

# Macro II

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# Announcements

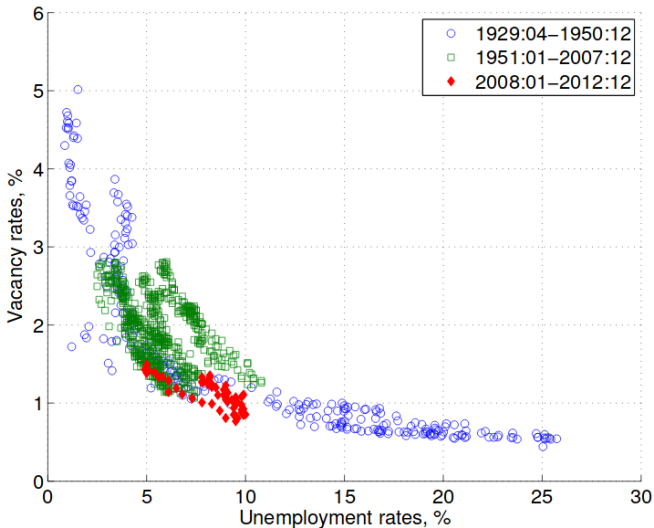
- ▶ Today: the Mortensen and Pissarides model (canonical equilibrium search)
- ▶ Heterogeneous agent code emailed this morning.

# Arrival Rates of Job Offers

- ▶ Last time: we assumed that the arrival rate of job offers is *exogenous*: regardless of equilibrium, the frequency with which you receive an offer is the same.
- ▶ Consider an example:
  1. There is a productivity downturn:
  2. How does a firm respond?
  3. McCall model: the quality of the offer distribution deteriorates, but searchers receive offers at the same rate.
- ▶ Essentially, slackness in the labor market is due to worker selectivity, not due to decisions made by the firm.
- ▶ Obviously, firms do respond.

# The Beveridge Curve

- ▶ Another implication: there is no relationship between unemployment and vacancy creation.



# The DMP Model (“Ch. 1 of Pissarides (2000)”)

▶ Agents:

1. Employed workers;
2. unemployed workers;
3. vacant firms;
4. matched firms.

▶ Linear utility ( $u = b$ ,  $u = w$ ) and production  $y = p > b$ .

▶ Matching function:

1. Determines *number* of meetings between firms & workers.
2. Args: levels searchers & vacancies ( $U = u \times L$ ,  $V = v \times L$ )
3. Constant returns to scale ( $L$  is lab. force):

$$M(uL, vL) = uL \times M\left(1, \frac{v}{u}\right) = uL \times p(\theta)$$

4. where  $\theta = \frac{v}{u}$  is “labor market tightness”
5. Match rates:

$$\underbrace{p(\theta)}_{\text{Worker}} = \theta \underbrace{q(\theta)}_{\text{Firm}}$$

# Worker Value Functions

- ▶ Value functions:
  1. Employed at wage  $w$ :  $W(w)$
  2. Unemployed:  $U$ .
- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

- ▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

## Firm Value Functions

- ▶ Value functions:
  1. Filled, paying wage  $w$ :  $J(w)$
  2. Vacant  $V$ .
- ▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V]$$

- ▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

- ▶ Free entry equilibrium condition:

$$\begin{aligned} rV &= 0 \\ \rightarrow \frac{\kappa}{E[J(w)]} &= q(\theta) \end{aligned}$$

- ▶ This is just a market clearing condition!

# Equilibrium Objects

- ▶ Three key equilibrium objects:
  1. Wages;
  2. unemployment;
  3.  $\theta = \frac{v}{u}$  (vacancies).
- ▶ How we determine each of these is largely a modeling decision.
- ▶ Steady-state: pin down unemployment via flow equation.
- ▶ Free-entry: Assume that firms always post vacancies so that free entry binds.
- ▶ Wages: Assume that wages are determined by a surplus-(profit) sharing rule.

# Steady-State Unemployment

- ▶ Flow of unemployment:

$$\dot{u} = \delta(1 - u) - p(\theta)u$$

- ▶ Steady-state:

$$\begin{aligned}0 &= \delta(1 - u) - p(\theta)u \\ p(\theta)u &= \delta(1 - u) \\ u &= \frac{\delta}{\delta + p(\theta)}\end{aligned}$$

- ▶ Same as McCall with  $\alpha = p(\theta)$ .
- ▶ (Note: no heterogeneity &  $p > b \rightarrow$  all wages accepted.)

## Free Entry

- ▶ Free entry  $V = 0$ :

$$rJ(w) = (p - w) + \delta[V - J(w)]$$
$$(r + \delta)J(w) = (p - w)$$

- ▶ Vacancy creation condition (i.e., free entry imposed):

$$q(\theta) = \frac{\kappa}{E[J(w)]}$$
$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$
$$\theta = q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)$$

- ▶ Thus, mapping between wages and  $\theta$ . 1 equation, 2 unknowns.
- ▶ Need equation to determine wages in equilibrium.

# Wage Determination

- ▶ Workers and firms bargain over the surplus of a match.
- ▶ Surplus of a match:

$$S(w) = W(w) + J(w) - U - \cancel{V}$$

$$S(w) = W(w) + J(w) - U$$

- ▶ Nash Bargaining splits this surplus according to a bargaining weight,  $\beta$ :

$$w = \underset{\text{Net Utility}}{\operatorname{argmax}_w (W(w) - U)^\beta} \underset{\text{Net Profits}}{(J(w) - V)^{1-\beta}}$$

- ▶ Insight from the interwebs: “When Nash Bargaining, you are really just geometrically maximizing expected utility with respect to your uncertainty about your identity”

## Wage Determination

- ▶ Nash Bargaining splits this surplus according to a bargaining weight,  $\beta$ :

$$w = \underset{\text{Net Utility}}{\operatorname{argmax}_w} \underbrace{(W(w) - U)^\beta}_{\text{Net Profits}} \underbrace{(J(w) - V)^{1-\beta}}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta} \frac{\partial W}{\partial w} + (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U) \frac{\partial J}{\partial w}$$

- ▶  $\frac{\partial W}{\partial w} = 1$ ,  $\frac{\partial J}{\partial w} = -1$  (no endogenous separations/OTJS):

$$\beta \left( \frac{J(w)}{W(w) - U} \right)^{1-\beta} = (1 - \beta) \left( \frac{W(w) - U}{J(w)} \right)^\beta$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

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## Wage Determination

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$$w = \underset{w}{\operatorname{argmax}} \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$w \text{ solves } (W(w) - U) = \beta(W(w) + J(w) - U) = \beta S(w)$$

- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

- ▶ Employed flow value:

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- ▶ Matched flow value:

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- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

- ▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

- ▶ Plug in for the worker v-funs:

$$(1 - \beta)[W(w) - U] = \beta J(w)$$

$$\beta J(w) = (1 - \beta)[w - \delta(U - V(w)) - b - p(\theta)(W(w) - U)]$$

$$(1 - \beta)(w - b) = \beta J(w) + (1 - \beta)(p(\theta) + \delta)[W(w) - U]$$

## Wage Determination

- ▶ Nash Bargaining splits this surplus according to a bargaining weight,  $\beta$ :

$$w = \underset{w}{\operatorname{argmax}} \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$w \text{ solves } (W(w) - U) = \beta(W(w) + J(w) - U) = \beta S(w)$$

- ▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

- ▶ Plug in:

$$\begin{aligned}\beta J(w) &= (1 - \beta)[w - \delta(U - V(w)) - b - p(\theta)(W(w) - U)] \\ (1 - \beta)(w - b) &= \beta J(w) + (1 - \beta)(p(\theta) + \delta)[W(w) - U] \\ (1 - \beta)(w - b) &= \beta(p - w - \delta J(w)) + (1 - \beta)(p(\theta) + \delta)[W(w) - U]\end{aligned}$$

## Wage Determination

- ▶ Note that  $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

- ▶ And  $(1 - \beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1 - \beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta \frac{J(w)}{1 - \beta} \\ w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

- ▶ Free entry condition:  $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

# Computation

- ▶ How would we solve this model?
- ▶ Need way to compute three equilibrium objects:
  1. Wages;
  2. unemployment;
  3.  $\theta = \frac{v}{u}$  (vacancies).
- ▶ How we determine each of these is largely a modeling decision.
- ▶ Steady-state: pin down unemployment via flow equation.
- ▶ Free-entry: Assume that firms always post vacancies so that free entry binds.
- ▶ Wages: Assume that wages are determined by a surplus-(profit) sharing rule.
- ▶ Computation:
  - ▶ Wages, vacancies: depend on surplus.
  - ▶ Unemployment: law of motion.
- ▶ Here: add aggregate shocks.

# Worker Value Functions

- ▶ Value functions:
  1. Employed at wage  $w$ :  $W(w)$
  2. Unemployed:  $U$ .
- ▶ Unemployed flow value:

$$rU(z) = b + p(\theta)E[W(w, z) - U(z)] + \gamma E[U(z') - U(z)]$$

- ▶ Employed flow value:

$$rW(w, z) = w(z) + \delta[U(z) - W(w, z)] \\ + \gamma E[W(w', z') - W(w, z)]$$

# Firm Value Functions

▶ Value functions:

1. Filled, paying wage  $w$ :  $J(w)$
2. Vacant  $V$ .

▶ Vacant flow value:

$$rV(z) = -\kappa + q(\theta(z))E[J(w, z) - V(z)] + \gamma[V(z') - V(w, z)]$$

▶ Matched flow value:

$$\begin{aligned} rJ(w, z) &= (z + p - w) + \delta[V(z) - J(w, z)] \\ &\quad + \gamma[J(w', z') - J(w, z)] \end{aligned}$$

▶ Free entry equilibrium condition:

$$\begin{aligned} rV &= 0 \\ \rightarrow \frac{\kappa}{E[J(w, z)]} &= q(\theta) \end{aligned}$$

## Computation

- ▶ Surplus of a match:

$$S(w, z) = W(w, z) + J(w, z) - U(z) - \cancel{V(z)}$$

$$S(w, z) = W(w, z) + J(w, z) - U(z)$$

- ▶ Plugging in and using  $\beta S(w, z)$  is workers surplus and  $(1 - \beta)S(w, z)$  is firm surplus:

$$S(z) = \frac{p + z}{r + \delta + \gamma} - \frac{b + \theta \kappa \frac{\beta}{1 - \beta}}{r + \delta + \gamma} + \frac{\gamma}{r + \delta + \gamma} \int_{z'} S(x) dF(x)$$

- ▶ This is just a contraction:  $\frac{\gamma}{r + \delta + \gamma} < 1$ .
- ▶ Pick  $S_0(z_i) = 0, \forall i$  and iterate.
- ▶ Yields vacancies  $q(\theta) = \frac{\kappa}{(1 - \beta)S(z)}$ .

## Hazard Rate (from last time)

- ▶ What is the hazard rate of unemployment?
- ▶ Rate of leaving unemployment at time  $t$ .

$$H_u(t) = \alpha \int_{w_R}^{\bar{w}} dF(w) \quad (1)$$

$$= \alpha(F(\bar{w}) - F(w_R)) \quad (2)$$

$$= \underbrace{\alpha}_{\text{MeetingRate}} \underbrace{(1 - F(w_R))}_{\text{Selectivity}} \quad (3)$$

- ▶ Note, almost every search model generates a hazard composed of the product of a meeting probability and worker selectivity.
- ▶ This is important to remember.
- ▶ Hazard rate of employment (leaving employment for unemployment)?

$$H_e(t) = \delta \quad (4)$$

- ▶ Because separations are independent of state.

## Dynamics of Unemployment

- ▶ Use hazard rates to understand dynamics and steady-state.
- ▶ What does the model predict about employment and unemployment?

$$\dot{u} = \delta(1 - u) - \alpha(1 - F(w_R))u \quad (5)$$

$$\dot{e} = \alpha(1 - F(w_R))(1 - e) - \delta e \quad (6)$$

- ▶ Steady-state:  $\dot{u} = 0$ ,  $\dot{e} = 0$ :

$$0 = \delta(1 - u) - \alpha(1 - F(w_R))u \quad (7)$$

$$\rightarrow u = \frac{\delta}{\delta + \alpha(1 - F(w_R))} \quad (8)$$

$$0 = \alpha(1 - F(w_R))(1 - e) - \delta e \quad (9)$$

$$\rightarrow e = \frac{\alpha(1 - F(w_R))}{\alpha(1 - F(w_R)) + \delta} \quad (10)$$

## Next Time

- ▶ One of:
  - ▶ Endogenous separations (probably);
  - ▶ Efficiency in search (Hosios Condition);
  - ▶ or Directed/competitive search.
- ▶ HW6 due 5/8