Macro II

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Introduction

- Today: consumption smoothing and permanent income.
- "The income fluctuation problem"
- HW2 due Thursday.
- HW3 posted.

Feb. 25th and 27th

- I will be in Japan. Time difference means class would be 11:30pm to 1:00am for me.
- Will post the slides for lecture 11 (stochastic neoclassical growth) and teach lecture 10 (market structure) on 3/4.
- Please review the lecture 11 slides, and focus on stochastic component.

Thinking about Uncertainty in Macroeconomic Models

- Uncertainty makes macroeconomic models more difficult to solve.
- We make assumptions about the environment (preferences, technology, etc.) to decrease complexity of problem.
- Euler Equation:

$$u'(c_t) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(c_{t+1})}_{Non-linear}] \tag{1}$$

- Each agent chooses consumption and savings based on a
  1. general equilibrium object (given by the decision rules of all other agents)
  2. (potentially highly) non-linear marginal utility.

Today

- Today: Think about how workers insure against income risk.
- Foundation for consumption smoothing.
- Explore using different preferences:
  1. Certainty Equivalence - Quadratic Utility.
  2. Constant Absolute Risk Aversion - Exponential Utility.
- These each imply different ways in which agents respond to income shocks and uncertainty.
- We will return to this when we study heterogeneous agents.

Risk

- How do we typically think about risk in economic models?
- Absolute Risk Aversion:

$$AR = -\frac{u''(c)}{u'(c)} \tag{2}$$

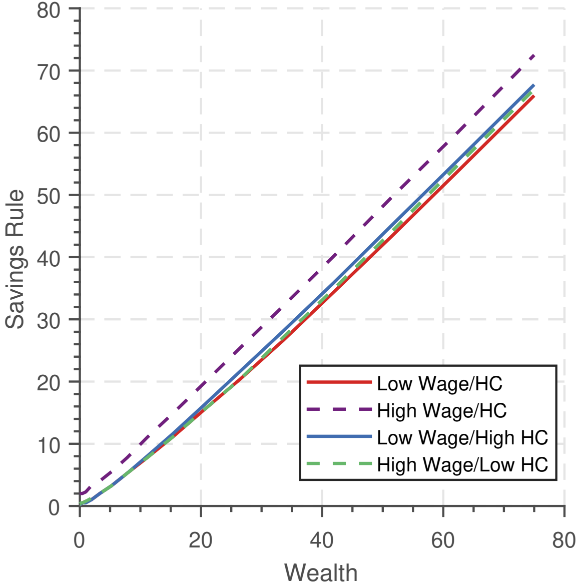
- A measure of the agent's risk aversion unconditional upon their level of wealth.
- Relative Risk Aversion:

$$RRA = -\frac{u''(c)c}{u'(c)} \tag{3}$$

- Conditioning upon an agent's wealth, how does his risk aversion change?
- Probably most reasonable are "DARA" "CRRA"
- These will have different implications for savings and consumption.

When approximations work

- For a lot of the distribution, decision rules are linear:



Introduction

- In the case of quadratic utility, we will see that agents don't change their consumption choices when faced with shocks.
- Uncertainty still decreases expected utility, but does not change choices.
- Why is this relevant? One solution technique (LQ) assumes that agents have a quadratic utility function (locally risk-neutral).
- We will see that this is sometimes not a great assumption.

Quadratic Utility

- Utility is given by the following:

$$\max E[\sum_{t=0}^{\infty} \beta^t (aC_t - bC_t^2)] \tag{4}$$

$$\begin{aligned} \text{s.t. } A_{t+1} &= (1+r)A_t + Y_t - C_t && (5) \\ Y_{t+1} &= \rho Y_t + \epsilon_{t+1} && (6) \end{aligned}$$

Euler Equation

- Do the usual steps to find the Euler Equation:

$$V(A) = \max_{C,A'} aC_t - bC_t^2 + \beta E[V(A')] \tag{7}$$

$$\text{s.t. } A' = (1+r)A + Y - C \tag{8}$$

$$Y' = \rho Y + \epsilon' \tag{9}$$

$$\frac{\partial V}{\partial C} = a - 2bC - \lambda \tag{10}$$

$$\frac{\partial V}{\partial A'} = -\lambda + \beta E[\frac{\partial V}{\partial A'}] \tag{11}$$

$$\frac{\partial V}{\partial A} = (1+r)\lambda \tag{12}$$

$$\Rightarrow C = \beta(1+r)E[C'] \tag{13}$$

Certainty Equivalence

- Suppose that $\beta = (1 + r)$:

$$C = E[C'] \tag{14}$$

- Suppose that there were two states of the world: high and low.

$$C = P_h C_h + P_l C_l \tag{15}$$

- This is equivalent to an agent receiving the mean income between both states:

$$C = C_m \tag{16}$$

- i.e., workers make savings decisions *as though they are receiving the average consumption with certainty*.

Prudence

- Agents in this economy are not "prudential."
- That is, they don't change their choices based upon uncertainty about the future.
- Another way to express this is in the third derivative of the utility function:

$$U''' = 0 \tag{17}$$

- This captures the response of marginal utility (i.e., decisions) to uncertainty.
- Marginal utility changes linearly, so any convex combination is equal to the expected value.

Random Walk

- Can show for the AR(1) case:

$$C_t - C_{t-1} = \frac{r}{1 + r - \rho}\epsilon \tag{18}$$

- Now, consider the case in which income shocks are iid:

$$Y_{t+1} = Y_t + \epsilon_{t+1} \tag{19}$$

- Then the difference in consumption becomes:

$$C_t - C_{t-1} = \epsilon_t \tag{20}$$

- In other words, the agent consumes all of the shock in each period (will also happen with CRRA and autarky).

- This is a martingale!

Conclusion

- In the quadratic utility world, uncertainty does not change an agents decision when compared with an identical income stream.
- In the case of CARA utility, we will see that agents have precautionary savings that result from curvature in the utility function.
- The choices are the same as they would be under complete markets.

Introduction to CARA World

- Now, use CARA preferences to think about world in which certainty equivalence does not hold.

- Now, we will allow agents to be prudential in their savings response to future uncertainty.

Constant Absolute Risk Aversion Utility

- The maximization problem is given by

$$\max E[\sum_{t=0}^{\infty} -\frac{1}{\alpha}\exp(-\alpha C_t)] \tag{21}$$

$$\begin{aligned} \text{s.t. } A_{t+1} &= A_t + Y_t - C_t && (22)\\ Y_t &= \rho Y_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma^2) && (23) \end{aligned}$$

- Key difference: first derivative (i.e., policy functions), no longer linear.

Euler Equation

- Bellman Equation (implicitly assume $\beta = \frac{1}{1+r}$):

$$V(A) = \max_{C,A'} -(\frac{1}{\alpha}) \exp(-\alpha C) + E[V(A')] \quad (24)$$

$$\begin{aligned} \text{s.t. } A' &= A + Y - C \quad (25) \\ Y' &= \rho Y + \epsilon' \quad (26) \end{aligned}$$

$$\frac{\partial V}{\partial C} = \exp(-\alpha C) - \lambda \quad (27)$$

$$\frac{\partial V}{\partial A'} = -\lambda + E[\frac{\partial V}{\partial A'}] \quad (28)$$

$$\frac{\partial V}{\partial A} = \lambda \quad (29)$$

$$\Rightarrow \exp(-\alpha C) = E[\exp(-\alpha C')] \quad (30)$$

Euler Equation

- Bellman Equation (implicitly assume $\beta = (1 + r)$):

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \tag{31}$$

- For normally distributed random variables, the following holds:

$$E[exp(x)] = exp(E[x] + \sigma_x^2/2) \tag{32}$$

- Thus, we can rewrite the Euler Equation as

$$\exp(-\alpha C) = E(exp(-\alpha C' + \alpha^2\sigma^2/2)) \tag{33}$$

$$\Rightarrow C' = C + \frac{\alpha\sigma^2}{2} + \nu \tag{34}$$

Policy Function

- Policy function:

$$\Rightarrow C' = C + \frac{\alpha\sigma^2}{2} + \nu \tag{35}$$

- This says that consumption is *increasing* ex-ante in response to uncertainty, measured by σ^2.

- What does this mean about life-cycle consumption?

- We would expect it to be upward-sloping, at least initially.

Consumption in time t

- Can show:

$$C_t = (\frac{1}{T-t})A_t + Y_t - \frac{\alpha(T-t-1)\sigma^2}{4} \tag{36}$$

- Certainty equivalence: last term is equal to zero. i.e., cake-eating problem.

- Agents consume less than they would if their income stream was certain!

Prudence

- What is different in this case?
- Agents are prudential: $U''' > 0$.
- The Euler Equation is given by:

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \tag{37}$$

- Suppose $C = C'$, then consider Jensen's Inequality:

$$exp(-\alpha E(C)) < E[\exp(-\alpha C)] \tag{38}$$

- This needs to hold in equilibrium, thus agents must decrease current consumption.
- Agents save in excess of what they would under certainty!

CARA Utility

- When CARA agents cannot perfectly insure, they change their choices from the certainty equivalence (quadratic utility) case.
- Unfortunately, CARA has some problems: Marginal utility is finite when consumption is equal to zero.
- CRRA utility will solve this problem, but is more challenging to solve.

Permanent Income Hypothesis

- Theory developed by Milton Friedman that describes how agents allocate resources over their lifetime.
- Consumption is based on not just current income, but expectations over future income as well.
- Implies that agents want to consumption smooth, rather than consume out of transitory income shocks.

Pemanent Income

- Lifetime budget, holds for any t:

$$\sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t c_t = A_0 + \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t y_t, \quad \text{(PVBC)}$$

- Define permanent income PDV of average future income

$$y_t^P \equiv \frac{1}{R_J} W_t$$

- Consumption smoothing & BC implies

$$c_t = \frac{1}{R_J} W_t \equiv y_t^P, W_t = \sum_{j=0}^{J} \left(\frac{1}{1+r}\right)^j y_t^P$$

- Therefore, consumption equals permanent income

Overview

- A temporary change in income leads to a permanent change in expected consumption: consumption smoothing extends the effects of income changes over time

- The effect of a change in current income on current consumption depends on its effect on permanent income

- Permanent changes in income have larger consumption effects than temporary changes

Empirical Implications (Friedman (1957))

- Consider the linear projection of consumption on total income

$$\widehat{c}_t = \alpha_1 + \alpha_2 y_t$$

  - For a cross-section of households at a point in time, $\alpha_1 > 0$, and α_2 is much less than 1
  - For a country over time, $\alpha_1 \approx 0$, and α_2 is closer to 1

- Define transitory income

$$y_t^T = y_t - y_t^P.$$

- Suppose $C\left(y_t^T, y_t^P\right) = 0$

Friedman (1957)

- The coefficient α_2 is given by

$$\begin{aligned} \alpha_2 &= \frac{C\left(y_t, c_t\right)}{V\left(y_t\right)} = \frac{C\left(y_t^T + y_t^P, y_t^P\right)}{V\left(y_t^T + y_t^P\right)} \\ &= \frac{V\left(y_t^P\right)}{V\left(y_t^P\right) + V\left(y_t^T\right)}. \end{aligned}$$

- Cross-section data: $V\left(y_t^T\right)$ is large because of wide variance of household transitory income implying small α_2

- Time-series data: $V\left(y_t^T\right)$ is small because transitory income averages out across households in the aggregate implying large α_2 close to one

Hall (1978)

- Consider an alternative equation

$$\widehat{c}_t = \alpha_1 + \alpha_2 c_{t-1} + \gamma x_{t-1},$$

  where x_{t-1} is some other variable

- Recall that under linear quadratic preferences

$$E_t\left(c_{t+1}\right) = \frac{a}{b}\left[1 - \frac{1}{\beta\left(1+r\right)}\right] + \frac{1}{\beta\left(1+r\right)}c_t,$$

  so that $\gamma = 0$. Nothing should predict consumption except lagged consumption

- There is some evidence that $\gamma \neq 0$
- Perhaps permanent income changes over time and the change takes time for agents to realize so that c_{t-1} is not affected, but c_t is

Conclusion

- Next: Cover Asset Pricing and Lucas Tree

- Please let me know if you can't access the cluster!

Appendix: Deriving Lifetime Budget Constraint

- Solve the flow budget constraint forward

$$
\begin{aligned}
A_0 &= \frac{1}{1+r} A_1 - (y_0 - c_0) \\
&= \frac{1}{1+r}\left(\frac{1}{1+r} A_2 - (y_1 - c_1)\right) - (y_0 - c_0) \\
&= -\sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t (y_t - c_t) \\
&\quad + \left(\frac{1}{1+r}\right)^{T+1} A_{T+1},
\end{aligned}
$$

- Impose No-Ponzi condition requiring, $A_{T+1} = 0$, to yield

$$
A_0 = -\sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t (y_t - c_t)
$$

Appendix: Deriving Lifetime Budget Constraint

- Rearrange to derive the present value budget constraint

$$\sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^{t} c_t\left(I_t\right) = A_0 + \sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^{t} y_t\left(I_t\right), \text{ (PVBC)}$$

- Holds for all realized $\{y_t\}$
- Not an expectation
- Right-hand side of (PVBC) is lifetime wealth
- (PVBC) does not imply that the time path of consumption is known in advance

Derivation

- In finite-horizon case with $J \equiv T$, expected present value budget constraint (EPVBC) becomes

$$E_t \left\{ \sum_{j=0}^{J} \left(\frac{1}{1+r} \right)^j c_{t+j} \right\} = W_t. \qquad \text{(EPVBC)}$$

$$W_t \equiv A_t + E_t \left\{ \sum_{j=0}^{J} \left(\frac{1}{1+r} \right)^j y_{t+j} \right\}.$$

- As $J \to \infty$, (EPVBC) follows from (FBC) and (ENPG)

- Equation (EE') and the law of iterated expectations imply that

$$
\begin{aligned}
E_t\left(c_{t+2}\right) &= E_t\left(E_{t+1}\left(c_{t+2}\right)\right) \\
&= E_t\left(c_{t+1}\right) \\
&= c_t,
\end{aligned}
$$

so that

$$
\begin{aligned}
E_t\left\{\sum\nolimits_{j=0}^{J}\left(\frac{1}{1+r}\right)^j c_{t+j}\right\} &= c_t \sum\nolimits_{j=0}^{J}\left(\frac{1}{1+r}\right)^j \\
&\equiv R_J c_t.
\end{aligned}
$$