AECO 701 Final

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1 [30] **Firm values and adjustment costs**. Suppose a firm has the production function $z = f(h)$, where factory space, $h \in R_+$, is the single input and $z \in R_+$ is output. Assume that f is continuously differentiable, strictly increasing, and strictly concave, and that

$$f(0) = 0, \qquad \lim_{h\to\infty} f'(h) = 0, \qquad \lim_{h\to 0} f'(h) = \infty$$

Normalize the price of output to $p = 1$ and let q be the price of capital, the interest rate, r, is constant over time, and the discount factor is given by $\beta = \frac{1}{1+r}$. Assume that factory space must be purchased one period in advance and depreciates at rate $\delta \in (0, 1)$. Thus, firm investment is given by

$$x_t = h_{t+1} - (1-\delta)h_t$$

The firm also faces an adjustment cost to factory size given by $c(x_t)$ where $c(\cdot)$ is strictly increasing, convex, and differntiable, with $c(0) = 0$.

a [15] Write down the firm's problem recursively. Be explicit about any and all choice/state variables.

Answer:
The firm's problem:

$$V(h) = \max_{x,h'\geq 0} \{f(h) - q\cdot h' - c(x) + \beta V(h')\}$$

$$s.t. \quad x = h' - (1-\delta)h$$

b [15] Show that this satisfies the conditions for a contraction or state the conditions under which it would satisfy a contraction. Answer:
For there to exist a unique solution in the space of bounded and continuous functions we need for the Theorem of the Maximum to hold and for $Tv = \max F(h, x) + \beta v$ to be a contraction. For the theroem of the maximum to hold we need that:

1) $\Gamma(h)$ is nonempty, compact, and continuous
2) $F(h, x)$ to be bounded and continuous

Next, we need for our operator to be a contraction. Hence, we will invoke Blackwell's sufficiency conditions:

1) T is monotone
2) T is discounting

15 Suppose that the firm's adjustment cost is linear: $c(x_t) = q \cdot x_t$. Find the exact solutions for the value and policy functions. BRIEFLY explain the dynamics of h from $h_0 \to h^*$. [Hint: which equations drives dynamics in our models?]

Answer:

$$V(h) = \max\{f(h) - q\cdot h' - q\cdot(h' - (1-\delta)h) + \beta V(h')\}$$

FOC[h]:

$$2q = \beta V'(h')$$

Envelope theorem:

$$V'(k) = f'(h) + (1-\delta)$$

Combining these equations, we get

$$2q = \beta\left[f'(h') + (1-\delta)q\right]$$

$$\Leftrightarrow f'(h') = \frac{2q}{\beta} - (1-\delta)q$$

Hence, our policy function is to always reach our target level of factory space as soon as possible. Moreover, notice that h' is independent of h. Thus, we will immediately jump to the steady-state level of factory space since there are no bounds on x.

$$\Rightarrow V(h) = f(h) - q \cdot h^* - q \cdot (h^* - (1-\delta)h) + \beta V(h^*)$$

$$\Leftrightarrow V(h) = f(h) - q \cdot (h^* - (1-\delta)h) + \frac{1}{1-\beta}[f(h^*) - 2q \cdot h^* + (1-\delta)q \cdot h^*]$$

Where h^* is defined by $f'(h^*) = \frac{2q}{\beta} - (1-\delta)$.

2 [50] **Liquidity effects vs. moral hazard** Consider a discrete time economy in which agents live for T periods and the discount rate is zero (i.e. $\beta = 1$). Agents may be employed or unemployed and searching for a job. Once employed, that worker is employed permanently. While searching, agents may choose their search intensity, s_t, which linearly increases the probability they will find a job, but results in disutility $\gamma(s_t)$. All agents make consumption and savings decisions and receive utility $u(c_t)$ from consumption, whose functional for will be specified later. The wage is identical for all jobs, w_t.

An employed agent faces the following problem for $t < T$:

$$V_t(a_t) = \max_{c_t, a_{t+1} \geq \underline{a}} u(c_t) + V_{t+1}(a_{t+1}) \tag{1}$$

$$\text{s.t.} c_t + a_{t+1} = a_t + w_t \tag{2}$$

An unemployed agent faces the following problem for $t < T$:

$$U_t(a_t) = \max_{c_t, a_{t+1} \geq \underline{a}} u(c_t) + J_{t+1}(a_{t+1}) \tag{3}$$

$$\text{s.t.} c_t + a_{t+1} = a_t + b_t \tag{4}$$

where

$$J_t(a_t) = \max_{s_t} s_t V(a_t) + (1 - s_t) U_t(a_t) - \gamma(s_t) \tag{5}$$

and s_t is their search intensity and $\gamma(s_t)$ their disutility from search. s_t is restricted to $[0, 1]$ and the function $\gamma(s_t)$ is convex with $\gamma'(s_t) > 0$, $\gamma''(s_t) > 0$ with $\gamma'(0) = 0$ and $\gamma'(1) = \infty$.

a [5] Find the first-order condition for optimal search intensity.

This is easy. Take the derivative of J_t with respect to s_t.

b [5] Show how search intensity changes as the unemployment benefit b_t changes.

Answer:

$$\gamma'(s_t) = V_t(a_t) - U_t(a_t) \tag{6}$$

Taking the derivative of this expression with respect to b_t yields

$$\gamma''(s_t) \frac{\partial s_t}{\partial b_t} = -\frac{\partial U_t(a_t)}{\partial b_t} \tag{7}$$

$$\implies \gamma''(s_t) \frac{\partial s_t}{\partial b_t} = -u'(c_t^u) \tag{8}$$

$$\implies \frac{\partial s_t}{\partial b_t} = -\frac{u'(c_t^u)}{\gamma''(s_t)} \tag{9}$$

c Show how search intensity changes as i) assets, a_t and ii) wages, w_t change (separately).

Again starting from the FOC in s_t:

$$\gamma'(s_t) = V_t(a_t) - U_t(a_t) \tag{10}$$

Taking the derivative of this expression with respect to a_t yields

$$\gamma''(s_t)\frac{\partial s_t}{\partial a_t} = \frac{\partial V_t}{\partial a_t} - \frac{\partial U_t(a_t)}{\partial a_t} \tag{11}$$

$$\implies \gamma''(s_t)\frac{\partial s_t}{\partial a_t} = u'(c_t^e) - u'(c_t^u) \tag{12}$$

$$\implies \frac{\partial s_t}{\partial a_t} = -\frac{u'(c_t^e) - u'(c_t^u)}{\gamma''(s_t)} \tag{13}$$

and for wages:

$$\gamma''(s_t)\frac{\partial s_t}{\partial w_t} = \frac{\partial V_t(a_t)}{\partial w_t} \tag{14}$$

$$\implies \gamma''(s_t)\frac{\partial s_t}{\partial w_t} = u'(c_t^e) \tag{15}$$

$$\implies \frac{\partial s_t}{\partial w_t} = \frac{u'(c_t^e)}{\gamma''(s_t)} \tag{16}$$

d Show that $\frac{\partial s_t}{\partial b_t} = \frac{\partial s_t}{\partial a_t} - \frac{\partial s_t}{\partial w_t}$.

This is clear from taking the difference between expressions in part (c).

e Now consider two possible utility functions: $u(c_t) = ln(c_t)$ and $u(c_t) = c_t$. Describe how the components that make-up $\frac{\partial s_t}{\partial b_t}$ would differ for an agent with $a_t \approx \underline{a}$ and for one with $a_t >> \underline{a}$ first for linear utility and then for log-utility. Describe means this is an open-ended question where appropriate use of intuition and mathematical expressions will be rewarded.

Answer:

Under linear utility, the effect of assets on search intensity, $\frac{\partial s_t}{\partial a_t} = 0$, because marginal utility is constant. Thus, only the *moral hazard* effect $\frac{\partial s_t}{\partial w_t}$ would remain. Under convex preferences, $c_t^e > c_t^u$ implies that $\frac{\partial s_t}{\partial a_t} > 0$ and could be quite large for the agent with nearly no capacity to borrow.

f If utility is $u(c_t) = ln(c_t)$, describe how optimal UI would differ if i) wealth was relatively equally distributed (i.e., few agents with $a_t \approx \underline{a}$) and ii) wealth was highly unequal (i.e., many agents with $a_t \approx \underline{a}$). Describe means this is an open-ended question where appropriate use of intuition and mathematical expressions will be rewarded.

Answer:

Answers could differ, but the welfare-maximizing response would be to increase b_t in the economy in which many agents are near the borrowing constraint, and decrease b_t in the economy in which few agents are near the borrowing constraint.