

Quantitative Macro-Labor: Wage Dispersion and Comparative Statics

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Announcements

- ▶ Present introduction/outline for research paper next Tuesday.
- ▶ This could be as little as a single slide (example on next slide).
- ▶ Introduction/outline due following week (2-3 pages).
- ▶ Will try to return comments to you today.
- ▶ I've been guaranteed that you have cluster access now.

Example Outline Project Slide

- ▶ Research Idea/Topic: The Impact of AI on Human Capital.
- ▶ Question: “AI use is difficult to detect and creates long-term learning deficiencies with undirected use. How will this affect primary, secondary, and higher education, and what will the impact on aggregate human capital be?”
- ▶ Data and Empirical Approach: Use IPEDS to estimate contemporaneous impact of AI on cognition. Use pandemic to infer longer-term effects.
- ▶ Theory: Build human capital model (Ben-Porath) with costly human capital acquisition. GE effects: expectations about future labor market?
- ▶ The written outline is just a longer version of this where I would expand the question, describe the data in detail as well as the empirical specification, and explicitly describe the trade-offs that I'd like the theory to capture.
- ▶ Just one or two slides.

Recap: The McCall Model

- ▶ Basic idea:
 1. Workers can be in one of two states: employed or unemployed, with value functions V, U .
 2. Receive job offers at exogenous rate α , no information about meeting prior.
 3. Once employed, workers remain at current job until unexogenously separated (no OTJS) at rate δ .
 4. Exogenous distribution of wages, $w \in [\underline{w}, \bar{w}]$, $w \sim F(\cdot)$.
 5. Linear utility: $u(c) = b$ or $u(c) = w$.
- ▶ Optimal policy is a “reservation strategy,” i.e., a lower bound on the wages a worker will accept out of unemployment.
- ▶ Why is $w_R > \underline{w}$?

Model and Reservation Strategy

- Generally, we will use the continuous time Bellman in its “asset value” formulation:

$$U = \frac{b + \alpha E[\max\{V, U\}]}{r + \alpha} \quad (1)$$

$$(r + \alpha)U = b + \alpha E[\max\{V, U\}] \quad (2)$$

$$rU = b + \alpha E[\max\{V - U, 0\}] \quad (3)$$

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V - U, 0\} dF(w) \quad (4)$$

- Employment:

$$rV(w) = w - \delta(V(w) - U) \quad (5)$$

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw \quad (6)$$

Reservation Strategy II

- ▶ Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw \quad (7)$$

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} (w - w_R) dF(w) \quad (8)$$

- ▶ Assume that the distribution of wage offers is uniform.
- ▶ What is the conditional expectation of a truncated uniform random variable? $E[w - w_R | w \geq w_R] = \frac{\bar{w} - w_R}{2}$
- ▶ What is the probability of drawing from the truncated part of the offer distribution? $P(w \geq w_R) = \frac{\bar{w} - w_R}{\bar{w} - \underline{w}}$.

$$w_R = b + \frac{\alpha}{r + \delta} \frac{\bar{w} - w_R}{2} \frac{\bar{w} - w_R}{\bar{w} - \underline{w}} \quad (9)$$

$$w_R = b + \frac{\alpha}{r + \delta} \frac{(\bar{w} - w_R)^2}{2(\bar{w} - \underline{w})} \quad (10)$$

- ▶ (yes, this is the same as if you integrate the option value)

Reservation Strategy II

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \frac{(\bar{w} - w_R)^2}{2(\bar{w} - \underline{w})} \quad (11)$$

$$w_R = b + \frac{\alpha}{r + \delta} \frac{\bar{w}^2 - \bar{w}w_R + w_R^2}{2(\bar{w} - \underline{w})} \quad (12)$$

$$0 = b + \frac{\alpha}{r + \delta} \frac{\bar{w}^2}{2(\bar{w} - \underline{w})} - \left(1 + \frac{\alpha}{r + \delta} \frac{\bar{w}}{2(\bar{w} - \underline{w})}\right) w_R \quad (13)$$

$$+ \frac{\alpha}{r + \delta} \frac{1}{2(\bar{w} - \underline{w})} w_R^2 \quad (14)$$

- Apply quadratic formula and choose root st $w_R \in [0, 1]$

$$\frac{\left(1 + \frac{\alpha}{r + \delta} \frac{\bar{w}}{2(\bar{w} - \underline{w})}\right) \pm \sqrt{\left(1 + \frac{\alpha}{r + \delta} \frac{\bar{w}}{2(\bar{w} - \underline{w})}\right)^2 - 4\left(b + \frac{\alpha}{r + \delta} \frac{\bar{w}^2}{2(\bar{w} - \underline{w})}\right)\left(\frac{\alpha}{r + \delta} \frac{1}{2(\bar{w} - \underline{w})}\right)}}{2\left(b + \frac{\alpha}{r + \delta} \frac{\bar{w}^2}{2(\bar{w} - \underline{w})}\right)} \quad (15)$$

Reservation Strategy III

- ▶ Reservation strategy:

$$\frac{\left(1 + \frac{\alpha}{r+\delta} \frac{\bar{w}}{2(\bar{w}-w)}\right) \pm \sqrt{\left(1 + \frac{\alpha}{r+\delta} \frac{\bar{w}}{2(\bar{w}-w)}\right)^2 - 4\left(b + \frac{\alpha}{r+\delta} \frac{\bar{w}^2}{2(\bar{w}-w)}\right)\left(\frac{\alpha}{r+\delta} \frac{1}{2(\bar{w}-w)}\right)}}{2\left(b + \frac{\alpha}{r+\delta} \frac{\bar{w}^2}{2(\bar{w}-w)}\right)} \quad (16)$$

- ▶ Let's just pick some values for the parameters (assume monthly calibration):
 1. $w \sim U[0, 1]$
 2. $\alpha = 0.43$: avg. mon. U-E (this isn't right. Why?)
 3. $\delta = 0.03$: avg. mon. E-U
 4. $r = 0.0041$: ann. int. rate
 5. $b = 0.4$: UI rep. rate
- ▶ $w_R \in \{1.31, 0.72\}$
- ▶ I'm a little skeptical of these results, but you get the idea.

Hazard Rate

- ▶ What is the hazard rate of unemployment?
- ▶ Rate of leaving unemployment at time t .

$$H_u(t) = \alpha \int_{w_R}^{\bar{w}} dF(w) \quad (17)$$

$$= \alpha (F(\bar{w}) - F(w_R)) \quad (18)$$

$$= \underbrace{\alpha}_{\text{MeetingRate}} \underbrace{(1 - F(w_R))}_{\text{Selectivity}} \quad (19)$$

- ▶ Note, almost every search model generates a hazard composed of the product of a meeting probability and worker selectivity.
- ▶ Hazard rate of employment (leaving employment for unemployment)?

$$H_e(t) = \delta \quad (20)$$

- ▶ Because separations are independent of state.

Dynamics of Unemployment

- ▶ Use hazard rates to understand dynamics and steady-state.
- ▶ What does the model predict about employment and unemployment?

$$\dot{u} = \delta(1 - u) - \alpha(1 - F(w_R))u \quad (21)$$

$$\dot{e} = \alpha(1 - F(w_R))(1 - e) - \delta e \quad (22)$$

- ▶ Steady-state: $\dot{u} = 0$, $\dot{e} = 0$:

$$0 = \delta(1 - u) - \alpha(1 - F(w_R))u \quad (23)$$

$$\rightarrow u = \frac{\delta}{\delta + \alpha(1 - F(w_R))} \quad (24)$$

$$0 = \alpha(1 - F(w_R))(1 - e) - \delta e \quad (25)$$

$$\rightarrow e = \frac{\alpha(1 - F(w_R))}{\alpha(1 - F(w_R)) + \delta} \quad (26)$$

What can we say about an increase in UI?

- ▶ Whenever we write down a model, we have created a laboratory.
- ▶ Let's run experiments with it!
- ▶ What will happen to wages and unemployment if UI b increases?
- ▶ For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial b} = 1 + \frac{\alpha}{r + \delta} \frac{\partial \int_{w_R}^{\bar{w}} [1 - F(w)] dw}{\partial w_R} \frac{\partial w_R}{\partial b} \quad (27)$$

- ▶ Leibniz's integral rule:

$$\frac{\partial}{\partial x} \int_{a(x)}^{b(x)} f(x, t) dt = f(x, b(x)) \frac{\partial b(x)}{\partial x} - f(x, a(x)) \frac{\partial a(x)}{\partial x} \quad (28)$$

$$+ \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt \quad (29)$$

$$\frac{\partial w_R}{\partial b}$$

- For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial b} = 1 + \frac{\alpha}{r + \delta} \frac{\partial \int_{w_R}^{\bar{w}} [1 - F(w)] dw}{\partial w_R} \frac{\partial w_R}{\partial b} \quad (30)$$

- Leibniz's integral rule:

$$\frac{\partial}{\partial x} \int_{a(x)}^{b(x)} f(x, t) dt = f(x, b(x)) \frac{\partial b(x)}{\partial x} - f(x, a(x)) \frac{\partial a(x)}{\partial x} + \int_{a(x)}^{b(x)} \frac{\partial f(x, t)}{\partial x} dt$$

$$\begin{aligned} \frac{\partial w_R}{\partial b} &= 1 + \frac{\alpha}{r + \delta} \left(\cancel{[1 - F(\bar{w})]} \frac{\partial \bar{w}}{\partial w_R} \right. \\ &\quad \left. - [1 - F(w_R)] \frac{\partial w_R}{\partial w_R} + \int_{w_R}^{\bar{w}} \frac{\partial [1 - F(w)]}{\partial w_R} \right) \end{aligned}$$

$$\frac{\partial w_R}{\partial b} = 1 - \frac{\alpha}{r + \delta} [1 - F(w_R)] \frac{\partial w_R}{\partial b}$$

$$\frac{\partial w_R}{\partial b} = \frac{r + \delta}{r + \delta + \alpha[1 - F(w_R)]} < 1 \quad (31)$$

What about unemployment?

- ▶ Now, $\frac{\partial u}{\partial b}$.
- ▶ Let's find the semi-elasticity: $\frac{\partial \log(u)}{\partial b}$

$$\ln(u) = \ln(\delta) - \ln(\delta + \alpha(1 - F(w_R))) \quad (32)$$

$$\frac{\partial \ln(u)}{\partial b} = \frac{\alpha f(w_R) \frac{\partial w_R}{\partial b}}{\delta + \alpha(1 - F(w_R))} \quad (33)$$

- ▶ Unemployment clearly increases.
- ▶ More interesting: separation rate (δ) and offer arrival rate (α)
- ▶ Why? Predictions are unclear.
- ▶ If $\alpha \uparrow$, find jobs faster, but also sample better jobs more often.

$$\frac{\partial w_R}{\partial \alpha}$$

- For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial \alpha} = \underbrace{\frac{1}{r + \delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw}_{\text{Match Rate}} - \underbrace{\frac{\alpha}{r + \delta} [1 - F(w_r)] \frac{\partial w_R}{\partial \alpha}}_{\text{Selectivity}} \quad (34)$$

$$\frac{\partial w_R}{\partial \alpha} = \frac{\int_{w_R}^{\bar{w}} [1 - F(w)] dw}{r + \delta + \alpha [1 - F(w_r)]} \quad (35)$$

- Now the semi-elasticity: $\frac{\partial \log(u)}{\partial \alpha}$

$$\ln(u) = \ln(\delta) - \ln(\delta + \alpha(1 - F(w_R))) \quad (36)$$

$$\frac{\partial \ln(u)}{\partial \alpha} = \underbrace{\frac{\alpha f(w_R) \frac{\partial w_R}{\partial \alpha}}{\delta + \alpha(1 - F(w_R))}}_{\text{Selectivity}} - \underbrace{\frac{(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))}}_{\text{Match Rate}} \quad (37)$$

Log-Concavity

$$\frac{\partial \ln(u)}{\partial \alpha} = \frac{\alpha f(w_R) \frac{\int_{w_R}^{\bar{w}} [1-F(w)] dw}{r+\delta+\alpha[1-F(w_R)]}}{\delta + \alpha(1 - F(w_R))} - \frac{(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))} \quad (38)$$

- ▶ Uh oh... how are we going to sign this?
- ▶ Properties of log-concave distributions (where $F(x)$ is log-concave):
 1. $F(x)$ log-concave $\rightarrow \int F(x)$ log-concave.
 2. $F(x)$ log-concave $\rightarrow \frac{\partial F}{\partial x}$ log-concave.
 3. $F(x)F''(x) \leq (F'(x))^2$

Log-Concavity

- Properties of log-concave distributions (where $F(x)$ is log-concave):

1. $F(x)$ log-concave $\rightarrow \int F(x)$ log-concave.

2. $F(x)$ log-concave $\rightarrow \frac{\partial F}{\partial x}$ log-concave.

3. $F(x)F''(x) \leq (F'(x))^2$

- $(\delta + \alpha(1 - F(w_R))) > 0$

$$\frac{\partial \ln(u)}{\partial \alpha} \propto \alpha f(w_R) \frac{\int_{w_R}^{\bar{w}} [1 - F(w)] dw}{r + \delta + \alpha[1 - F(w_r)]} - (1 - F(w_R)) \quad (39)$$

$$\propto \alpha f(w_R) \int_{w_R}^{\bar{w}} [1 - F(w)] dw - (1 - F(w_R))^2 < 0 \quad (40)$$

- By the third property of log-concave distributions.

“Estimation”/Calibration

- ▶ Earlier, I picked some parameters from Hornstein, Krusell, and Violante (“Calibrated Example”)
- ▶ If we want to match this model to the data, what targets can we use?
- ▶ Unconditional moments (i.e., population averages):
 - ▶ Hazard rates (U-E, U-E)
 - ▶ Employment rates (e, u)
 - ▶ Wage distribution
- ▶ How many moments do we need?
 - ▶ δ : separation rate
 - ▶ α : match rate
 - ▶ $F(w)$: distribution function
- ▶ Can we separately (ex-ante) identify them?
- ▶ Particularly, what can we use to identify α and $F(w)$?

“Estimation”/Calibration II

- ▶ What targets can we use to discipline model?
- ▶ Unconditional moments (i.e., population averages):
 - ▶ Hazard rates (U2E, E2U)
 - ▶ Employment rates (e, u)
 - ▶ Wage distribution
- ▶ What should we match?
- ▶ Depends on what we are after:
 - ▶ Transition rates: don't target the transition rates.
 - ▶ Wage distribution: don't target the wage distribution.
- ▶ What time period should we use?
- ▶ Steady-state: time-independent .
- ▶ We could pick any time interval and get same steady-state.
- ▶ But, pick monthly.
- ▶ Return to calibration momentarily.

Why are Similar Workers Paid Differently?

- ▶ Posed by Dale Mortensen in his book “Wage Dispersion”
- ▶ Abowd, Kramarz, and Margolis (1999): “That... observably equivalent individuals earn markedly different compensation and have markedly different employment histories—is one of the enduring features of empirical analyses of labor markets...”
- ▶ What are some possible reasons?
 1. Ability
 2. Selectivity
- ▶ What does the McCall model say is the source of wage dispersion?

A Notion of Wage Dispersion

- ▶ Extremely clever paper: Hornstein, Krusell, Violante (2011).
- ▶ Basic idea: use the mean-min (Mm) ratio for wage dispersion.
- ▶ Almost every search model has expression for the Mm ratio.
- ▶ Compare model Mm with data Mm.
- ▶ Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} (w - w_R) dF(w) \quad (41)$$

$$w_R = b + \frac{\alpha(1 - F(w_R))}{r + \delta} \int_{w_R}^{\bar{w}} (w - w_R) \frac{dF(w)}{1 - F(w_R)} \quad (42)$$

$$w_R = b + \frac{\alpha(1 - F(w_R))}{r + \delta} \int_{w_R}^{\bar{w}} (w - w_R) \frac{dF(w)}{1 - F(w_R)} \quad (43)$$

- ▶ Exp. of a truncated random variable: $E[w|w \geq w_R] = \hat{w}$

$$\rightarrow w_R = b + \frac{\alpha(1 - F(w_R))}{r + \delta} [\hat{w} - w_R] \quad (44)$$

The Mean-Min Ratio

- ▶ Average UI in the economy: $b = \rho \hat{w}$
- ▶ Reservation strategy:

$$w_R = \rho \hat{w} + \frac{\alpha(1 - F(w_R))}{r + \delta} [\hat{w} - w_R] \quad (45)$$

- ▶ What is minimum wage in this economy? w_R of course!

$$\left(\rho + \frac{\alpha(1 - F(w_R))}{r + \delta}\right) \hat{w} = \left(1 + \frac{\alpha(1 - F(w_R))}{r + \delta}\right) w_R \quad (46)$$

$$\rightarrow \frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1 - F(w_R))}{r + \delta}}{\rho + \frac{\alpha(1 - F(w_R))}{r + \delta}} \quad (47)$$

- ▶ Good news: $\rho < 1 \rightarrow$ mean wage is greater than w_R .
- ▶ What is *incredibly* useful (empirically) about this formulation?

Calibration

$$\frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1-F(w_R))}{r+\delta}}{\rho + \frac{\alpha(1-F(w_R))}{r+\delta}} \quad (48)$$

- ▶ It's hard to separately identify the *offer* distribution and the *accepted* offer or wage distribution.
- ▶ This expression ignores the distinction: $\alpha(1 - F(w_R)) = H_u$.
- ▶ We just need the observed hazard, and can plug in for values.

Calibration II

- Parameters can be calibrated directly from observed data (or observed from HKV):
 - $\alpha(1 - F(w_R)) = 0.43$: avg. mon. U-E (HKV)
 - $\delta = 0.03$: avg. mon. E-U (HKV)
 - $r = 0.0041$: ann. int. rate (HKV)
 - $\rho = 0.4$: UI rep. rate (HKV)

$$\frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1-F(w_R))}{r+\delta}}{\rho + \frac{\alpha(1-F(w_R))}{r+\delta}} = \frac{1 + \frac{0.43}{0.0341}}{0.4 + \frac{0.43}{0.0341}} = 1.046 \quad (49)$$

- Great! What does this tell us?
- The McCall model predicts Mm wage dispersion of 4.6%.

Wage Dispersion

- ▶ The McCall model predicts Mm wage dispersion of 4.6%.



- ▶ HKV: Mm ratio is roughly 2.
- ▶ What does this mean?

Next Time

- ▶ Extensions of the McCall model: On-the-Job Search.
- ▶ Between now and then:
 1. Access the campus storage/cluster.
 2. Run some example code.
 3. Start research proposal and one slide presentation.