Instructor: *Professor Griffy*
Due: *Mar., 6th 2025*
AECO 701

Problem Set 3

Problem 1: The Lucas Critique and Rational Expectations) Suppose that a household faces the following problem:

$$\max_{c_1,c_2} u(c_1) + \beta u(c_2) \tag{1}$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r} + T_1 \tag{2}$$

where $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, and (for now), w_1, w_2, and T_1 are constants. Assume that the interest rate, r, is fixed throughout.

a) Solve for the household's first order conditions and derive the expressions for consumption c_1 and c_2.

b) Assume that workers have log-utility $u(c) = ln(c)$, and that the parameters take on the following values: $r = 0.04$, $\beta = \frac{1}{1+r}$, $T_1 = 0.0$. Now assume that w_1 and w_2 are independently and identically distributed according to $w_t \sim LN(0, 1)$. Complete the following

  (a) Formulate the problem using the appropriate expectations operator and budget constraints, and then derive expressions for c_1 and c_2.

  (b) Simulate the model using the decision rules you found in part (a) and the wage distributions above and plot c_1 for the 10th, 30th, 50th, 70th, and 90th percentiles of the lifetime income ($w_1 + \frac{w}{}21 + r$) distribution.

  (c) Calculate the slope of the consumption function with respect to w_1.

c) Now suppose that a policymaker would like to estimate the impact of wages on consumption for future use in stimulating the economy. Run the following regression on your simulated data: $c_t = \gamma w_t + \epsilon_t$. How does γ compare to the slope of the consumption function?

d) Now suppose that wages are an AR(1) process. Simulate your model again, but use the following for wages: $w_1 \sim LN(0, 1)$, $ln(w_2) = \rho ln(w_1) + \epsilon_2$, where $\rho = 0.95$ and $\epsilon \sim N(0, 1)$.

  (a) Now, draw a random sample of log-normal wages for period 1. Then draw an equal-sized sample of normally-distributed errors for period 2 and calculate wages in period 2 as $ln(w_2) = \rho ln(w_1) + \epsilon_2$. Now plot c_1 for the 10th, 30th, 50th, 70th, and 90th percentiles of the lifetime income ($w_1 + \frac{w}{}21 + r$) distribution.

  (b) Calculate the slope of the consumption function with respect to w_1.

e) Now suppose that a policymaker would like to use a reduced-form estimate to stimulate the economy. Run the following regression on your simulated data: $c_t = \gamma w_t + \epsilon_t$. Now Suppose that the policymaker implements a one-time payment of $T_1 = 0.5$, which they see as equivalent to w_1. Simulate the model with this transfer and compare the impact on consumption to the estimate $\hat{\gamma}$.