

# Macro II

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# Announcements

- ▶ Today:
  1. Random vs. directed search
  2. Change primitive: workers can observe wage offers prior to match.
  3. i.e., Moen (1997)
- ▶ Homework due next Thursday.
- ▶ Two more classes (no class 4/30).

# Random vs. Directed Search

- ▶ How do workers find jobs?
- ▶ How much information do they have about a job before applying?
- ▶ Two extremes:
  1. Random Search: *no* information about a job prior to receiving offer.
  2. Directed Search: *all* information about a job prior to application.
- ▶ Why does this matter?
  1. Random search is generically inefficient: one worker may reject a job offer than another would accept.
  2. Directed search is generically efficient: by applying for a job, a worker signals that the job is already acceptable.
- ▶ As we will see next time, it also changes computational complexity.

## Random vs. Directed Search II

- ▶ Empirically, how can we tell them apart?
- ▶ Hazard rate to wage  $w$  generically:

$$H_U(w) = \underbrace{\lambda(w)}_{\text{Arrival Rate}} \times \underbrace{f(w)}_{\text{Prob. Offer} = W} \quad (1)$$

- ▶ Random search:

$$H_U(w) = \underbrace{\lambda}_{\text{Arrival Rate}} \underbrace{[1 - F(w_R)]}_{\text{Selectivity}} \underbrace{f(w)}_{P(\text{Offer} = W)} \quad (2)$$

- ▶ Directed search:

$$H_U(w) = \underbrace{\lambda(w)}_{\text{Arrival}} \underbrace{[1 - F(w_R)]}_{F(w_R)=0} \underbrace{f(w)}_{P(\text{Offer} = w)} \quad (3)$$

$$= \underbrace{\lambda(W)}_{\text{Arrival}} \underbrace{f(w)}_{P(w=w_j)=1} \quad (4)$$

$$= \underbrace{\lambda(w)}_{\text{Arrival Rate of Wage } w} \quad (5)$$

## Some Evidence

- ▶ (First couple Borrowed from Shouyong Shi)
- ▶ Hall and Krueger (08):
  1. 84% had information on wage prior to first interview.
- ▶ Holzer, Katz, and Krueger (91)
  1. Firms in high-wage industries receive more applications than low-wage industries, controlling for observables.
- ▶ Braun, Engelhardt, Griffy, and Rupert: unemployment insurance changes  $\lambda(w) \rightarrow$  inconsistent with random search.

## Mortensen and Pissarides Model

- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U] \quad (6)$$

- ▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)] \quad (7)$$

- ▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V] \quad (8)$$

- ▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)] \quad (9)$$

- ▶ Free entry equilibrium condition:

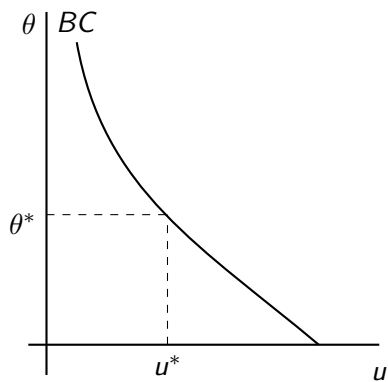
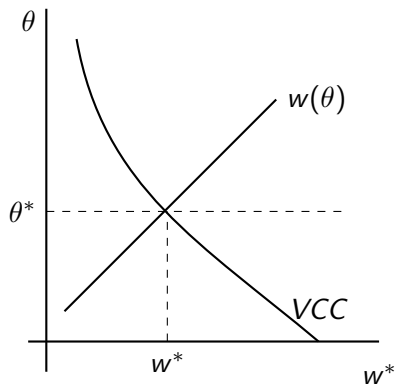
$$V = 0 \quad (10)$$

$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta) \quad (11)$$

# Equilibrium

- ▶ The equilibrium we have described is a steady-state equilibrium characterized by value functions  $U, W, J, V$ , a wage function  $w$ , a market tightness function  $\theta$ , and steady-state level unemployment  $u$ , such that
  1. A steady-state level of unemployment, derived from the flow unemployment equation.
  2. A wage rule that splits the surplus of a match according to a sharing rule with bargaining weight  $\beta$
  3. A free entry condition that determines  $\theta$  given wages and steady-state unemployment.
- ▶ What were these policy functions?
  1.  $w = (1 - \beta)b + \beta p + \beta \theta \kappa$
  2.  $\theta = q^{-1} \left( \frac{\kappa(r + \delta)}{p - w} \right)$
  3.  $u = \frac{\delta}{\delta + p(\theta)}$

# Equilibrium



## Directed/Competitive Search

- ▶ In DMP, wages are negotiated/revealed after meeting.
- ▶ This can create inefficiency:
  1. Consider example with unemployed workers A and B.
  2.  $w_R^A = 10$ ,  $w_R^B = 15$
  3. Firm pays a cost  $\kappa$  to open a vacancy and posts a wage 12.
  4. Both worker A and B apply for the job. Firm randomly picks worker B.
  5. Worker B rejects job that would have been acceptable to worker A.
- ▶ Directed search: Worker B applies for different job with  $w \geq w_R^B$ .
- ▶ (Directed and competitive search generally used interchangeably).

# The Competitive Search Model (Moen, 1997)

- ▶ Agents:
  1. Employed workers employed in submarket  $i$ ;
  2. unemployed workers considering searching in  $i \in \{1, \dots, N\}$ ;
  3. unmatched firms indexed by productivity  $y_i \in y_1, \dots, y_N$ ;
  4. matched firms indexed by productivity  $y_i \in y_1, \dots, y_N$ ;
  5. “Market Maker”: benevolent overlord who announces eqm.  $w_i$ .
- ▶ Linearity: ( $u = z, u = w_i$ ) and  $y = y_i > z$  in open submarkets.
- ▶ Matching function:
  1. Determines *number* of meetings between firms & workers in submarket  $i$ :

$$M(u_i L_i, v_i L_i) = u_i L_i \times M\left(1, \frac{v_i}{u_i}\right) = u_i L_i \times p(\theta_i) \quad (12)$$

2. where  $\theta_i = \frac{v_i}{u_i}$  is “submarket tightness”
3. Match rates:

$$\underbrace{p(\theta_i)}_{\text{Worker wage } i} = \theta_i \underbrace{q(\theta_i)}_{\text{Firm wage } i} \quad (13)$$

- ▶  $i$  indexes both the productivity and wage.

# Worker Value Functions

- ▶ Value functions:
  1. Employed in submarket  $i$ :  $W_i$
  2. Unemployed and searching in submarket  $i$ :  $U_i$ .
  3. Unemployed:  $U = \max\{U_1, \dots, U_N\}$ .
- ▶ Unemployed flow value in submarket  $i$ :

$$rU_i = z + p(\theta_i)(W_i - U_i) \quad (14)$$

- ▶ Employed flow value in submarket  $i$ :

$$rW_i = w_i + \delta(U_i - W_i) \quad (15)$$

- ▶ Both problems are stationary: optimal choice of  $i$  true  $\forall t$ .

## Worker Value Functions II

- ▶ We can solve for match rates:

$$rU_i = z + p(\theta_i)(W_i - U_i) \quad (16)$$

$$(r + p(\theta_i))U_i = z + p(\theta_i)\frac{w_i + \delta U_i}{r + \delta} \quad (17)$$

$$(r + \delta)(r + p(\theta_i))U_i - p(\theta_i)\delta U_i = (r + \delta)z + p(\theta_i)w_i \quad (18)$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))} \quad (19)$$

$$p(\theta_i) = \frac{rU_i - z}{w_i - rU_i}(r + \delta) \quad (20)$$

$$(21)$$

- ▶  $U = \max\{U_1, \dots, U_N\}$  and ex-ante homogeneity among workers implies

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + \delta) \quad (22)$$

$$(23)$$

## Firm Value Functions

- ▶ Pays a cost  $\chi$  to draw productivity.
- ▶ Firm observes own productivity, chooses to open vacancy given submarkets  $(w, \theta)$ .
- ▶ Value functions:
  1. Vacant with productivity  $y_i$ :  $V(y_i, w, \theta)$
  2. Filled with productivity  $y_i$ , paying wage  $w$ :  $J(y_i, w)$
- ▶ Vacant flow value:

$$rV(y_i, w, \theta) = -\kappa + q(\theta)(J(y_i, w) - V(y_i, w, \theta)) \quad (24)$$

- ▶ Matched flow value:

$$rJ(y_i, w) = y_i - w + \delta(V(y_i, w, \theta) - J(y_i, w)) \quad (25)$$

## Firm Value Functions II

- ▶ Value functions:
  1. Vacant with productivity  $y_i$ :  $V(y_i, w, \theta)$
  2. Filled with productivity  $y_i$ , paying wage  $w$ :  $J(y_i, w)$
- ▶ In equilibrium  $V(y_i, w, \theta) = 0$ :

$$rJ(y_i, w) = y_i - w - \delta J(y_i, w) \quad (26)$$

- ▶ Asset value of vacancy in submarket  $(y_i, w, \theta)$ :

$$(r + q(\theta))V(y_i, w, \theta) = q(\theta) \frac{y_i - w}{r + \delta} - \kappa \quad (27)$$

# Equilibrium

- ▶ We will be interested in the same equilibrium objects, but now for each submarket  $i$ :
  1. Wages  $w_i$ ;
  2. unemployment  $u_i$ ;
  3.  $\theta_i = \frac{v_i}{u_i}$  vacancies in each submarket.
- ▶ Before, 1 & 3 were separate equilibrium conditions.
- ▶ New equilibrium objects
  1. set of open submarkets,  $\mathcal{I}$ ;
  2. value of unemployment  $\bar{V}(U)$
- ▶ Market maker sets wages according to

$$\max_w V(y_i, w, \theta(w; U)) \quad (28)$$

- ▶ Given  $p(\theta)$  from worker's problem, find  $w$  that maximizes value of vacancy.

## Free Entry

- ▶ Free entry implies that the expected value of opening a vacancy will be equal to the cost of opening it  $\chi$
- ▶ The expected value of opening a vacancy

$$\bar{V}(U) = \sum_{i=l(U)}^n f_i V(y_i, w_i^*(U), \theta_i^*(U))$$

- ▶ equilibrium:

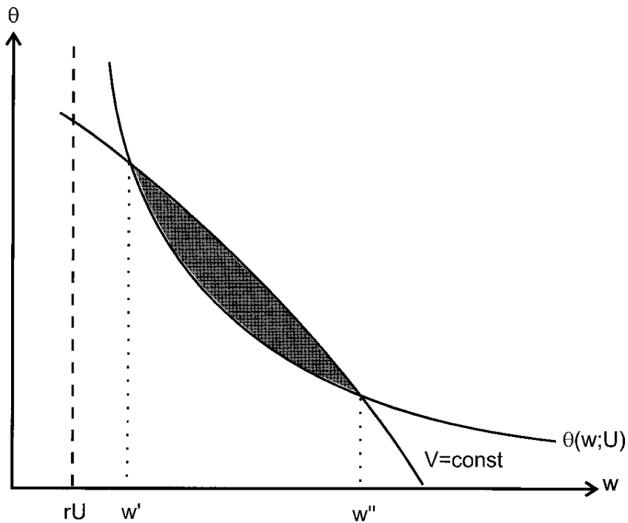
$$\bar{V}(U) = \kappa$$

# Equilibrium Number of Markets

- ▶ We know that each productivity will form a separate market
- ▶ There are  $n$  productivities in the distribution
- ▶ All submarkets such that  $w_i \geq rU$  will remain open
- ▶ Let  $\iota$  denote the lowest submarket open

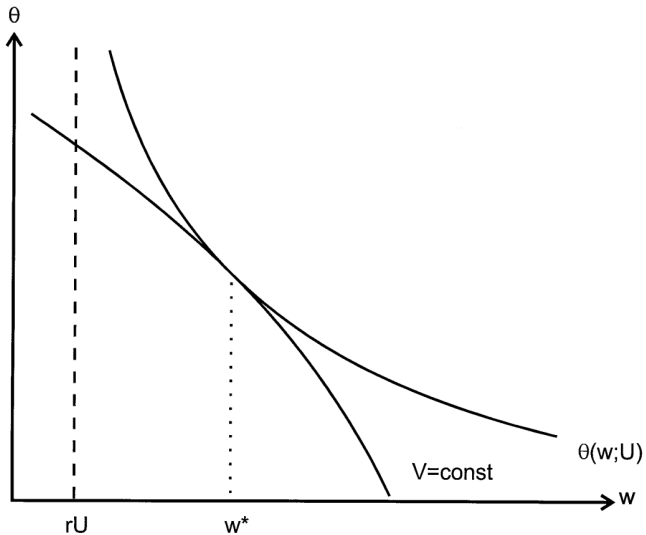
# “Competitive” Search

- ▶ What is shaded region?



# “Competitive” Search

- ▶ Inefficiency (rejected matches in DMP)



# Equilibrium

- ▶ The resulting competitive equilibrium with frictional labor markets is characterized by the following equations

$$\bar{V}(U) = \chi \quad (29)$$

$$w_i = \arg \max V(y_i, w, \theta(w; U)), i \geq i_R \quad (30)$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))}, i \geq i_R \quad (31)$$

$$\dot{u}_i = 0; u_i p(\theta_i) = e_i \delta \quad (32)$$

$$\sum_i u_i = u \quad (33)$$

## What is “valuable” about directed search?

- ▶ Submarkets are individually priced.
- ▶ i.e., contracts  $(w, \theta)$  are known given the state of the worker and firm.
- ▶ Then, assuming that free entry binds in every open submarket, no longer need to condition on aggregate distributions as state variables (Menzio and Shi, 2011).
- ▶ So models are computationally tractable.
- ▶ Makes it possible to easily incorporate heterogeneity.

# Conclusion

- ▶ Two more classes (no class 4/30).
- ▶ Next class: block recursivity