

Macro II

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Introduction

- ▶ There is a new homework online.
- ▶ All should have access to the cluster.
- ▶ Today:
 - ▶ Talk about Lucas Critique and Rational Expectations
 - ▶ Use simple two period model
 - ▶ Show intuition behind Lucas Critique.
- ▶ Lecture largely based on Eric Sims' (Notre Dame) notes.

Lucas Critique Overview

- ▶ Some history:
 - ▶ Prior to the late 1970s, macroeconomists had no systematic way of modeling consumer expectations.
 - ▶ They found *empirical relationships* between *equilibrium objects* and interpreted these as causal.
 - ▶ This is a problem!
 - ▶ (Old) Phillips Curve: inverse relationship between inflation and unemployment
 - ▶ more money → more demand → more employment.
 - ▶ This led policy makers to institute persistent inflation.
 - ▶ But this broke down in the 70s: we had stagflation: inflation and unemployment.
 - ▶ The reason is that consumers came to expect an increase in prices and adjusted their demand.
- ▶ Lucas Critique (broadly):
 - ▶ Need to use “deep” (structural) parameters to inform policy.
 - ▶ Otherwise policy may affect these parameters.

Basic two-period model

- ▶ A (very) basic consumption-savings model:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \quad (1)$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = w_1 + T_1 + \frac{w_2}{1+r} + \frac{T_2}{1+r} \quad (2)$$

- ▶ Simple set-up:
 - ▶ Household receives permanent income (w_1, w_2) , which might have a dependence structure.
 - ▶ They also may receive transitory income (T_1, T_2) , which does not have a dependence structure.
 - ▶ Define $y_t = w_t + T_t$
 - ▶ r is fixed over time, household takes as given.
 - ▶ Standard definitions for u : $u' > 0$, $u'' < 0$, $u'(0) = \infty$

Basic two-period model

- ▶ A (very) basic consumption-savings model:

$$V = \max_{c_1, c_2} u(c_1) + \beta u(c_2) \quad (3)$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad (4)$$

- ▶ Solve by first finding the Euler Equation:

$$\frac{\partial V}{\partial c_1} = u'(c_1) - \lambda = 0 \quad (5)$$

$$\frac{\partial V}{\partial c_2} = \beta u'(c_2) - \frac{\lambda}{1+r} = 0 \quad (6)$$

$$\rightarrow u'(c_1) = \beta(1+r)u'(c_2) \quad (7)$$

- ▶ We know dynamics, now need to pin down c_t using budget constraint (boundary condition).

Basic two-period model

- ▶ Dynamics and budget:

$$u'(c_1) = \beta(1+r)u'(c_2) \quad (8)$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad (9)$$

- ▶ Assume log utility: $u(c) = \ln(c)$.

- ▶ This yields

$$\frac{1}{c_1} = \beta(1+r)\frac{1}{c_2} \quad (10)$$

$$c_1 = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right) \quad (11)$$

- ▶ What does this tell us?

Basic two-period model

$$c_1 = \frac{1}{1 + \beta} \left(y_1 + \frac{y_2}{1 + r} \right) \quad (12)$$

- ▶ This tells us that consumption today is a function of
 - ▶ income today (not surprising)
 - ▶ income in the future (possibly a problem)
- ▶ Suppose there is a recession.
- ▶ A policymaker wants to implement a tax cut based on empirical evidence

Basic two-period model

- ▶ (Assume $\beta = \frac{1}{1+r}$ so that $c_2 = c_1 = c_t$.

$$c_t = \frac{1}{1+\beta} \left(y_t + \frac{y_{t+1}}{1+r} \right) \quad (13)$$

- ▶ Policymaker:

- ▶ Run the following regression:

$$c_t = \alpha + \gamma y_t + u_t \quad (14)$$

- ▶ Want to stimulate the economy.
 - ▶ Give people money, consumption will increase by γ !

Basic two-period model

$$c_t = \frac{1}{1+\beta} \left(w_t + T_t + \frac{w_{t+1}}{1+r} + \frac{T_{t+1}}{1+r} \right) \quad (15)$$

$$\hat{c}_t = \alpha + \gamma y_t \quad (16)$$

- ▶ Policy maker changes transitory income, T_t . Problem?
- ▶ Assume $\frac{\partial w_2}{\partial w_1} = 0$ (i.e., uncorrelated). Then

$$\frac{\partial c_t}{\partial w_t} = \frac{\partial c_t}{\partial T_t} = \frac{1}{1+\beta} \quad (17)$$

$$\frac{\partial \hat{c}_t}{\partial w_t} = \frac{\partial \hat{c}_t}{\partial T_t} = \gamma \quad (18)$$

- ▶ In this context, $\gamma = \frac{1}{1+\beta}$. We're good!

Basic two-period model

$$c_t = \frac{1}{1 + \beta} \left(w_t + T_t + \frac{w_{t+1}}{1 + r} + \frac{T_{t+1}}{1 + r} \right) \quad (19)$$

$$\hat{c}_t = \alpha + \gamma y_t \quad (20)$$

- Assume $\frac{\partial w_2}{\partial w_1} \neq 0$. Then

$$\frac{\partial c_t}{\partial y_t} = \frac{1}{1 + \beta} \left(1 + \frac{\frac{\partial w_{t+1}}{\partial w_t} \frac{\partial w_t}{\partial y_t}}{1 + r} \right) \quad (21)$$

$$\frac{\partial \hat{c}_t}{\partial y_t} = \gamma > \frac{\partial c_t}{\partial T_t} \quad (22)$$

$$\frac{\partial c_t}{\partial T_t} = \frac{1}{1 + \beta} \quad (23)$$

- If income was positively correlated (AR, etc.), we're not going to get the response we want.

Lucas critique overview

- ▶ In this context, policymakers might over predict the response of consumption.
- ▶ Why? Because consumers understand that this is a temporary increase in income.
- ▶ They won't believe that w_2 will increase.
- ▶ Therefore, they will respond less than predicted by the model.
- ▶ This is the crux of the Lucas Critique: that you need to find deep parameters that don't change with consumer behavior.

Lucas critique

- ▶ Lucas made his critique in the context of monetary policy.
- ▶ There had been multiple decades of inflation, aimed at reducing unemployment.
- ▶ Consumers eventually built in the expectation of inflation and this empirical relationship no longer held.
- ▶ Let's use a simple monetary policy model to understand what happened.

Phillips Curve

- ▶ Suppose that inflation is characterized by the following difference equation:

$$\pi_t = \theta(u_t - u^*) + \beta\mathbb{E}(\pi_{t+1}) \quad (24)$$

- ▶ What does this tell us?
 - ▶ If we hold expectations fixed,
 - ▶ an increase in current inflation, π_t ,
 - ▶ leads to a θ reduction in unemployment (in percentage points).
- ▶ θ was observed to be negative, ie inflation reduced unemployment.

Policymaker

$$\pi_t = \theta(u_t - u^*) + \beta\mathbb{E}(\pi_{t+1}) \quad (25)$$

- ▶ Suppose that an econometrician ran the following specification:

$$\pi_t = \gamma(u_t - u^*) + \epsilon_t \quad (26)$$

- ▶ They conclude that $\gamma < 0$.
- ▶ They tell the policymaker to raise inflation to reduce unemployment.

What happens?

$$\pi_t = \theta(u_t - u^*) + \beta\mathbb{E}(\pi_{t+1}) \quad (27)$$

$$\pi_t = \gamma(u_t - u^*) + \epsilon_t \quad (28)$$

- ▶ Well, as long as expectations don't change, the empirical specification will appear to hold.
- ▶ But if they change, consequences!
- ▶ An increase in inflation can lead to one of two things:
 1. a decrease in unemployment (good!) or
 2. an increase in expected future inflation
- ▶ and the equation will still hold.
- ▶ This is what we saw in the 1970s/1980s.

Log linearization

- ▶ Non-linear difference equations are tricky to solve.
- ▶ Macroeconomists often log-linearize these difference equations to make them easier to solve.
- ▶ Basic idea:
 - ▶ In some area around the steady state, deviations are small.
 - ▶ Can approximate using a log-linearized version of the model.
 - ▶ Will be “wrong,” but close as long as deviations stay small.
- ▶ Today, short refresher.

Log linearization II

- ▶ Take a generic difference equation with a single variable x :

$$x_{t+1} = Ax_t \quad (29)$$

- ▶ Suppose that $A = 1 + g$:

$$x_{t+1} = (1 + g)x_t \quad (30)$$

- ▶ Taking logs of both sides:

$$\ln(x_{t+1}) = \ln(1 + g) + \ln(x_t) \quad (31)$$

Taylor Series Approximation

- ▶ A first-order Taylor approximation of a function $f(x)$ around a point x^* is given by

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*) \quad (32)$$

- ▶ If $(x - x^*)$ is small and f'' is not too large, this approximation is reasonable.
- ▶ Idea: we know the value of a function at a particular point
- ▶ We can also find the derivative at that point.

Applying this to log-linear approximation

- ▶ Taylor series approximation of growth rate $(1 + g)$ at $g = 0$:

$$\ln(1 + g) \approx \ln(1 + 0) + \frac{1}{1 + 0}(1 + g - 1) \quad (33)$$

$$\approx g \quad (34)$$

- ▶ This means that we can approximate our difference equation as

$$\ln(x_{t+1}) = \ln(1 + g) + \ln(x_t) \quad (35)$$

$$\approx \ln(x_t) + g \quad (36)$$

- ▶ This insight will prove very useful in solving macro models.

Next Time

- ▶ Start dynamic programming.
- ▶ Homework due 2/19 (next Thursday).