Macro II

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Spring 2025

Introduction

- Hand back midterms.
- A few people forgot Blackwell's Sufficient Conditions.
- Everyone demonstrated they understood the material.
- Scheduling:
  - No class next Thursday (4/10)
  - I'm hosting a macro conference here that weekend (4/11-4/12).
  - I know first year is busy, but please come if you are interested!

Motivation I

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- Extra, extra bonus points if he's wearing his Red Sox hat.

Motivation II

- Goal of RBC literature: quantitatively understand the macroeconomy.
- Interpret previous phenomena; make predictions about future phenomena.
- To do this, we need a credible way of parametrizing the model:
  1. Historically, macroeconomists have used calibration: just-identified method of moments.
  2. Recently, economists have begun to employ alternative techniques like maximum likelihood.
  3. With knowledge of these more advanced techniques, we might be able to explore more important issues, like identification.
- Here: provide background for maximum likelihood estimation and calibration and compare the results.

Motivation

- Reintroduce Hansen's model:
  - Standard RBC: all fluctuations of hours worked on the *intensive* margin, i.e. average number of hours worked.
  - Data: little fluctuation in average hours worked; lots of fluctuation in whether or not people are working (*extensive* margin).
  - Standard RBC: missed badly on labor fluctuations (Frisch Elasticity, i.e. response of labor to change in wage too low).
  - Solution: Modify model to have extensive margin with high Frisch Elasticity.
  - Now: households pick the *probability* of working, but have to work a set number of hours.
  - This is a *nonconvexity* in that it forces individuals to work either 0 or h hours.

Hansen (1985)

- Neoclassical growth model with labor-leisure lottery.
- A social planner maximize the following:

$$E(\sum_{t=0}^{\infty} \beta^t [ln(C_t) - \gamma H_t] \tag{1}$$

- Subject to the following constraints:

$$Y_t = A_t K_t^{\theta} (\eta^t H_t)^{1-\theta} \tag{2}$$

$$ln(A_t) = (1-\rho) ln(A) + \rho ln(A_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{3}$$

- The goods market clears and capital evolves in a predetermined fashion.
- Here, we assume that per capita labor productivity grows at rate η.

Equilibrium

- First step: detrend appropriate variables by per capita growth to get stationarity: i.e. $y_t = Y_t/\eta^t$.
- The system of equations that characterize the equilibrium are:

$$y_t = a_t k_t^{\theta} h_t^{1-\theta} \tag{4}$$

$$ln(a_t) = (1-\rho)ln(A) + \rho ln(a_{t-1}) + \epsilon_t \tag{5}$$

$$y_t = c_t + i_t \tag{6}$$

$$\eta k_{t+1} = (1-\delta)k_t + i_t \tag{7}$$

- Combine FOC[c] and FOC[h]:

$$\gamma c_t h_t = (1-\theta)y_t \tag{8}$$

- Euler Equation:

$$\frac{\eta}{c_t} = \beta E_t[\frac{1}{c_{t+1}}(\theta(\frac{y_{t+1}}{k_{t+1}}) + 1 - \delta)] \tag{9}$$

Formally

- Calibration is mathematically equivalent to just-identified GMM.
- Select a set of moments that we believe have a "high signal-to-noise" ratio.
- Generally, choose parameter so that steady-state variables match well-known quantities.

$$\Omega(\{X_t^M\}_{t=1}^T) = \Omega(\{X_t\}_{t=1}^T) \tag{10}$$

- Informally, use other implied moments to consider the "fit" of these parameters.

Selecting Moments for Hansen's Model

- We will start by considering a relationship between wages and output:

$$w_t = \frac{\partial y_t}{\partial h_t} = (1-\theta)a_t(\frac{k_t}{h_t})^\theta \tag{11}$$

$$\Rightarrow \frac{w_t h_t}{y_t} = (1-\theta) \tag{12}$$

- That is, our theory implies that the ratio of real wages to output should equal $1-\theta$, or the share of income paid to workers.

$$ln(Y_{t+1}) - ln(Y_t) \approx (1-\theta)ln(\eta) \tag{13}$$

- If we assume that the capital stock is approximately constant quarter to quarter, then this might be a reasonable approximation, given that A and H have little trend.

Selecting Moments for Hansen's Model - Cont.

- Cooley (1995) suggests that the steady-state capital-output ratio is 3.32 yearly:
- Then β^4 solves equation 9:

$$3.32 = \frac{\theta}{\frac{4\eta}{\beta} - 1 + 4\delta} \tag{14}$$

- We also take $\delta = 0.012$ from Cooley.
- Hours have been observed to be roughly trendless, thus we can find γ from the following:

$$h^* = (\frac{1-\theta}{\gamma})[1 - (\frac{\theta(\eta - 1 + \delta)}{\frac{\eta}{\beta} - 1 + \delta})]^{-1} \tag{15}$$

- From the following, we can estimate TFP and its associated parameters, ρ and σ_ϵ:

$$\Delta ln(Y_t) - [(1-\theta)[\Delta ln(H_t) + ln(\eta)] \approx \Delta ln(A_t) \tag{16}$$

Readying the Data

- We must match theoretical moments to the correct empirical moments:
  1. Our model doesn't include government or international trade, so these need to be removed from GDP.
  2. Use personal consumption and private investment.
  3. We have no prices, so each variable needs to be in real terms.
  4. Each of the variables is defined to be per-capita, so we need to divide by population.
- Further preparations are needed:
  1. Series decomposed into trend and cycle using Hodrick-Prescott Filter.
  2. Solving for θ requires further detrending: divide per-capita variable by η^t.
- Most of the data taken from BEA.
- Real wages per capita are taken from FRED, and not explicitly in the model.
- Really lean into the model as the "true" model of the world

Calibration Results

Table: Calibration Estimates

Preferences			Technology			
β	γ	θ	η	δ	ρ	σ_ϵ
0.9903	0.0076	0.3739	1.0061	0.0120	0.9972	0.0129

Table: Steady-States

y^*	c^*	i^*	h^*	k^*	a^*
8,834	6,694	2,140	108.61	118,320	17.8309

Maximum Likelihood

- An alternative approach to estimation is maximum likelihood via the Kalman Filter.
- With equation (40), we can now write the system in state-space form:

$$f_t = \Pi_1 s_t + \eta_t \tag{17}$$

$$s_{t+1} = \Pi_2 s_t + \epsilon_t \tag{18}$$

- We typically include η_t as measurement errors for the observed variables to avoid stochastic singularity.
- Having written the model like this, we can apply the Kalman Filter for different parameter values to find the likelihood maximizing parameter vector.

Log-Linearizing the System

- We can now write the system as:

$$\Psi_1 \zeta_t = \Psi_2 \xi_t + \Psi_3 \tilde{a}_t \tag{19}$$

$$\Psi_4 E_t(\xi_{t+1}) = \Psi_5 \xi_t + \Psi_6 \zeta_t + \Psi_7 \tilde{a}_t \tag{20}$$

- ζ_t are static predetermined and nonpredetermined variables, $[\tilde{y}_t, \tilde{h}_t, \tilde{i}_t]'$.
- ξ_t are dynamic predetermined and nonpredetermined variables, $[\tilde{k}_t, \tilde{c}_t]'$.
- $\tilde{a}_t$ is the technology process.

Matrices

$$\kappa = \eta/\beta - 1 + \delta$$

$$\lambda = \eta - 1 + \delta$$

$$\zeta_t = \begin{bmatrix} \tilde{y}_t & \tilde{i}_t & \tilde{h}_t \end{bmatrix}', \quad \xi_t = \begin{bmatrix} \tilde{k}_t & \tilde{c}_t \end{bmatrix}'$$

$$\Psi_1 = \begin{bmatrix} 1 & 0 & \theta - 1 \\ \kappa & -\theta\lambda & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} \theta & 0 \\ 0 & \kappa - \theta\lambda \\ 0 & 1 \end{bmatrix}, \quad \Psi_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Psi_4 = \begin{bmatrix} \eta & 0 \\ \kappa & \eta/\beta \end{bmatrix}, \quad \Psi_5 = \begin{bmatrix} 0 & 0 & 0 \\ -\kappa & 0 & 0 \end{bmatrix}, \quad \Psi_6 = \begin{bmatrix} 1 - \delta & 0 \\ 0\eta/\beta & \end{bmatrix}, \quad \Psi_7 = \begin{bmatrix} 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Comparing Results

Table: MLE Results Fixing β and δ

Preferences			Technology			
β	γ	θ	η	δ	ρ	σ_ϵ
0.99	0.0045	0.2292	1.0051	0.0250	0.9987	0.0052

Table: Calibration Estimates

Preferences			Technology			
β	γ	θ	η	δ	ρ	σ_ϵ
0.9903	0.0076	0.3739	1.0061	0.0120	0.9972	0.0129

Rios-Rull et al. (2012)

- Attempt to compare calibrated and Bayesian results.
- Estimate Hansen's model with investment shocks and different labor supply elasticities.
- Three different calibration approaches to identifying elasticity:

    1. Use long-run hours worked: elasticity around 2.
    2. Use lotteries (equivalent to what we have done here): elasticity of ∞.
    3. Use estimates from microeconomic studies: between 0.2 - 0.76.

- The models result in around the same results if identifying assumption 3 is used.
- They conclude that identification is more important than estimation technique.

Criticism 1: no independent evidence for technology shocks

- Hard to identify specific shocks
- Alexopoulos (AER 2011). Publication of new technology books seem to signal changes in TFP. But this explains only a fraction of Solow residual
- Negative shocks – Are they technological regress?
- Oil price shocks act like technology shocks in some ways, but are best modeled separately

Criticism 2: Solow residual is correlated with demand shocks

- Measured labor hours don't account for intensity of effort
- During recessions, reduce effort rather than hours
- During expansions, increase effort, rather than hours
- Can reflect matching and training costs, implicit insurance

- Suppose output is given by

$$Y_t = K_t^{\alpha} \left(A_t L_t U_t \right)^{1-\alpha}$$

  where U denotes utilization (effort)
- The true Solow Residual is

$$SR_t = (1-\alpha)\left(\ln A_t + \ln U_t\right) = \ln Y_t - \alpha \ln K_t - (1-\alpha)\ln L_t$$

  implying that changes in utilization affect the Solow Residual
- Increases in utilization are mistaken for increases in productivity
- Addresses Criticism 2: Demand shocks increase utilization and thus increase utilization-unadjusted Solow residual
- Keynesian AD-AS model with sticky wages, demand driven fluctuations and no labor hoarding implies $corr(y, y/\ell) < 0$. Data shows $corr(y, y/\ell) \approx 1/2$. Adding labor hoarding can generate a positive correlation

Propagation Mechanisms: Economic dynamics that extend and transform the effects of an exogenous shock

- Intertemporal substitution of labor: higher productivity today induces more work today
- Capital accumulation
- Problem 1: Without indivisible labor, small IES_L implies small labor propagation
- Problem 2: Capital accumulation generates little propagation
- Even with indivisible labor, the dynamics of output are the dynamics of technology

RBC models that use measured Solow residuals cannot produce (Cogley and Nason 1995)

- Positive autocorrelation in output growth (not output)
- Note: Solow residuals follow AR(1) with $\rho \leq 1$
- A 'hump-shaped" impulse response function for transitory shocks

Generating persistence: need to slow down the economy's response to the initial shock

- Labor search (Merz, 1995; Andolfatto, 1996); Employers and workers need time to make matches
- Finance constraints (Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1999): Over time, higher productivity allows firms to borrow more
- Factor hoarding (Burnside, Eichenbaum and Rebelo, 1996): Firms first increase effort and capital utilization, then increase hours and capital

Conclusion

- Baseball extra credit!
- Calibration uses model implied restrictions and estimates on data.
- Maximum likelihood (and related techniques) use all variation in the data.
- MLE and calibration provide estimates that are relatively similar in this context.
- Others have shown similar results for more complex models (Rios et al., 2012).
- Next: Most likely, Heterogeneous agents
- No class next Thursday (come to conference that weekend instead!)