AECO 701 Final

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1 [30] **Firm values and adjustment costs**. Suppose a firm has the production function $z = f(h)$, where factory space, $h \in R_+$, is the single input and $z \in R_+$ is output. Assume that f is continuously differentiable, strictly increasing, and strictly concave, and that

$$f(0) = 0, \qquad \lim_{h \to \infty} f'(h) = 0, \qquad \lim_{h \to 0} f'(h) = \infty$$

Normalize the price of output to $p = 1$ and let q be the price of capital, the interest rate, r, is constant over time, and the discount factor is given by $\beta = \frac{1}{1+r}$. Assume that factory space must be purchased one period in advance and depreciates at rate $\delta \in (0, 1)$. Thus, firm investment is given by

$$x_t = h_{t+1} - (1 - \delta)h_t$$

The firm also faces an adjustment cost to factory size given by $c(x_t)$ where $c(\cdot)$ is strictly increasing, convex, and differntiable, with $c(0) = 0$.

a [15] Write down the firm's problem recursively. Be explicit about any and all choice/state variables.

b [15] Show that this satisfies the conditions for a contraction or state the conditions under which it would satisfy a contraction.

c [15] Suppose that the firm's adjustment cost is linear: $c(x_t) = q \cdot x_t$. Find the exact solutions for the value and policy functions. BRIEFLY explain the dynamics of h from $h_0 \to h^*$. [Hint: which equations drives dynamics in our models?]

2 [50] **Liquidity effects vs. moral hazard** Consider a discrete time economy in which agents live for T periods and the discount rate is zero (i.e. $\beta = 1$). Agents may be employed or unemployed and searching for a job. Once employed, that worker is employed permanently. While searching, agents may choose their search intensity, s_t, which linearly increases the probability they will find a job, but results in disutility $\gamma(s_t)$. All agents make consumption and savings decisions and receive utility $u(c_t)$ from consumption, whose functional for will be specified later. The wage is identical for all jobs, w_t.

An employed agent faces the following problem for $t < T$:

$$V_t(a_t) = \max_{c_t, a_{t+1} \geq \underline{a}} u(c_t) + V_{t+1}(a_{t+1}) \tag{1}$$

$$\text{s.t.} c_t + a_{t+1} = a_t + w_t - \tau \tag{2}$$

An unemployed agent faces the following problem for $t < T$:

$$U_t(a_t) = \max_{c_t, a_{t+1} \geq \underline{a}} u(c_t) + J_{t+1}(a_{t+1}) \tag{3}$$

$$\text{s.t.} c_t + a_{t+1} = a_t + b_t \tag{4}$$

where

$$J_t(a_t) = \max_{s_t} s_t V(a_t) + (1 - s_t) U_t(a_t) - \gamma(s_t) \tag{5}$$

and s_t is their search intensity and $\gamma(s_t)$ their disutility from search. s_t is restricted to $[0, 1]$ and the function $\gamma(s_t)$ is convex with $\gamma'(s_t) > 0$, $\gamma''(s_t) > 0$ with $\gamma'(0) = 0$ and $\gamma'(1) = \infty$.

a [5] Find the first-order condition for optimal search intensity.

b [5] Show how search intensity changes as the unemployment benefit b_t changes.

c [10] Show how search intensity changes as i) assets, a_t and ii) wages, w_t change (separately).

d [10] Show that $\frac{\partial s_t}{\partial b_t} = \frac{\partial s_t}{\partial a_t} - \frac{\partial s_t}{\partial w_t}$.

e [10] Now consider two possible utility functions: $u(c_t) = ln(c_t)$ and $u(c_t) = c_t$. Describe how the components that make-up $\frac{\partial s_t}{\partial b_t}$ would differ for an agent with $a_t \approx \underline{a}$ and for one with $a_t >> \underline{a}$ first for linear utility and then for log-utility. Describe means this is an open-ended question where appropriate use of intuition and mathematical expressions will be rewarded.

f [10] If utility is $u(c_t) = ln(c_t)$, describe how optimal UI would differ if i) wealth was relatively equally distributed (i.e., few agents with $a_t \approx \underline{a}$) and ii) wealth was highly unequal (i.e., many agents with $a_t \approx \underline{a}$). Describe means this is an open-ended question where appropriate use of intuition and mathematical expressions will be rewarded.