Macro II:
Inequality in Heterogeneous Agent Models

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Announcements

- Final: 5/13 10:30-12:30.
- 2 Short Answer Qs.
- Today: using macro-labor models to answer questions with lots of heterogeneity.
- Extension of Block Recursive model with human capital and assets
- How does this affect inequality? (Griffy, 2021)

Wealth and Borrowing Constraints

- Low wealth limits ability to borrow early in the life-cycle.
- Feared or were denied credit (ages 20-30):
  - 1st quartile (Survey of Consumer Finances, 2013): 50%
  - Rest of population (SCF, 2013): 33%
- Less likely to be able to borrow in the future (ages 20-30):
  - 1st quartile (SCF, 2013): unsecured 80% of total debt
  - Population Average (SCF, 2013): unsecured 41% of total debt
- Wealth and earnings are correlated:
  - Low wealth, lower initial earnings;
  - Lower slope over life-cycle.

Question

- How do **differences** in **wealth**, **human capital**, and **learning ability** at **labor market entry** impact life-cycle
  - job search behavior?
  - human capital accumulation?
  - consumption?
- What channels are quantitatively important?

What I Do

- Construct quantitative general equilibrium life-cycle model:
  - search and matching in the labor market;
  - risk-aversion and borrowing constraints;
  - endogenous human capital accumulation.
- Estimate model using indirect inference.
- Consider counterfactual initial conditions.
- Decompose effect into interaction between wealth, search, and human capital.

Model Environment

- Life-cycle model: age discrete, indexed by t; retire at $T+1$.
- Agents:
  - Employed and unemployed workers.
  - Matched and unmatched firms.
- Technology:
  - Frictional matching in labor markets.
  - Endogenous human capital accumulation.
  - Borrowing constraints.
- Initial heterogeneity:
  - Initial wealth (a_0), human capital (h_0), and learning ability (ℓ).

Agents

- Risk-averse workers indexed by (a, h, ℓ, t):
  - Employed (μ), unemployed w/ UI (b_{UI}) or w/o UI (b_L).
  - Search on and off job.
  - Consume & save s.t. borrowing constraint $a' \geq \underline{a}_t$.
  - Emp.: portfolio allocation (HC inv. & precautionary savings).
  - Unemployed & employed: stochastic HC depreciation.
- Continuum of profit maximizing firms:
  - Risk neutral. Produce using human capital.
  - Post vacancies that specify piece-rate μ.
- World risk-free rate r_F; common discount rate β.
- Type-distribution $\phi' = \Phi(\phi)$ (suppressed throughout).

Search and Matching Technology

- Directed search (Moen, 1997):
  - Submarket: homogeneous workers (a, h, ℓ, t) and firms (μ)
  - Workers apply to job in submarket w/ known piece-rate μ.
- Matching technology:
  - # of matches in submkt (μ, a, h, ℓ, t): $M_t = M(s_t, v_t)$ (CRS).
  - Submarket tightness: $\theta_t(\cdot) = \frac{v_t}{s_t}$
  - Worker finding rate: $q(\theta_t) = \frac{M(s_t, v_t)}{v_t}$
  - Job finding rates: $p(\theta_t) = \frac{M(s_t, v_t)}{s_t} = \theta_t q(\theta_t)$

Firms

- States: $s_J = (\mu, a, h, \ell), s' = (\mu', a', h', \ell), s'_J = (\mu, a', h', \ell)$
- Matched firms:
  - produce $(1-\tau)h$, pay $\mu(1-\tau)h$
  - separate exog. w/ prob. δ; endog. w/ prob. $\lambda_E p(\theta_t(s'))$
  - continue w/ value $J_{t+1}(s'_J)$
- Value of filled vacancy with age-t type-s_J worker:

$$J_t(s_J) = (1-\mu)(1-\tau)h + \beta E[(1-\delta)(1-\lambda_E p(\theta_t(s')))J_{t+1}(s'_J)]$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

- Worker decisions: μ', a', h', τ.

Free Entry and Equilibrium Job-Finding Rates

- Unmatched firms:
  - Pay κ to post (profitable) vacancies.
  - Match w/ prob. $q(\theta_t(s_J))$.
- Value of vacancy with age-t type-s_J worker:

$$V_t(s_J) = -\kappa + q(\theta_t(s_J))J_t(s_J)$$

- Free Entry ($V_t(s_J) = 0$):

$$q(\theta_t(s_J)) = \frac{\kappa}{J_t(s_J)}$$
$$\theta_t(s_J) = q^{-1}(\frac{\kappa}{J_t(s_J)})$$

- Eqm. job finding rate: $p(\theta_t) = \theta_t q(\theta_t)$ determined by J_t, κ
- Eqm.: $\frac{\partial P}{\partial \mu} < 0$

Unemployed Searcher's Problem

- States (w/ UI): $s_U = (b_{UI}, a, h, \ell)$, $s'_E = (\mu', a, h, \ell)$
- States (w/o UI): $s_U = (b_L, a, h, \ell)$, $s'_E = (\mu', a, h, \ell)$
- Unemployed searcher's problem:
  - Apply for job w/ piece-rate μ'.
  - Transition to employment w/ prob. $p(\theta_t(s'_E))$.
  - Continue w/ value $W_t(s'_E)$ if offered job.
  - Continue w/ value $U_t(s_U)$ if no offer.
- Value of searching while unemployed:

$$R_t^U(s_U) = \max_{\mu'} p(\theta_t(s'_E))W_t(s'_E) + (1 - p(\theta_t(s'_E)))U_t(s_U)$$

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- Competitive labor market:
  - Paid marginal product $\rightarrow$ inc. inequality because of diffs in HC
  - Idiosyncratic shocks $\rightarrow$ consumption risk. Insurance via $a - \underline{a}$.
- Frictional labor market:
  - Frictions $\rightarrow \mu < 1$.
  - Employment risk $\rightarrow$ consumption risk.
  - Precautionary savings (& UI) only explicit insurance.
  - Alternative: decrease μ. $\rightarrow$ (low) wealth can impact earnings.

Unemployed Worker's Problem

- States:
  - Unemp. w/ UI: $s_U = (b_{UI}, a, h, \ell)$, $s'_{UI} = (b_{UI}, a', h', \ell)$
  - Unemp w/o UI: $s_U = (b_L, a, h, \ell)$, $s'_L = (b_L, a', h', \ell)$
- Consumption and savings problem:
  - Consume & save s.t. $a' \geq \underline{a}_t$.
  - Lose benefits w/ prob. γ.
  - Human Capital depreciates: $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$.
- Value of unemployment (w/ UI):

$$U_t(s_U) = \max_{c, a' \geq \underline{a}_t} u(c) + \beta E[(1-\gamma) R^U_{t+1}(s'_{UI}) + \gamma R^U_{t+1}(s'_L)]$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI}$$

$$h' = e^{\epsilon'} h$$

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$$h' = e^{\epsilon'} h$$

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Employed Worker's Problem

- States:
  - Emp.: $s_E = (\mu, a, h, \ell)$, $s'_E = (\mu, a', h', \ell)$
  - Unemp. w/ UI: $s'_U = (b_{UI}, a', h', \ell)$
- Employed Worker's Problem:
  - Portfolio alloc.: $(a' \geq a_t, \tau)$, τ to HC inv. & $(1-\tau)$ to work.
  - Stochastic HC depreciation $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$
  - Lose job w/ prob. δ, receive $b(1-\tau)\mu h$.
- Value of employment:

$$W_t(s_E) = \max_{c, a' \geq \underline{a}_t, \tau} u(c) + \beta E[(1-\delta)R^E_{t+1}(s'_E) + \delta R^U_{t+1}(s'_U)]$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + (1-\tau)\mu h$$

$$b_{UI} = b(1-\tau)\mu h$$

$$h' = e^{\epsilon'}(h + \ell(h\tau)^\alpha), \quad \epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

Employed Worker's Problem

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- Human capital inv. is risky:
  1. Rate of return uncertain: stochastic dep., unknown ex-ante.
  2. Illiquid: no consumption smoothing value when unemployed.
- Rate of return risk determines allocation for "wealthy-enough."
- Separation while low-wealth $\rightarrow$ take low-μ job.
- $\rightarrow$ Exposure to unemployment risk distorts allocation.

Equilibrium

A *Block Recursive Equilibrium* (BRE) in this model is a set of value functions, $U_t, W_t, R_t^E, R_t^U, J_t, V_t$, associated policy and market tightness functions, a', c, μ', τ, and θ_t, which satisfy

1. The policy functions $\{c, \mu', a', \tau\}$ solve the workers problems, W_t, U_t, R_t^E, R_t^U.
2. $\theta_t(\mu, a, h, \ell)$ satisfies the free entry condition for all submarkets (μ, a, h, ℓ, t).
3. The aggregate law of motion is consistent with all policy functions.

Estimation

- Indirect Inference (conditional MoM) (Gourieux et al, 1993):
  - Select reduced-form analogs to structural model.
  - Objective: match coefs. for regs. w/ data & simulated data.
  - Minimize by changing structural parameters.
- Basic approach:
  - Estimate effect of wealth on job search behavior.
  - Match age-earnings regs (eqm. outcome) by initial heterogeneity.
  - Match observable marginal distributions.

Empirical Preliminaries

- Quarterly model, ages 23-64, retire at 65.
- Model parameters: $\sigma = 2, r_F = 0.012, \beta = \frac{1}{1+r_F}$
- Power utility + unemp leisure: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$
- HC Evolution: $h' = e^{\epsilon}(h + H(h, \ell, \tau)) = e^{\epsilon}(h + \ell \times (h\tau)^{\alpha})$
- Natural borrowing constraint: $\underline{a}_t = \sum_{j=t}^{T} \frac{b_L}{(1+r_F)^j}$
- Initial conditions:
  - $(a_0, h_0, \ell) \sim LN(\psi, \Sigma)$
  - Correlations $\rho_{AH}, \rho_{AL}, \rho_{HL}$
- Full list of preset values:

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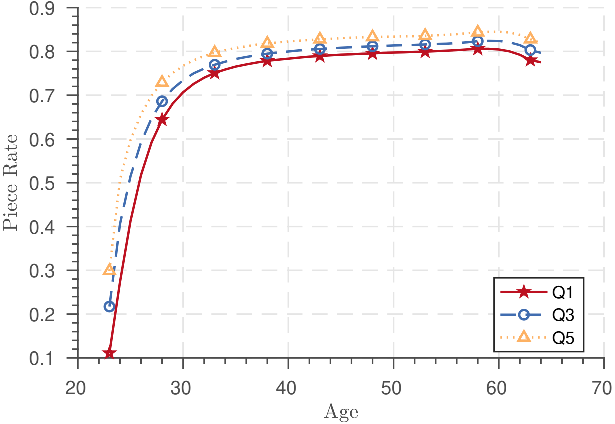
Key Estimated Parameters and Coefficients

- Parameter Estimates
  - Age-23 constraint: $\underline{a}_0 = -\$6,378$ (2011\$)
  - HC curvature: $\alpha = 0.5687$.
  - HC dep.: $(\mu_\epsilon, \sigma_\epsilon)$ $-0.0249, 0.0621)$.
  - Corrs.: $\rho_{AH} = 0.3253$ $\rho_{A\ell} = 0.4642$ $\rho_{H\ell} = 0.6915$.
- Coefficient Estimates
  - $\frac{\partial ln(W_{i,j+1})}{\partial ln(UI_i)}$: Data: 0.4652; Model: 0.2918,
  - $\frac{\partial ln(W_{i,j+1})}{\partial ln(UI_i)}(q > 1)$: Data: -0.4425; Model: -0.2731
  - $\frac{\partial ln(H_{i,j+1})}{\partial ln(UI_i)}(q = 1)$: Data: -0.8664; Model: -0.932,
  - $\frac{\partial ln(H_{i,j+1})}{\partial ln(UI_i)}(q > 1)$: Data: -0.4542; Model: -0.3336
  - ρ_{AH}: intercepts by wealth underpredicts higher quintiles.
  - ρ_{AL}: overpredicts slopes by wealth in higher quintiles.
  - ρ_{HL}: slopes by AFQT score quintile close.

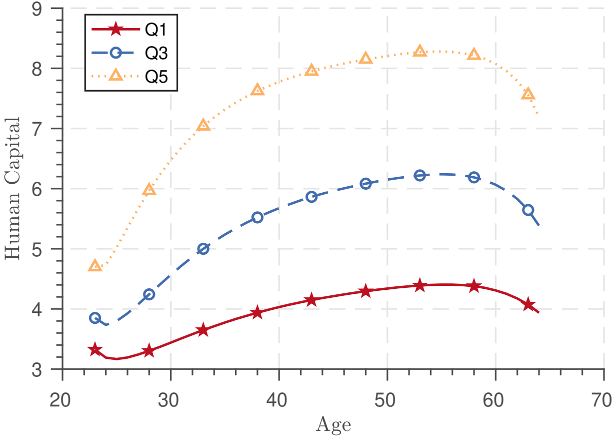
Findings

- Mechnisms & life-cycle earnings growth $w_t = \mu_t(1-\tau_t)h_t$
- Two sources of earnings growth:
  - Movement up job (piece-rate) ladder. μ_t
  - Investment in human capital. h_t
- Consider two experiments, compare Inc., Cons., etc.:
  1. Decrease initial conditions of median worker by 1 SD for each (a_0, h_0, ℓ).
  2. Eliminate initial dispersion for each (a_0, h_0, ℓ).
- Decompose interaction between wealth, search, and human capital.

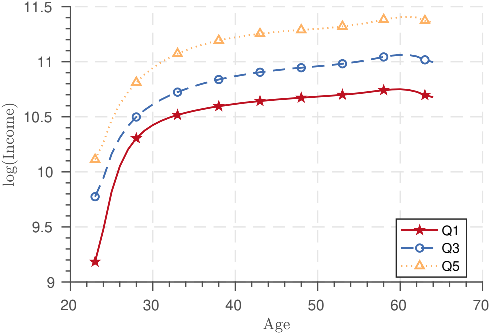
Job Ladder



Human Capital



Income

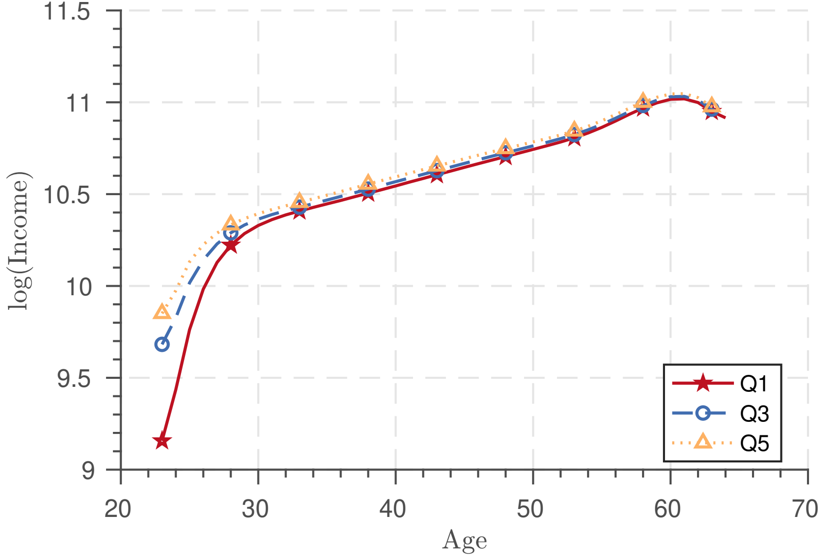


- Job ladder: important early.
- Human capital: important mid/late.

Sources of Inequality

- Explore 3 ways:
  1. Set h_0, ℓ to median inital value.
     - i.e., resulting variation due to wealth heterogeneity **only**.
     - Compare to previous figures.
  2. Subject median worker to -1 SD in each (a_0, h_0, ℓ).
     - Same experiment as HVY (2011).
  3. Eliminate dispersion in initial conditions (separately).
- Focus on changes in average outcomes & by wealth.

Income



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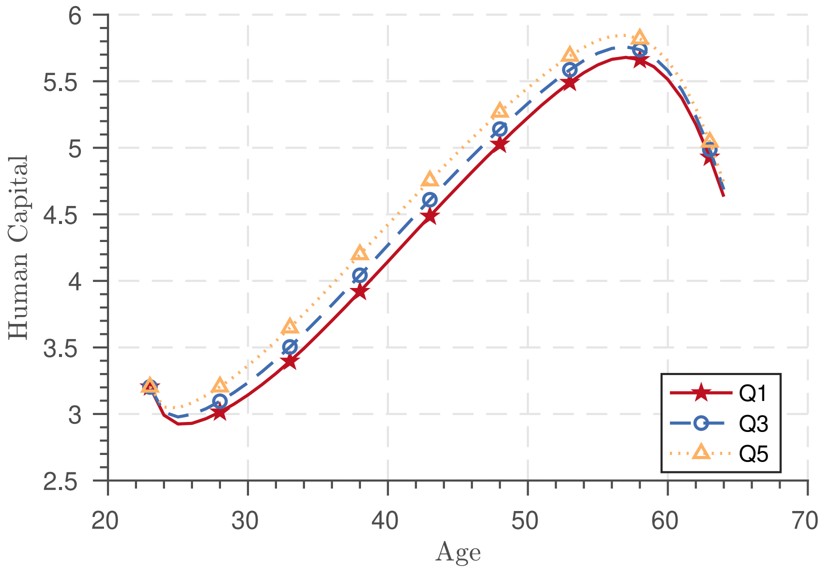

Findings: Median Worker

Change	Δ Consumption (%)	HVY (%)	Δ Earnings (%)	Δ h (%)	Δ τ (%)	Δ μ' (%)
Wealth	-6.4	-1.6	-5.8	-2.5	-5.7	-4.8
Human Capital	-3.8	-28.3	-3.6	-4.8	-5.9	-0.4
Learning Ability	-15.5	-2.6	-16.8	-29.1	-96.3	0.3

Findings: No Dispersion

	Δ Income (%)				Δh (%)				$\Delta \mu$ (%)			
Counterfactual	1st	3rd	5th	Ave	1st	3rd	5th	Ave	1st	3rd	5th	Ave
$a_0 = E[a_0]$	5.79	1.09	−2.06	1.03	1.50	0.44	−1.33	0.12	5.44	0.89	−1.84	1.42
$h_0 = E[h_0]$	1.74	−0.65	−3.40	−1.10	3.16	0.69	−2.14	0.23	0.69	−0.16	−0.52	−0.01
$\ell = E[\ell]$	24.85	1.24	−17.97	−1.07	37.75	11.32	−8.37	9.65	1.26	−0.51	−1.35	−0.29

Decomposing the Interaction

- How does interaction between wealth, search, and human capital affect inequality?
- Compare outcomes in baseline model to 3 restrictions.
- Restrictions:
  - R1: exogenous portfolio $\tilde{\tau}_t(\mu, a, h, \ell) = \tau_t(\mu, \bar{a}_t, h, \ell) \forall\ t$ and $\tilde{a'}_t(\mu, a, h, \ell) = a_t(\mu, \bar{a}_t, h, \ell) \forall\ t$.
  - Bewley model: frictionless labor market, still human capital & savings decision.
  - R2: Bewley + exogenous portfolio $\tilde{\tau}_t(\mu, a, h, \ell) = \tau_t(\mu, \bar{a}_t, h, \ell) \forall\ t$ and $\tilde{a'}_t(\mu, a, h, \ell) = a_t(\mu, \bar{a}_t, h, \ell) \forall\ t$.

Decomposing the Interaction

- R1: exogenous portfolio $\tilde{\tau}_t(\mu, a, h, \ell) = \tau_t(\mu, \bar{a}_t, h, \ell) \forall\ t$ and $\tilde{a'}_t(\mu, a, h, \ell) = a_t(\mu, \bar{a}_t, h, \ell) \forall\ t$.
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- $R1$ - Base: precautionary effect on human capital by wealth in baseline model.
- $R2$ - Bewley: precautionary effect on human capital by wealth without frictional labor markets.
- Difference between these comparisons: interaction between wealth, search, human capital.

Findings: Exogenous Human Capital Comparison

	$\Delta\tau$ (%)				Δh (%)			
Counterfactual	1st	3rd	5th	Ave	1st	3rd	5th	Ave
%Δ(Base$\rightarrow$R1)	33.18	17.84	6.42	16.51	6.01	4.90	1.36	4.09

Findings: Frictionless Labor Markets Comparison

	$\Delta\tau$				Δh			
Counterfactual	1st	3rd	5th	Ave	1st	3rd	5th	Ave
%Δ(Bewley→R2)	15.15%	12.49%	6.80%	11.16%	3.29%	3.75%	2.16%	3.19%
Effect of Wealth x Search	18.03pp	5.35pp	−0.37pp	5.35pp	2.72pp	1.16pp	−0.80pp	0.90pp

Findings: Interaction

Counterfactual	1st	3rd	5th
%ΔIncome (Base$\rightarrow$R1)	41.11%	3.24%	$-$26.87%
% Explained by Interaction	6.61%	35.69%	2.98%

Conclusion

- Constructed quantitative life-cycle model:
  - Risk-averse agents who face borrowing constraints.
  - General equilibrium labor market frictions.
  - Endogenous earnings growth through human capital choice.
- Estimated using indirect inference.
- Findings:
  - Borrowing constraints & search impact low-wealth individuals.
  - Wealth dynamically alters the earnings process through search behavior and human capital accumulation.
  - Initial wealth causes larger life-cycle changes than initial human capital (and sometimes learning ability).
- Final exam next; good luck!