

# Macro II

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# Announcements

- ▶ Hand back exams on Tuesday.
- ▶ Homework 5 due next Thursday, homework 6 due April 30th.
- ▶ Today: Start heterogeneous agent model.
- ▶ First: Huggett and Aiyagari.
- ▶ Schedule: no class on Thursday (4/30)

# Thinking about Uncertainty in Macroeconomic Models

- ▶ Typical assumptions in macroeconomics are a convex combination of

1. certainty equivalence:

$$u'(\bar{c}_{i,t}) = \beta E \left[ \underbrace{(1 + r_{t+1})}_{GE} \underbrace{u'(\bar{c}_{i,t+1})}_{\text{Closer to Linear}} \right] \quad (1)$$

2. linearized decision rules:

$$\sum_{i=1}^N ((1 + r_{t+1})a_{i,t+1} + w_{i,t+1} - c_{i,t+1} - a_{i,t+2}) = 0 \quad (2)$$

$$\sum_{i=1}^N ((1 + r_{t+1})\beta_a \hat{S}_{i,t+1} + \beta_w (\hat{S}_{i,t+1}) - \beta_c \hat{S}_{i,t+1} - \beta_a \hat{S}_{i,t+2}) = 0 \quad (3)$$

- ▶ Trick in Krusell-Smith: assume that workers make a linear prediction about prices in the future.
- ▶ i.e., workers use OLS to predict future prices.

## Heterogeneous Agent Models

- ▶ Workers change their behavior in response to uncertainty.
- ▶ First wave of heterogeneous agent models: how do aggregates change when *individual idiosyncratic* uncertainty is uninsurable.
- ▶ In other words: when agents must accumulate *precautionary savings* to insure against income shocks.
- ▶ Key “first wave” papers (no particular order):
  - ▶ Huggett (1993): Incomplete markets exchange economy with GE interest rate.
  - ▶ Imrohroglu (1989): Individual and aggregate uncertainty with fixed interest rate.
  - ▶ Aiyagari (1994): Incomplete markets production economy with GE interest rate.
  - ▶ Bewley (1986): Individual uncertainty with fixed interest rate.
- ▶ Krusell and Smith (1998): individual and aggregate uncertainty with GE interest rate.
- ▶ Do this using an approximation to the aggregate evolution of capital.

# Heterogeneous Agent Models

- ▶ We can write a generic worker's problem as

$$\max_{\{c_t, i_t, l_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (4)$$

$$\text{s.t. } c_t + i_t \leq r_t a_t + w_t l_t \quad (5)$$

$$a_{t+1} = (1 - \delta) a_t + i_t \quad (6)$$

$$a_{t+1} \geq \underline{a}_t \quad (7)$$

$$w_t \sim F \quad (8)$$

$$c_t \geq 0, l_t \geq 0, a_0 \text{ given} \quad (9)$$

- ▶ How we deal with prices  $r_t$ ,  $w_t$  and choices  $c_t$ ,  $i_t$ ,  $l_t$  is central to equilibrium.

## Recursive Formulation

- ▶ Can be written as

$$V(a) = u(c) + \beta E[V(a')] \quad (10)$$

$$\text{s.t. } c + i \leq ra + wl \quad (11)$$

$$a' = (1 - \delta)a + i \quad (12)$$

$$a' \geq \underline{a} \quad (13)$$

$$w \sim F \quad (14)$$

$$c \geq 0, l \geq 0, a_0 \text{ given} \quad (15)$$

- ▶ Under fairly general conditions, this inherits same properties as non-stochastic version.

## Huggett (1993)

- ▶ Endowment economy, no aggregate risk.
- ▶ Setup:
  - ▶ Discrete time;
  - ▶ Continuum of heterogeneous agents;
  - ▶ Idiosyncratic endowment risk (labor income stochastic).
- ▶ Single bond,  $a$ , can be borrowed or saved.
- ▶ Borrowing limit,  $\underline{a} \leq 0 \leq a_{it}$

# Idiosyncratic Markov Income Uncertainty

- ▶ Suppose  $wl = e$ ,  $F[e'] = \pi(e'|e)$
- ▶ Two states:  $e_l, e_h$
- ▶ Can be written as

$$V(a, e) = u(c) + \beta \sum_{e'} \pi(e'|e) V(a', e') \quad (16)$$

$$\text{s.t. } c + a' \leq (1 + r)a + e \quad (17)$$

$$a' \geq \underline{a} \quad (18)$$

$$c \geq 0, a_0 \text{ given} \quad (19)$$

- ▶ Agents want to build precautionary savings against idiosyncratic risk.

# Equilibrium

- ▶ Define a distribution of agents over assets  $a$  and endowments  $e, \psi$ .
- ▶ Stationary equilibrium: aggregate state ( $\psi$ ) is unchanging.
- ▶ Agents move around distribution, but LLN  $\rightarrow \psi' = \psi$
- ▶ Define  $\psi(B)$  such that given transition function  $P$ :

$$\psi(B) = \int_S P(x, B) d\psi \quad (20)$$

- ▶  $P(x, B)$  the probability that an agent with state  $x$  will have state  $B \in \beta_S$  next period.
- ▶  $B$  is a subset of the state space.

# Stationary Equilibrium

- ▶ Roughly summarizing Huggett, 1993: A stationary equilibrium for this economy is a tuple  $(c, a', r, \psi)$  that satisfy
  1.  $c$  and  $a'$  solve the workers problem taking prices as given.
  2. Markets clear:
    - 2.1 consumption = production:  $\int c(x)d\psi = \int ed\psi$
    - 2.2 no net savings:  $\int a(x)d\psi = 0$
  3.  $\psi$  is stationary:

$$\psi(B) = \int_S P(x, B)d\psi \quad (21)$$

for all  $B \in \beta_S$

## Aiyagari (1994)

- ▶ Production economy, no aggregate risk.
- ▶ Firms employ capital, households save using capital (really assets loaned/borrowed from firm).
- ▶ Setup:
  - ▶ Discrete time;
  - ▶ Continuum of heterogeneous agents;
  - ▶ Idiosyncratic hours shocks (labor supply stochastic).
- ▶ Capital,  $k$ , can be borrowed or saved.
- ▶ Borrowing limit,  $\underline{k} \leq 0 \leq k_{it}$

# Heterogeneous Agent Production Economy

- ▶ In a production economy, the agent's problem is given by

$$V(k, \epsilon; \psi) = u(c) + \beta E[V(k' \epsilon'; \psi')] \quad (22)$$

$$\text{s.t. } c + k' \leq (1 + r(K, L) - \delta)k + w(K, L)\epsilon \quad (23)$$

$$k' \geq \underline{k} \quad (24)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon) \quad (25)$$

$$\psi' = \Psi(\psi) \quad (26)$$

$$c \geq 0, k \geq 0, k_0 \text{ given} \quad (27)$$

- ▶  $\epsilon$  is a markov process that yields hours worked.
- ▶  $\Psi$  is an unspecified evolution of the aggregate state  $(k, \epsilon)$ .
- ▶ Prices are determined from the firm's problem

## Prices - The Firm's Problem

- ▶ How we handle prices determines the difficulty of this problem.
- ▶ In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} F(K, L) - wL - rK \quad (28)$$

- ▶ This yields standard competitive prices for the rental rates.

# Information

- ▶ What information do workers need in order to be able to solve this problem?
- ▶ Current period:
  - ▶ interest rate,  $r(K, L)$ . This is known from being told the aggregates at the beginning of the period.
  - ▶ wage rate,  $w(K, L)$ . This is known from being told the aggregates at the beginning of the period.
- ▶ Future:
  - ▶ interest rate and wage rate next period.
  - ▶ These depend on capital and labor next period.
  - ▶ Thus, workers need to predict capital and labor in future.
- ▶ Rep. Agent model: just need to know their own decision rule.
- ▶ Here: need to know distribution across workers, and their decision rules.

# Stationary Recursive Competitive Equilibrium

- A stationary RCE is given by pricing functions  $r, w$ , a worker value function  $V(k, \epsilon; \psi)$ , worker decision rules  $k', c$ , a type-distribution  $\psi(k, \epsilon)$ , and aggregates  $K$  and  $L$  that satisfy
1.  $k'$  and  $c$  are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules:  $\psi$  is the stationary distribution implied by worker decision rules.
  4. Aggregates are consistent with individual policy rules:  
$$K = \int k d\psi, L = \int \epsilon d\psi$$

# Return to Capital

- ▶ How does return to capital vary by
  - ▶ serial corr. ( $\rho$ ) in labor income (think AR1 process)
  - ▶ and CRRA ( $\mu$ )?

TABLE II

A. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.2$ )			
$\rho \backslash \mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

  

B. Net return to capital in %/aggregate saving rate in % ( $\sigma = 0.4$ )			
$\rho \backslash \mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	-0.3456/37.63

- ▶ Higher  $\rho$  or  $\mu$ , more saving, lower return.

## Krussell-Smith (1998)

- ▶ In the previous model, we relied on the aggregate *certainty* of  $\psi(k, \epsilon)$  for a solution by appealing to the law of large numbers.
- ▶ i.e., individuals move around the distribution, but those shocks offset and in the aggregate the distribution is unchanged.
- ▶ But what happens if there is aggregate *uncertainty*?
- ▶ Now the distribution changes in the equilibrium, and we need a way to incorporate this into worker decision rules.
- ▶ Krussell-Smith: Aiyagari + aggregate shocks.
- ▶ Some details:
  - ▶ Idiosyncratic labor shock  $\{0,1\}$  markov.
  - ▶ Aggregate shocks.
  - ▶ Idiosyncratic shock prob. changes with agg. shocks.

# Aggregate Uncertainty

- ▶ In a production economy, the agent's problem is given by

$$V(k, \epsilon, z; \psi) = u(c) + \beta E[V(k'\epsilon', z'; \psi')] \quad (29)$$

$$\text{s.t. } c + k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon \quad (30)$$

$$k' \geq \underline{k} \quad (31)$$

$$z' = \text{Markov}P(z'|z) \quad (32)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \quad (33)$$

$$\psi' = \Psi(\psi, z, z') \quad (34)$$

$$c \geq 0, k \geq 0, k_0 \text{ given}, z_0 \text{ given} \quad (35)$$

- ▶  $\epsilon$  is a markov process for employment  $\epsilon \in \{0, 1\}$
- ▶  $\Psi$  is an unspecified evolution of the aggregate state.
- ▶  $z$  *also* evolves as a markov process.
- ▶ Prices are determined from the firm's problem.

## Prices - The Firm's Problem

- ▶ How we handle prices determines the difficulty of this problem.
- ▶ In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} zF(K, L) - wL - rK \quad (36)$$

- ▶ This yields standard competitive prices for the rental rates.

# Laws of Motion

- ▶ The future aggregate state enters the probability of employment.
- ▶ This means that it impacts **all** of the laws of motion:

$$z' = \text{Markov}P(z'|z) \quad (37)$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \quad (38)$$

$$k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon - c \quad (39)$$

$$\psi' = \Psi(\psi, z, z') \quad (40)$$

- ▶ Because shocks to  $z$  change employment status and prices.

# Recursive Competitive Equilibrium

- ▶ An RCE is given by pricing functions  $r, w$ , a worker value function  $V(k, \epsilon, z; \psi)$ , worker decision rules  $k', c$ , a type-distribution  $\psi(k, \epsilon)$ , and aggregates  $K$  and  $L$  that satisfy
  1.  $k'$  and  $c$  are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules:  $\psi$  is the distribution implied by worker decision rules given the aggregate state.
  4. Aggregates are consistent with individual policy rules:  
$$K = \int k d\psi, L = \int \epsilon d\psi$$

# Type Distribution

- ▶ The type distribution is a problem.
- ▶ Each policy function and transition depends on the type distribution.
- ▶ But the type distribution is time-varying in response to aggregate shocks.
- ▶ Alternative: use a smaller number of moments that can be calculated quickly to characterize the type distribution.
- ▶ Like a “sufficient statistic” for the type distribution.
- ▶ Discuss the solution to this next time.

# Business Cycle Effects

- ▶ This model is built to handle stochastic shocks.
- ▶ How do heterogeneous agents respond over a business cycle?

TABLE 2  
AGGREGATE TIME SERIES

Model	Mean ( $k_t$ )	Corr( $c_t, y_t$ )	Standard Deviation ( $i_t$ )	Corr( $y_t, y_{t-4}$ )
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$ :				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic- $\beta$ :				
Incomplete markets	11.78	.825	.027	.459

## Conclusion

- ▶ Next time: Solving heterogeneous agent models.