Macro II

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Introduction

- Today: the Hosios Condition
- Efficiency in search and matching models.
- (Note: largely derived from Christine Braun's lecture on the DMP model).
- Homework on my website.
- Code for Aiyagari w/ labor-leisure choice on the cluster.

Efficiency

- Is zero unemployment efficient? No
  - higher unemployment incentivizes firms to post vacancies
  - but high unemployment is costly, less production
- Is a high vacancy rate efficient?
  - vacancy creation is costly
  - but lots of vacancies reduces unemployment
- So what is the efficient level of θ?

Efficiency

- Congestion externality
  - one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off
  - one more searching worker makes hiring firms better off and makes all other searching workers worse off
- Appropriability
  - firm pays a cost κ to post vacancy but does not get to keep the entire output p

Efficiency

- What value of θ would the social planer choose to maximize total output/utility if he is constrained by the same matching frictions?
  - does not care about wage b/c it's a linear transfer from the firm to the worker
- Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?
- Can we achieve this wage with the Nash solution?

The DMP Model ("Ch. 1 of Pissarides (2000)")

- Agents:
  1. Employed workers;
  2. unemployed workers;
  3. vacant firms;
  4. matched firms.
- Linear utility ($u = b, u = w$) and production $y = p > b$.
- Matching function:
  1. Constant returns to scale (L is lab. force):

$$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta)$$

  2. where $\theta = \frac{v}{u}$ is "labor market tightness"
  3. Match rates:

$$\underbrace{p(\theta)}_{Worker} = \theta \underbrace{q(\theta)}_{Firm}$$

- Social planner: pick θ optimally, no need to respect free entry condition.

Social Planner's Problem

$$\int_0^\infty e^{-rt}[p(1-u) + bu - \kappa\theta u]\; dt$$
$$\text{s.t.} \quad \dot{u} = \delta(1-u) - p(\theta)u$$

- Social planner's problem
  - $p(1-u)$: social output of employment
  - bu: leisure enjoyed by unemployed workers
  - $\kappa\theta u$: cost of jobs
- Social planner is subject to the same transition equation for unemployment

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- FOCs

$$\begin{aligned} H_u = -\dot{\mu} + r\mu \Rightarrow & \quad -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0 \\ H_\theta = 0 \Rightarrow & \quad -e^{-rt}\kappa u - \mu u(q(\theta) + \theta q'(\theta)) = 0 \end{aligned}$$

- μ: marginal value of an extra unemployed worker.

Social Planner's Problem

- Optimal θ

$$H_\theta = 0 \Rightarrow \qquad -e^{-rt}\kappa u - \mu u q(\theta)(1 + \frac{\theta q'(\theta)}{q(\theta)}) = 0$$

- What is $\frac{\theta q'(\theta)}{q(\theta)}$?

$$m(u,v) = vq(\theta)$$
$$\rightarrow \frac{\partial m(u,v)}{\partial u} = vq'(\theta)\frac{-v}{u^2}$$
$$\rightarrow \frac{\partial m(u,v)}{\partial u} = -\theta^2 q'(\theta)$$
$$\rightarrow \frac{\frac{\partial m(u,v)}{\partial u}}{m(u,v)} = -\frac{\theta^2 q'(\theta)}{vq(\theta)}$$
$$\rightarrow u\frac{\frac{\partial m(u,v)}{\partial u}}{m(u,v)} = -\frac{\theta q'(\theta)}{q(\theta)}$$

- $\frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function wrt u.

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- FOCs

$$\begin{aligned} H_u = -\dot{\mu} + r\mu \Rightarrow & \quad -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0 \\ H_\theta = 0 \Rightarrow & \quad -e^{-rt}\kappa u - \mu u q(\theta)(1 - \eta(\theta)) = 0 \end{aligned}$$

- $\eta(\theta)$: elasticity of match fun. wrt u.

Optimal θ

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$):

$$\frac{p - b + \kappa\theta}{\delta + r + p(\theta)} = \frac{\kappa}{q(\theta)(1 - \eta(\theta))}$$

$$(p - b)(1 - \eta(\theta)) + \kappa(1 - \eta(\theta))\frac{p(\theta)}{q(\theta)} = \frac{(\delta + r + p(\theta))\kappa}{q(\theta)}$$

$$\rightarrow (1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \qquad (1)$$

- This is optimal θ

Decentralized solution

- Can the decentralized solution achieve the same level of θ?
- i.e., can the decentralized level of unemployment be *efficient*?

Decentralized θ

- Free entry $V = 0$:

$$\begin{aligned} rJ(w) &= (p - w) + \delta[\cancel{V} - J(w)] \\ (r + \delta)J(w) &= (p - w) \end{aligned}$$

- Vacancy creation condition (i.e., free entry imposed):

$$\begin{aligned} q(\theta) &= \frac{\kappa}{E[J(w)]} \\ q(\theta) &= \frac{\kappa(r+\delta)}{(p-w)} \\ \theta &= q^{-1}(\frac{\kappa(r+\delta)}{(p-w)}) \end{aligned}$$

- Thus, mapping between wages and θ. 1 equation, 2 unknowns.
- Need equation to determine wages in equilibrium.

Wage Determination

- Recall Nash Bargained wages:

$$w = argmax_w \underbrace{(W(w) - U)^\beta}_{Net\ Utility} \underbrace{(J(w) - V)^{1-\beta}}_{Net\ Profits}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta}\frac{\partial W}{\partial w}$$
$$+ (1-\beta)(J(w) - V)^{-\beta}(W(w) - U)\frac{\partial J}{\partial w}$$

- $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\beta(\frac{J(w)}{W(w) - U})^{1-\beta} = (1-\beta)(\frac{W(w) - U}{J(w)})^\beta$$
$$\beta(J(w) + W(w) - U) = W(w) - U$$
$$\beta S(w) = W(w) - U$$

Wage Determination

- Note that $\beta S(w) = [W(w) - U]$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) + (1-\beta)(p(\theta)+\delta)\beta S(w)$$

- And $(1-\beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1-\beta}$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) + (1-\beta)(p(\theta)+\delta)\beta \frac{J(w)}{1-\beta}$$

$$w = (1-\beta)b + \beta p + p(\theta)\beta J(w)$$

- Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1-\beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

- Job creation curve:

$$
\begin{aligned}
(r+\delta)J(w) &= (p-w) \\
q(\theta) &= \frac{\kappa}{J(w)} \\
q(\theta) &= \frac{\kappa(r+\delta)}{(p-w)} \\
p - w - \frac{\kappa(r+\delta)}{q(\theta)} &= 0
\end{aligned}
$$

- Now, plug in using wages we just found:

$$w = (1-\beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

- Job creation curve:

$$p - ((1-\beta)b + \beta p + \beta\theta\kappa) - \frac{\kappa(r+\delta)}{q(\theta)} = 0$$

- identities: $p(\theta) = \theta q(\theta) \to \theta = \frac{p(\theta)}{q(\theta)}$

$$\to p - ((1-\beta)b + \beta p + \beta\frac{p(\theta)}{q(\theta)}\kappa) - \frac{\kappa(r+\delta)}{q(\theta)} = 0$$

$$(1-\beta)(p-b) - \beta\frac{p(\theta)}{q(\theta)}\kappa) - \frac{\kappa(r+\delta)}{q(\theta)} = 0$$

$$(1-\beta)(p-b) - \frac{r+\delta+\beta p(\theta)}{q(\theta)}\kappa = 0$$

- Looks familiar?

Social Planner's Problem

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$)

$$(1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta) p(\theta)}{q(\theta)} \kappa = 0 \tag{2}$$

- From the decentralized solution, plug the wage curve into the Job creation curve

$$(1 - \beta)(p - b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)} \kappa = 0 \tag{3}$$

Efficiency

- Comparing (1) and (2) we see that we have efficiency in the decentralized market if $\beta = \eta(\theta)$. The workers bargaining power is equal to the elasticity of the matching function with respect to u.
- This is a general result: we have efficiency when

$$\eta(\theta) = \beta$$

- This is called the Hosios (1990) condition

A couple more classes

- Two more classes, likely Directed Search and Block Recursive Equilibrium.

Cats!