_t = z^* + \epsilon_t$, ϵ_t is iid noise term.
- Learning rule:

$$z_t = z_{t-1} + \gamma(\nu_t - z_{t-1}) = (1-\gamma)z_{t-1} + \gamma\nu_t$$

  where γ is learning parameter on "surprise" $(\nu_t - z_{t-1})$
- Iterate forward:

$$z_t = (1-\gamma)^t z_0 + \gamma \sum_{j=1}^{t} (1-\gamma)^{t-j} \nu_j$$

- Past matters less and less for current beliefs

back

Comparison with Mueller et al (2021)

2013-2022 SCE and CPS results on prime-age workers		
	Perceived	Realized
SCE results on 20-65 yrs (2013-2019m6)		
Mueller et al (2021)		
Unemployed, All durations	0.49	0.41
Unemployed, < 27 weeks	0.59	0.59
SCE Public-use		
Unemployed, All durations	0.49	0.39
Unemployed, < 27 weeks	0.57	0.53

back

Stylized Fact 1: Average beliefs are optimistic

- 8pp gap (all), 6pp gap (equal weight employed and unemployed) ▸ mueller ▸ roadmap

2013-2022 SCE and CPS results on prime-age workers		
	Perceived	Realized
All		
SCE	0.57	-
CPS	-	0.49
Unemployed		
SCE	0.54	0.45
Employed		
SCE	0.57	-
All durations, CPS	-	0.49
$<$ 27wks, CPS	-	0.54
Mueller et al (2021) results on 20-65 yrs (2013-2019)		
Unemployed	0.49	0.41

Initial non-employment rates of group also affect expectations

[◦] Regress expected job-finding on initial non-employment rate

	Expected Job Finding			
	(1)	(2)	(3)	(4)
Local current NE/POP	0.09	0.02	0.09	−0.02
	(0.20)	(0.28)	(0.33)	(0.60)
Local initial NE/POP	−0.31	0.12	0.17	−0.19
	(0.19)	(0.28)	(0.34)	(0.49)
Local initial NE/POP (18-24)	-	-0.30^{***}	-0.36^{***}	-0.32^{*}
	-	(0.08)	(0.12)	(0.16)
Local current u	-	-	-0.60^{**}	−0.22
	-	-	(0.41)	(0.40)
Aggregate u (25-54yrs)	-2.38^{***}	-2.02^{***}	-1.55^{***}	-1.81^{***}
	(0.35)	(0.39)	(0.51)	(0.55)
Controls	Yes	Yes	Yes	Yes
State Fixed Effects	No	No	No	Yes
Observations	50,581	40,023	40,023	40,023

back

Extensions

Extension: learning with private outcomes and public signals

- Private signal s_i at date 0, observe noisy public signal and own outcome at every t
  - Both private and public information evolving over time
- Need to keep track of *distribution* of distribution of beliefs
- Simplification: binary $z \in \{z_H, z_L\}$ where $z_H > z_L$, $h_{it} =$ probability z_H
  - Individual's belief summarized by 1 variable: h_{it}. Track distribution of h
- By construction, mean belief lies between $[z_L, z_H]$, focus on persistence of beliefs

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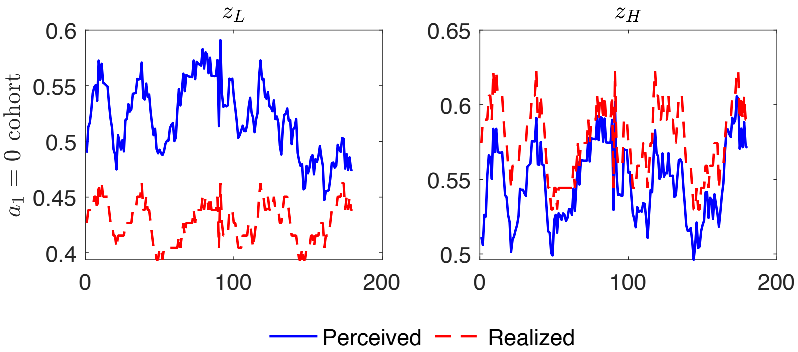
Extension: learning with private outcomes and public signals

- Persistence in beliefs even when allow learning from private outcomes



- Gap larger for z_L. Although same threshold belief $\widetilde{h}_1^*$ in period 1, p_1 lower since s_i is unbiased noisy signal around true z $\Rightarrow$ less informative public signal