Macro II

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Announcements

- Today:
  1. Random vs. directed search
  2. Change primitive: workers can observe wage offers prior to match.
  3. i.e., Moen (1997)
- Homework due next Thursday.
- One more classes.

Random vs. Directed Search

- ▶ How do workers find jobs?
- ▶ How much information do they have about a job before applying?
- ▶ Two extremes:
  1. Random Search: *no* information about a job prior to receiving offer.
  2. Directed Search: *all* information about a job prior to application.
- ▶ Why does this matter?
  1. Random search is generically inefficient: one worker may reject a job offer than another would accept.
  2. Directed search is generically efficient: by applying for a job, a worker signals that the job is already acceptable.
- ▶ As we will see next time, it also changes computational complexity.

Random vs. Directed Search II

▶ Empirically, how can we tell them apart?

▶ Hazard rate to wage w generically:

$$H_U(w) = \underbrace{\lambda(w)}_{Arrival\ Rate} \times \underbrace{f(w)}_{Prob.Offer = W} \tag{1}$$

▶ Random search:

$$H_U(w) = \underbrace{\lambda}_{Arrival\ Rate} \underbrace{[1 - F(w_R)]}_{Selectivity} \underbrace{f(w)}_{P.(Offer = W)} \tag{2}$$

▶ Directed search:

$$H_U(w) = \underbrace{\lambda(w)}_{Arrival} \underbrace{[1 - F(w_R)]}_{F(w_R)=0} \underbrace{f(w)}_{P(Offer = w)} \tag{3}$$

$$= \underbrace{\lambda(W)}_{Arrival} \underbrace{f(w)}_{P(w=w_j)=1} \tag{4}$$

$$= \underbrace{\lambda(w)}_{Arrival\ Rate\ of\ Wage\ w} \tag{5}$$

Some Evidence

- ▶ (First couple Borrowed from Shouyong Shi)
- ▶ Hall and Krueger (08):
    1. 84% had information on wage prior to first interview.
- ▶ Holzer, Katz, and Krueger (91)
    1. Firms in high-wage industries receive more applications than low-wage industries, controlling for observables.
- ▶ Braun, Engelhardt, Griffy, and Rupert: unemployment insurance changes $\lambda(w) \rightarrow$ inconsistent with random search.

Mortensen and Pissarides Model

▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U] \qquad (6)$$

▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)] \qquad (7)$$

▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V] \qquad (8)$$

▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)] \qquad (9)$$

▶ Free entry equilibrium condition:

$$V = 0 \qquad (10)$$

$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta) \qquad (11)$$

Equilibrium

- ▶ The equilibrium we have described is a steady-state equilibrium characterized by value functions U, W, J, V, a wage function w, a market tightness function θ, and steady-state level unemployment u, such that
    1. A steady-state level of unemployment, derived from the flow unemployment equation.
    2. A wage rule that splits the surplus of a match according to a sharing rule with bargaining weight β
    3. A free entry condition that determines θ given wages and steady-state unemployment.
- ▶ What were these policy functions?
    1. $w = (1 - \beta)b + \beta p + \beta \theta \kappa$
    2. $\theta = q^{-1}(\frac{\kappa(r+\delta)}{(p-w)})$
    3. $u = \frac{\delta}{\delta + p(\theta)}$

Equilibrium
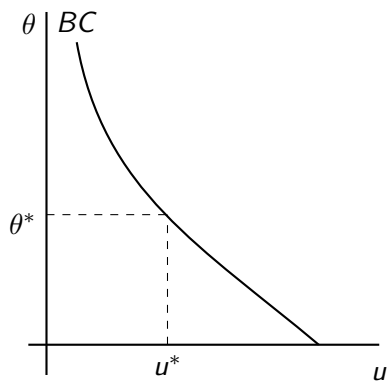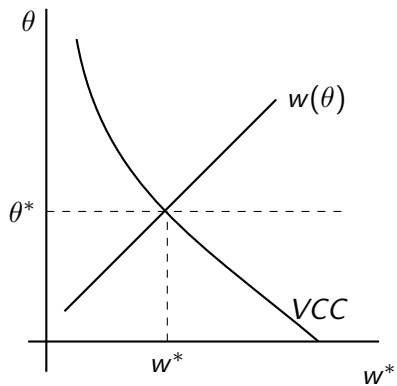
Directed/Competitive Search

► In DMP, wages are negotiated/revealed after meeting.
► This can create inefficiency:
  1. Consider example with unemployed workers A and B.
  2. $w_R^A = 10$, $w_R^B = 15$
  3. Firm pays a cost κ to open a vacancy and posts a wage 12.
  4. Both worker A and B apply for the job. Firm randomly picks worker B.
  5. Worker B rejects job that would have been acceptable to worker A.
► Directed search: Worker B applies for different job with $w \geq w_R^B$.
► (Directed and competitive search generally used interchangeably).

The Competitive Search Model (Moen, 1997)

▶ Agents:
  1. Employed workers employed in submarket i;
  2. unemployed workers considering searching in $i \in \{1, ..., N\}$;
  3. unmatched firms indexed by productivity $y_i \in y_1, ..., y_N$;
  4. matched firms indexed by productivity $y_i \in y_1, ..., y_N$;
  5. "Market Maker": benevolent overlord who announces eqm. w_i.

▶ Linearity: $(u = z, u = w_i)$ and $y = y_i > z$ in open submarkets.

▶ Matching function:
  1. Determines *number* of meetings between firms & workers in submarket i:

  $$M(u_i L_i, v_i L_i) = u_i L_i \times M(1, \frac{v_i}{u_i}) = u_i L_i \times p(\theta_i) \qquad (12)$$

  2. where $\theta_i = \frac{v_i}{u_i}$ is "submarket tightness"
  3. Match rates:

  $$\underbrace{p(\theta_i)}_{\text{Worker wage } i} = \theta_i \underbrace{q(\theta_i)}_{\text{Firm wage } i} \qquad (13)$$

▶ i indexes both the productivity and wage.

Worker Value Functions

- ▶ Value functions:
  1. Employed in submarket i: W_i
  2. Unemployed and searching in submarket i: U_i.
  3. Unemployed: $U = \max\{U_1, ..., U_N\}$.

- ▶ Unemployed flow value in submarket i:

$$rU_i = z + p(\theta_i)(W_i - U_i) \tag{14}$$

- ▶ Employed flow value in submarket i:

$$rW_i = w_i + \delta(U_i - W_i) \tag{15}$$

- ▶ Both problems are stationary: optimal choice of i true $\forall\, t$.

Worker Value Functions II

- We can solve for match rates:

$$rU_i = z + p(\theta_i)(W_i - U_i) \quad (16)$$

$$(r + p(\theta_i))U_i = z + p(\theta_i)\frac{w_i + \delta U_i}{r + \delta} \quad (17)$$

$$(r + \delta)(r + p(\theta_i))U_i - p(\theta_i)\delta U_i = (r + \delta)z + p(\theta_i)w_i \quad (18)$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))} \quad (19)$$

$$p(\theta_i) = \frac{rU_i - z}{w_i - rU_i}(r + \delta) \quad (20)$$

$$(21)$$

- $U = \max\{U_1, ..., U_N\}$ and ex-ante homogeneity among workers implies

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + \delta) \quad (22)$$

$$(23)$$

Firm Value Functions

- ▶ Pays a cost χ to draw productivity.
- ▶ Firm observes own productivity, chooses to open vacancy given submarkets (w, θ).
- ▶ Value functions:
  1. Vacant with productivity y_i: $V(y_i, w, \theta)$
  2. Filled with productivity y_i, paying wage w: $J(y_i, w)$
- ▶ Vacant flow value:

$$rV(y_i, w, \theta) = -\kappa + q(\theta)(J(y_i, w) - V(y_i, w, \theta)) \qquad (24)$$

- ▶ Matched flow value:

$$rJ(y_i, w) = y_i - w + \delta(V(y_i, w, \theta) - J(y_i, w)) \qquad (25)$$

Firm Value Functions II

- Value functions:
  1. Vacant with productivity y_i: $V(y_i, w, \theta)$
  2. Filled with productivity y_i, paying wage w: $J(y_i, w)$

- In equilibrium $V(y_i, w, \theta) = 0$:

$$rJ(y_i, w) = y_i - w - \delta J(y_i, w) \qquad (26)$$

- Asset value of vacancy in submarket (y_i, w, θ):

$$(r + q(\theta))V(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + \delta} - \kappa \qquad (27)$$

Equilibrium

▶ We will be interested in the same equilibrium objects, but now for each submarket i:

1. Wages w_i;
2. unemployment u_i;
3. $\theta_i = \frac{v_i}{u_i}$ vacancies in each submarket.

▶ Before, 1 & 3 were separate equilibrium conditions.

▶ New equilibrium objects

1. set of open submarkets, $\mathcal{I}$;
2. value of unemployment $\bar{V}(U)$

▶ Market maker sets wages according to

$$\max_w V(y_i, w, \theta(w; U)) \tag{28}$$

▶ Given $p(\theta)$ from worker's problem, find w that maximizes value of vacancy.

Free Entry

▶ Free entry implies that the expected value of opening a vacancy will be equal to the cost of opening it χ

▶ The expected value of opening a vacancy

$$\bar{V}(U) = \sum_{i=\iota(U)}^{n} f_i V(y_i, w_i^*(U), \theta_i^*(U))$$
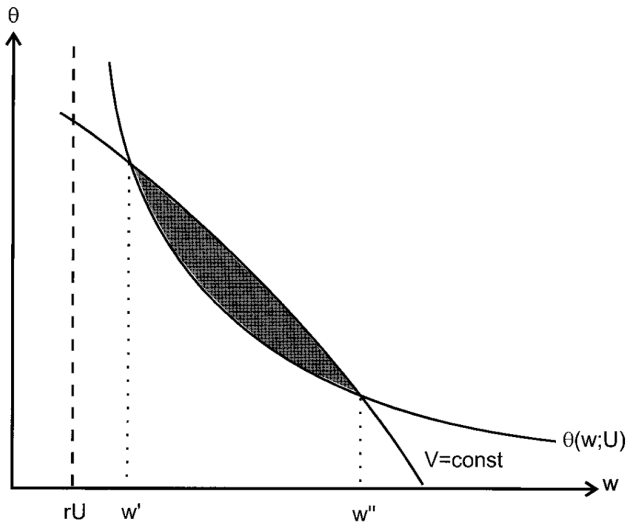
▶ equilibrium:

$$\bar{V}(U) = \kappa$$

Equilibrium Number of Markets

► We know that each productivity will form a separate market

► There are n productivities in the distribution

► All submarkets such that $w_i \geq rU$ will remain open
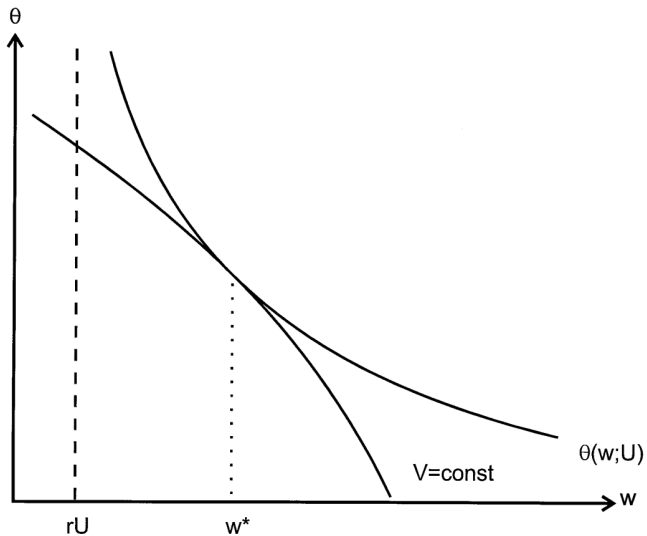
► Let ι denote the lowest submarket open

"Competitive" Search

▶ What is shaded region?

"Competitive" Search

▶ Inefficiency (rejected matches in DMP)

Equilibrium

▶ The resulting competitive equilibrium with frictional labor markets is characterized by the following equations

$$\bar{V}(U) = \chi \tag{29}$$

$$w_i = \arg\max V(y_i, w, \theta(w; U)), i \geq i_R \tag{30}$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))}, i \geq i_R \tag{31}$$

$$\dot{u}_i = 0; u_i p(\theta_i) = e_i \delta \tag{32}$$

$$\sum_i u_i = u \tag{33}$$

What is "valuable" about directed search?

- ▶ Submarkets are individually priced.
- ▶ i.e., contracts (w, θ) are known given the state of the worker and firm.
- ▶ Then, assuming that free entry binds in every open submarket, no longer need to condition on aggregate distributions as state variables (Menzio and Shi, 2011).
- ▶ So models are computationally tractable.
- ▶ Makes it possible to easily incorporate heterogeneity.

Conclusion

▶ Next class: block recursivity