AECO 701 Final

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2 [50] **Training and Frictions**. Consider a discrete time environment in which firms may optionally invest in their worker's human capital. Workers live for two periods, have linear utility $u(c) = c$ in both periods, and do not have access to savings technology (although it doesn't matter with linear utility). They start period 1 employed at a piece-rate wage $w \times h$, where w is the piece-rate earned out of total productivity and h is a worker's human capital. The piece-rate is distributed $F(\mu_w, \sigma_w)$, and $w \in [0, 1]$. Workers may differ in either h or w. Firms employ workers in a one-worker-one-firm match and receive $h - wh - c(\tau)$, where wh is the wage and $c(\tau)$ is a convex function that takes into account the possibility that the firm may train their worker. For simplicity, let $c(\tau) = \tau^2$. Prior to production in the second period, workers are able to search for a new job and find one with probability λ_E. They accept this job if it offers $w_R > w$. If they find and accept a job, the match dissolves. Otherwise, they produce at their original firm and the match dissolves at the end of the period. Workers and firms both discount the future at a common rate $\beta \in (0, 1)$. For the worker, this yields the following value function:

$$V_1(h, w) = wh + \beta\{\lambda_E \int_{\underline{w}}^{\bar{w}} \max\{V_2(h', w'), V_2(h', w)dF(w') + (1 - \lambda_E)V_2(h', w)\} \quad (1)$$

$$h' = g(w, h) \quad (2)$$

$$V_2(h, w) = wh \quad (3)$$

The firm's problem is given by

$$J_1(h, w) = \max_{\tau} h - wh - \tau^2 + \beta\{(1 - \lambda_E(1 - \int_{w}^{\bar{w}} dF(w')))J_2(h', w)\} \quad (4)$$

$$h' = \begin{cases} h + 1 & : with\ \tau\ probability \\ h & : with\ (1 - \tau)\ probability \end{cases} \quad (5)$$

$$J_2(h, w) = h - wh \quad (6)$$

note that w in the lower limit of integration is the wage at the worker's current firm.

a) (15) Write out the firm's optimization problem by substituting their period 2 problem into their period 1 problem. Now, write out the FOC for the firm in τ. Either explicitly or intuitively explain how this decision depends on the worker's current wage, and the availability of wages higher than their current wage (i.e., $(1 - F(W))$).

b) (10) Suppose that labor markets are competitive, which in this context means that workers are paid their marginal product, $w = 1$, $w \sim F(1, 0)$ and they always find a job if they are looking, $\lambda_E = 1$. Rewrite the firm's problem and then show that optimal training provision, τ is zero.

c) (15) Now assume that wages are uniformly distributed $w \sim U(0, 1)$. Rewrite the worker and firms problems with this distribution.

d) (10) How much does output change moving from a frictional world in part 3 to a frictionless world in part 2. For simplicity, we will assume that all workers are ex-ante identical. In the perfectly competitive world, $w = 1$, and $h = 1$. In the frictional world $w = \frac{1}{2}$, and $h = 1$. Also assume that in the frictional world, $w \sim U(0, 1)$ and $\lambda_E = 1$. Is the competitive equilibrium pareto optimal? Why or why not?

2 [50] **Temporary unemployment compensation**. At the beginning of each period an unemployed worker draws one offer to work forever at wage w (which she may accept or reject). Wages are i.i.d. draws from the c.d.f. F, where $F(\underline{w}) = 0$ and $F(\overline{w}) = 1$. The worker seeks to maximize $\sum_{t=0}^{\infty} \beta^t y_t$, where y_t is the worker's wage or unemployment compensation, if any. The worker is entitled to unemployment compensation in the amount $\gamma > 0$ only during the *first* period that she is unemployed. After one period on unemployment compensation, the worker receives none.

a) (15) Write the Bellman equations for this problem.

b) (10) Show and explain how the worker's reservation wage and her "hazard of leaving unemployment" (i.e. the probability of accepting a job offer) varies with the duration of unemployment.

For parts (c) and (d), assume that the worker is also entitled to unemployment compensation if she quits a job. As before, the worker receives unemployment compensation in the amount of γ during the first period of an unemployment spell, and zero during the remaining part of the spell. (In order to re-qualify for the benefits, the worker must find a job and work at least one period.)

The timing of events is as follows. At the very beginning of a period, a worker who was employed in the previous period must decide whether or not to quit. If she quits, she draws a new wage offer as described previously, and if she accepts the offer she immediately starts earning that wage without suffering any period of unemployment.

c) (15) Write the Bellman equations for this problem. [Hint: Define a value function $V_E(w)$ for a worker who was employed the previous period with wage w, before any decision to quit (and receive some new draw w') occurs.]

d) (10) Characterize the reservation strategy of an employed worker and then prove that the reservation wage for someone unemployed longer than one period is equal to 0 (if you cannot prove it, give some intuition as to why it must be the case).