

Macro II

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Introduction

- ▶ Today: Market structure
- ▶ Complete markets:
 - ▶ Arrow-Debreu structure (time-0 contingent claims);
 - ▶ Arrow securities (sequentially traded one-period claims).
- ▶ Exam: 3/27.
- ▶ Homework 3 (called HW 4 on class website) due before Spring Break (3/12).

Complete markets

- ▶ Individuals in the economy have access to a comprehensive set of risk-sharing contracts:
 - ▶ They can contract to insure against any event or sequence of events.
 - ▶ They write these contracts with other agents in the economy.
- ▶ Will lead to
 - ▶ Perfect risk sharing
 - ▶ i.e., representative agent.
- ▶ Note: This isn't an empirical statement. Not intended to exactly represent how individuals write contracts.
- ▶ We will see that a more realistic sequential formulation can provide the same allocation.

Complete markets

- ▶ Define unconditional probability of sequence of shocks $s^t = [s_0, s_1, \dots, s_t]$ to be $\pi_t(s^t)$.
- ▶ Assume there are $i = 1, \dots, I$ consumers, each of whom receives a stochastic endowment $y_t^i(s^t)$.
- ▶ They purchase a consumption plan that stipulates consumption for any history of shocks and yields:

$$U_i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u_i[c_t^i(s^t)] \pi_t(s^t)$$

- ▶ These contracts yield expected lifetime utility, where $\lim_{s \rightarrow 0} u'_i(c) = +\infty$

Complete markets

- ▶ They purchase a consumption plan that stipulates consumption for any history of shocks and yields:

$$U_i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u_i[c_t^i(s^t)] \pi_t(s^t)$$

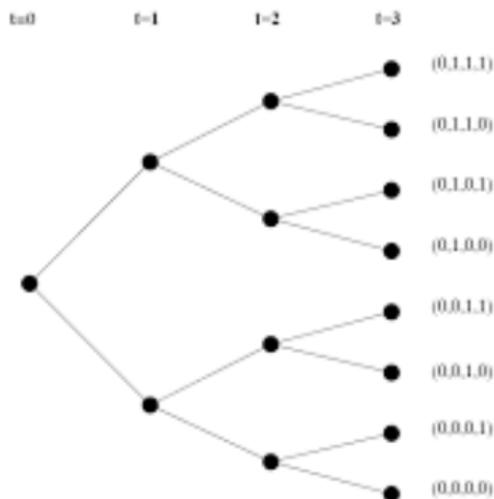
- ▶ And are subject to a feasibility constraint:

$$\sum_i c_t^i(s^t) \leq \sum_i y_t^i(s^t) \quad \forall t, s^t$$

- ▶ These contracts determine how to split resources at each t .
- ▶ i.e., they insure individuals ex-ante against income risk.

Contingent claims trading structure

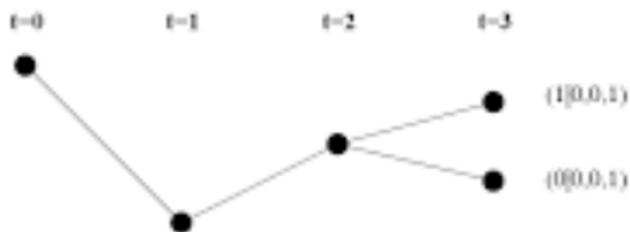
- ▶ Arrow-Debreu structure: contract at time $t = 0$ on every possible sequence of shocks.



- ▶ Each node represents a possible sequence of shocks.
- ▶ A consumption plan would specify consumption at each node at each time.

Sequential trading structure

- ▶ Arrow securities: re-contract at every t given the history of shocks s^t .



- ▶ At $t = 2$, contract for two possible realizations.

Trading structure

- ▶ Arrow-Debreu structure: contract at time $t = 0$ on every possible sequence of shocks.
- ▶ Arrow securities: re-contract at every t given the history of shocks s^t .
- ▶ Do these trading structures yield the same equilibrium allocation? Yes.
- ▶ Important property:
 - ▶ Under either structure, allocations are a function of the aggregate state only (& initial conditions).
 - ▶ i.e., allocation depends only on $\sum_{i=1}^I y_t^i(s^t)$
- ▶ Leads to representative agent structure.

Planner's Problem

- ▶ First, we will find the Pareto optimal allocation.
- ▶ i.e., the allocation from solving the Social Planner's problem:

$$\max_{c^i} W = \sum_{i=1}^I \lambda_i U_i(c^i)$$

- ▶ where λ_i is a "Pareto weight," i.e., how much Planner values individual i relative to others.
- ▶ Constrained maximization:

$$L = \sum_{t=0}^{\infty} \sum_{s^t} \left\{ \sum_{i=1}^I \lambda_i \beta^t u_i(c_t^i) \pi_t(s^t) + \theta_t(s^t) \sum_{i=1}^I [y_t^i(s^t) - c_t^i(s^t)] \right\}$$

- ▶ i.e., maximize weighted expected utility subject to the feasibility constraint (multiplier θ)

Planner's Problem

- ▶ Constrained maximization:

$$L = \sum_{t=0}^{\infty} \sum_{s^t} \left\{ \sum_{i=1}^I \lambda_i \beta^t u_i(c_t^i) \pi_t(s^t) + \theta_t(s^t) \sum_{i=1}^I [y_t^i(s^t) - c_t^i(s^t)] \right\}$$

- ▶ FOC in c_t^i :

$$\beta^t u_i'(c_t^i(s^t)) \pi_t(s^t) = \lambda_i^{-1} \theta_t(s^t)$$

- ▶ How is this allocated across consumers?

$$\begin{aligned} \frac{u_i'(c_t^i(s^t))}{u_1'(c_t^1(s^t))} &= \frac{\lambda_1}{\lambda_i} \\ \rightarrow c_t^i(s^t) &= u_i'^{-1}(\lambda_i^{-1} \lambda_1 u_1'(c_t^1(s^t))) \end{aligned}$$

- ▶ Often, assume $\lambda_i = \lambda_1 \forall i \rightarrow c_t^i(s^t) = u_i'^{-1}(u_1'(c_t^1(s^t)))$

Planner's Problem

- ▶ Allocation:

$$c_t^i(s^t) = u_i'^{-1}(\lambda_i^{-1} \lambda_1 u_1'(c_t^1(s^t)))$$

- ▶ Sub into resource constraint:

$$\sum_i u_i'^{-1}(\lambda_i^{-1} \lambda_1 u_1'(c_t^1(s^t))) = \sum_i y_t^i(s^t)$$

- ▶ i.e., the resource allocation depends only on aggregate endowment and weights of each consumer.

Decentralized allocations

- ▶ We know that the optimal allocation is given by

$$\sum_i u_i'^{-1}(\lambda_i^{-1} \lambda_1 u_1'(c_t^1(s^t))) = \sum_i y_t^i(s^t)$$

- ▶ Can we achieve the same allocation under different trading regimes?

- ▶ Specifically, does the decentralized economy achieve the same allocation?

Consumer's problem

- ▶ Consumer's problem: maximize

$$U_i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u_i[c_t^i(s^t)] \pi_t(s^t)$$

- ▶ subject to

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t)$$

Consumer's problem

- ▶ Yields the following:

$$\beta^t u'_i[c_t^i(s^t)] \pi_t(s^t) = \mu_i q_t^0(s^t)$$
$$\frac{u'_i(c_t^i(s^t))}{u'_1(c_t^1(s^t))} = \frac{\mu_i}{\mu_1}$$

- ▶ which implies

$$\sum_i u_i'^{-1}(\mu_1^{-1} \mu_i u_1'(c_t^1(s^t))) = \sum_i y_t^i(s^t)$$

Competitive Equilibrium

Definition A competitive equilibrium is a price system $\{q_t^0(s^t)\}_{t=0}^{\infty}$ and allocation $\{c^{i*}\}_{i \in \mathcal{I}}$ such that

1. Given a price system, each individual $i \in \mathcal{I}$ solves the following problem:

$$\begin{aligned} \{c_t^{i*}(s^t)\}_{t=0}^{\infty} = \arg \max_{\{c_t^i(s^t)\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t)) \pi_t(s^t) \\ \text{s.t.} & \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t) \end{aligned}$$

2. On every history s^t at time t , market clears

$$\sum_{i \in \mathcal{I}} c_t^{i*}(s^t) = \sum_{i \in \mathcal{I}} y_t^i(s^t)$$

Rules out economies with externalities, incomplete markets, etc.

First Welfare Theorem

- ▶ First welfare theorem:
Let c be a competitive equilibrium allocation. Then c is pareto efficient.

- ▶ Equivalence: Competitive equilibrium is a specific Pareto optimal allocation in which $\lambda_i = \mu_i^{-1}$.

Sequential trading

- ▶ Now, we will consider an economy with sequential trades.
- ▶ i.e., each period agents meet and trade state-contingent bonds
- ▶ Recall from asset pricing:

$$p_t = \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right)$$

- ▶ where the expectation is over realizations of s_{t+1} , which determines d_{t+1} .
- ▶ Price is determined by payout of asset across all different realizations.
- ▶ i.e., asset that provides good return across all realizations: expensive.

Market clearing

- ▶ Recall from asset pricing that the net bond position of the economy equaled zero.
- ▶ i.e., $\sum_i b_{t+1}^i = 0$.
- ▶ Same in this context.
- ▶ Some are borrowing and some are saving (in principle, if there were heterogeneity).
- ▶ This must net to zero.

Restriction: No Ponzi Schemes

- ▶ Must ensure that agents never take out too much debt.
- ▶ Natural debt limit:

$$A_t^i(s^t) = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} q_\tau^t(s^\tau) y_\tau^i(s^\tau)$$

- ▶ This is the amount that the agent could borrow and still commit to repay.
- ▶ Rules out Ponzi schemes.

Sequential problem

- ▶ Consumer's problem: maximize

$$U_i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u_i[c_t^i(s^t)] \pi_t(s^t)$$

- ▶ subject to

$$c_t^i + \sum_{s^{t+1}} Q_t(s_{t+1}|s^t) a_{t+1}^i(s_{t+1}, s^t) \leq y_t^i(s^t) + a_t^i(s^t)$$
$$-t + 1^i(s^{t+1}) \geq -A_{t+1}^i(s^{t+1})$$

- ▶ where Q_t is a pricing kernel: price of one unit of consumption given realization s_{t+1} and history s^t .

Sequential allocation

- ▶ Solving the previous problem yields the following Euler Equation:

$$Q_t(s_{t+1}|s^t) = \beta \left(\frac{u'(c_{t+1}^i(s^{t+1}))}{u'(c_t^i(s^t))} \pi_t(s^{t+1}|s^t) \right)$$

- ▶ Same as the asset pricing specification from earlier.
- ▶ Taking the expectation of this expression across all possible realizations of s^{t+1} yields the price, Q .

Sequential Trading - Competitive Equilibrium

Definition A competitive equilibrium is a price system

$\{\{Q_t(s_{t+1}|s^t)\}_{s_{t+1} \in S}\}_{t=0}^{\infty}$, an allocation

$\left\{ \left\{ \tilde{c}_t^i(s^t), \left\{ \tilde{a}_{t+1}^i(s_{t+1}, s^t) \right\}_{s_{t+1} \in S} \right\}_{t=0}^{\infty} \right\}_{i \in \mathcal{I}}$, an initial distribution of wealth

$\{a_0^i(s_0) = 0\}_{i \in \mathcal{I}}$, and a collection of natural borrowing limits

$\left\{ \left\{ \left\{ A_{t+1}^i(s_{t+1}, s^t) \right\}_{s_{t+1} \in S} \right\}_{t=0}^{\infty} \right\}_{i \in \mathcal{I}}$ such that

1. Given a price system, an initial distribution of wealth, and a collection of natural borrowing limits, each individual $i \in \mathcal{I}$ solves the workers problem.
2. On every history s^t at time t , markets clear.

$$\sum_{i \in \mathcal{I}} c_t^i(s^t) = \sum_{i \in \mathcal{I}} y_t^i(s^t) \quad (\text{Commodity market clearing})$$

$$\sum_{i \in \mathcal{I}} a_{t+1}^i(s_{t+1}, s^t) = 0 \forall s_{t+1} \in S \quad (\text{Asset market clearing})$$

Equivalence of allocations

- ▶ Is this allocation also a time-0 trading allocation?
- ▶ Yes. Suppose that the pricing kernel takes the following form

$$q_{t+1}^0(s^{t+1}) = Q_t(s_{t+1}|s^t)q_t^0(s^t)$$
$$\frac{q_{t+1}^0(s^{t+1})}{q_t^0(s^t)} = Q_t(s_{t+1}|s^t)$$

- ▶ That is, the price of 1 unit of consumption in period $t + 1$ is the same regardless of whether you purchased that consumption last period or in period 0.
- ▶ When this holds, sequential allocation coincides with time-0 trading allocation, subject to initial distribution.
- ▶ Formal proof (check on your own): [▶ formal proof](#)

Conclusion

- ▶ Next: Neoclassical growth and solution methods!

- ▶ Check website for homework.

Equivalence of allocations

$$\begin{aligned} Q_t(s_{t+1}|s^t) &= \frac{q_{t+1}^0(s^{t+1})}{q_t^0(s^t)} \Rightarrow \beta \frac{u'(\tilde{c}_{t+1}^i(s^{t+1}))}{u'(\tilde{c}_t^i(s^t))} \pi_t(s^{t+1}|s^t) \\ &= \beta \frac{u'(c_{t+1}^{i*}(s^{t+1}))}{u'(c_t^{i*}(s^t))} \pi_t(s^{t+1}|s^t) \end{aligned}$$

◀ back

Guess for portfolio

On every history s^t at time t ,

$$\tilde{a}_{t+1}^i(s_{t+1}, s^t) = \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q_\tau^0(s^\tau)}{q_{t+1}^0(s^{t+1})} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) \forall s_{t+1} \in S$$

Value of this portfolio expressed in terms of the date t , history s^t consumption good is $\sum_{s_{t+1} \in S} \tilde{a}_{t+1}^i(s_{t+1}, s^t) Q_t(s_{t+1} | s^t) =$

$$\begin{aligned} &= \sum_{s_{t+1} \in S} \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q_\tau^0(s^\tau)}{q_{t+1}^0(s^{t+1})} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) Q_t(s_{t+1} | s^t) \\ &= \sum_{s_{t+1} \in S} \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | (s_{t+1}, s^t)} \frac{q_\tau^0(s^\tau)}{q_{t+1}^0(s^{t+1})} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) \frac{q_{t+1}^0(s^{t+1})}{q_t^0(s^t)} \\ &= \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) \end{aligned}$$

Verify portfolio

On history $s^0 = s_0$ at time $t = 0$, assume that $a_0^i(s_0) = 0$. Then

$$\tilde{c}_0^i(s_0) + \sum_{s_1 \in S} \tilde{a}_1^i(s_1, s_0) Q_1(s_1 | s_0) = y_0^i(s_0) + 0$$

$$\tilde{c}_0^i(s_0) + \sum_{\tau=1}^{\infty} \sum_{s^\tau | s_0} \frac{q_\tau^0(s^\tau)}{q_0^0(s_0)} (c_\tau^{i*}(s^\tau) - y_t^i(s^\tau)) = y_0^i(s_0) + 0$$

$$q_0^0(s_0) c_0^{i*}(s_0) + \sum_{\tau=1}^{\infty} \sum_{s^\tau | s_0} q_\tau^0(s^\tau) (c_\tau^{i*}(s^\tau) - y_t^i(s^\tau)) = q_0^0(s_0) y_0^i(s_0)$$

$$(\text{ if } \tilde{c}_0^i(s_0) = c_0^{i*}(s_0))$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^{i*}(s^t)$$

Therefore, given $\tilde{c}_0^i(s_0) = c_0^{i*}(s_0)$, portfolio $\{\tilde{a}_1^i(s_1, s_0)\}_{s_1 \in S}$ is affordable.

Verify portfolio

On history s^t at time t , assume that

$\tilde{a}_t^i(s^t) = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right)$. Then

$$\begin{aligned} \tilde{c}_t^i(s^t) + \sum_{s_{t+1} \in S} \tilde{a}_{t+1}^i(s_{t+1}, s^t) Q_t(s_{t+1} | s^t) &= y_t^i(s^t) \\ &+ \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) \end{aligned}$$

$$\begin{aligned} \tilde{c}_t^i(s^t) + \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) &= y_t^i(s^t) \\ &+ \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} \left(c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau) \right) \end{aligned}$$

Verify portfolio

On history s^t at time t , assume that

$\tilde{a}_t^i(s^t) = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \frac{q_\tau^0(s^\tau)}{q_t^0(s^t)} (c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau))$. Then

$$\begin{aligned} q_t^0(s^t) c_t^{i*}(s^t) + \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} q_\tau^0(s^\tau) (c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau)) &= q_t^0(s^t) y_t^i(s^t) \\ &+ \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} q_\tau^0(s^\tau) (c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau)) \quad (\text{if } \tilde{c}_t^i(s^t) = c_t^{i*}(s^t)) \\ &\rightarrow \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} q_\tau^0(s^\tau) (c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau)) = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} q_\tau^0(s^\tau) (c_\tau^{i*}(s^\tau) - y_\tau^i(s^\tau)) \end{aligned}$$

Therefore, given $\tilde{c}_t^i(s^t) = c_t^{i*}(s^t)$, portfolio $\{\tilde{a}_{t+1}^i(s_{t+1}, s^t)\}_{s_{t+1} \in S}$ is affordable.

Conclusion

- ▶ Exam: 3/27.
- ▶ Homework 3 due next Thursday.
- ▶ Homework 4 due Tuesday before exam (3/25).