AECO 701 Final

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1 [50] **Training and Hold-ups**. Consider an economy where there is a measure one of infinitely lived agents. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is consumption in period t. Agents can accumulate human capital h_t. All agents have the same initial human capital stock, h_0. Let H_t denote the average human capital in the population in period t. Human capital can be used either to produce output, or to increase future human capital. Let n_t be the part of human capital allocated to market activities (individual's labor input) and $h_t - n_t$ be the part allocated to enhancing human capital. Assume that human capital evolves according to

$$h_{t+1} = h_t + \sigma(h_t - n_t)^{\frac{1}{2}}$$

People are not allowed to borrow or save and there is no physical capital in the economy. The production technology is thus given by

$$Y_t = z_t N_t H_t^{\gamma} \qquad 0 < \gamma < 1$$

where Y_t is output, N_t is labor input, and z_t is a finite state first order Markov process that takes values from $Z = \{z_1, z_2, \ldots, z_n\}$ with all values being strictly positive.

a) (15) Write down the household's problem recursively. Be explicit about which variables are choice variables, which are state variables, and any constraints faced by households. Does the household's problem change if z where a second order Markov process? If so, show how and explain why. If not, exlpain why not.

b) (10) Define the rational expectations recursive competitive equilibrium for the economy.

c) (10) Write down the social planner's problem recursively. Be explicit about which variables are choice variables, which are state variables, and any constraints that the social planner faces. Has anything changed from part (a)? If so, show how and explain why. If not, explain why not.

d) (15) Argue whether or not the Second Welfare Theorem holds. That is, show explicitly whether or not the solution to the social planner's problem and household's problem are the same. If they are the same, explain why. If they are not the same, explain why not. If you cannot show that it holds or does not hold explicitly, a correct argument will receive most of the points.

2 [50] **Training and Frictions**. Consider a discrete time environment in which firms may optionally invest in their worker's human capital. Workers live for two periods, have linear utility $u(c) = c$ in both periods, and do not have access to savings technology (although it doesn't matter with linear utility). They start period 1 employed at a wage $w(h)$, where $w(h)$ is a function of the workers human capital and will take different functional forms during different parts of the question. Workers may differ in either h or w. Firms employ workers in a one-worker-one-firm match and receive $h - w(h) - c(\tau)$, where $w(h)$ is the wage and $c(\tau)$ is a convex function that takes into account the possibility that the firm may train their worker. For simplicity, let $c(\tau) = \tau^2$. Prior to production in the second period, workers are able to search for a new job and find one with probability λ_E. They accept this job if it offers $w_R(h) > w(h)$. If they find and accept a job, the match dissolves. Otherwise, they produce at their original firm and the match dissolves at the end of the period. Workers and firms both discount the future at a common rate $\beta \in (0, 1)$. For the worker, this yields the following value function:

$$V_1(h, w) = w(h) + \beta\{\lambda_E \int_{\underline{w}}^{\bar{w}} \max\{V_2(h', w'), V_2(h', w)\} dF(w') + (1 - \lambda_E)V_2(h', w) \tag{1}$$

$$h' = g(w, h) \tag{2}$$

$$V_2(h, w) = w(h) \tag{3}$$

The firm's problem is given by

$$J_1(h, w) = \max_{\tau} h - w(h) - \tau^2 + \beta\{(1 - \lambda_E(1 - \int_{w}^{\bar{w}} dF(w')))J_2(h', w) \tag{4}$$

$$h' = \begin{cases} h + 1 & : \textit{with } \tau \textit{ probability} \\ h & : \textit{with } (1 - \tau) \textit{ probability} \end{cases} \tag{5}$$

$$J_2(h, w) = h - w(h) \tag{6}$$

a) (15) Write the Flow Bellman equations each agent in this model (unemployed worker, employed worker, unmatched firm, matched firm). Define the unemployed value function as U, the employed value function as $W(\epsilon)$, the unmatched firm value function as V and the matched firm value function as $J(\epsilon)$.

b) (10) Define a steady-state equilibrium in this model. You do not need to solve for the policy functions, just note the appropriate policy functions and steady state variables of the equilibrium.

c) (5) Use the free entry condition (i.e., that firms open vacancies until it is no longer profitable to do so in expectation, $rV = 0$) to derive θ. Assume that $q(\cdot)$ is invertible.

d) (20) Write out the surplus equation, i.e., $S(\epsilon) = W(\epsilon) + J(\epsilon) - U - V$. Solve for the separation threshold, ϵ_d, given by $S(\epsilon_d) = 0$. Note that Nash Bargained wages imply that $\beta S(\epsilon) = W(\epsilon) - U$ and $(1 - \beta)S(\epsilon) = J(\epsilon)$.